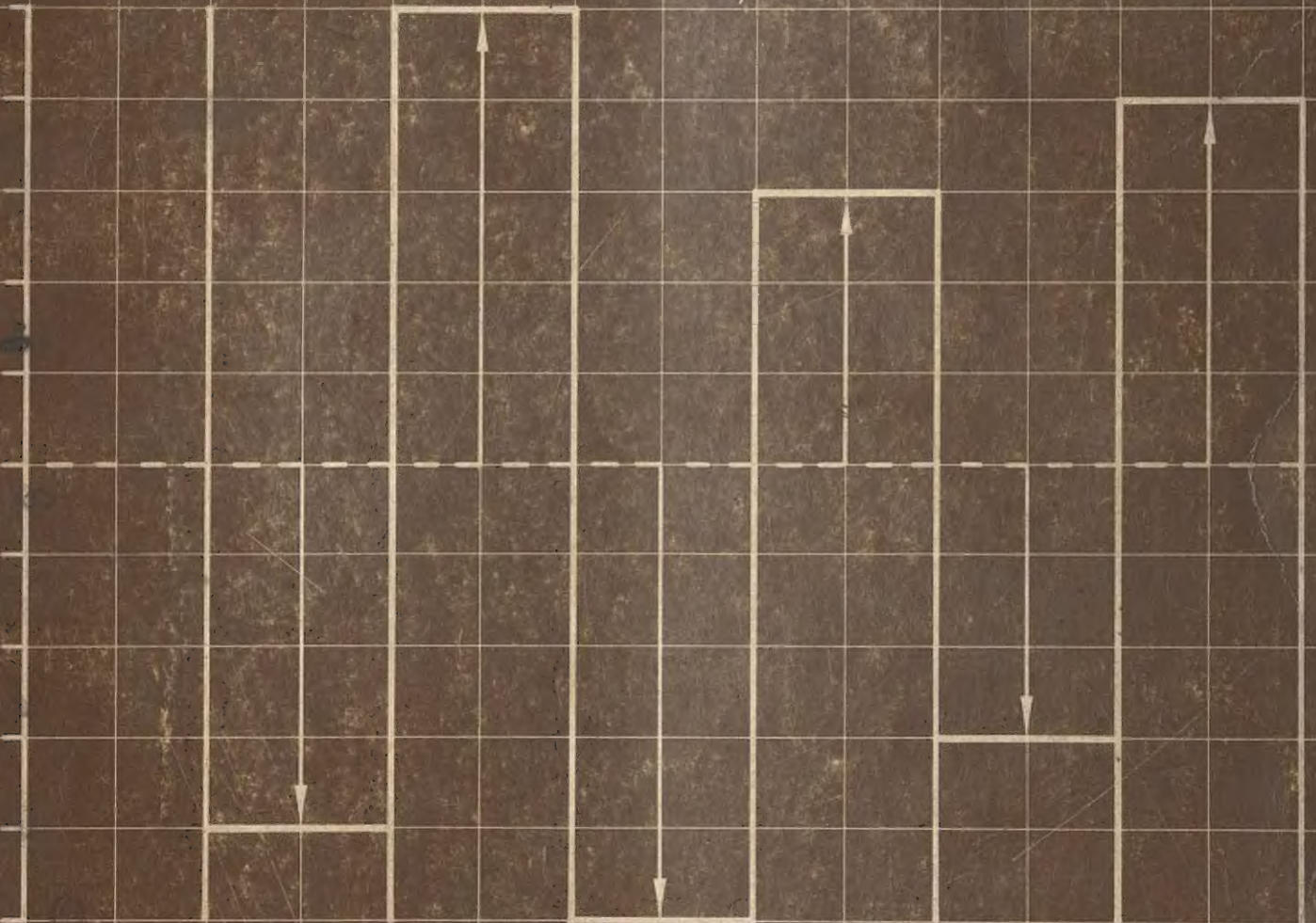


STATISTICS

a unit for INTRODUCTORY PSYCHOLOGY

A BEHAVIORAL RESEARCH LABORATORIES PROGRAM / ronald a. kinchla



AN ADDISON-WESLEY PROGRAMMED TEXT

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A PROGRAMMED TEXT

STATISTICS

a unit for INTRODUCTORY PSYCHOLOGY

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Under the auspices of

BEHAVIORAL RESEARCH LABORATORIES

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Ronald A. Kinchla

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TO THE INSTRUCTOR

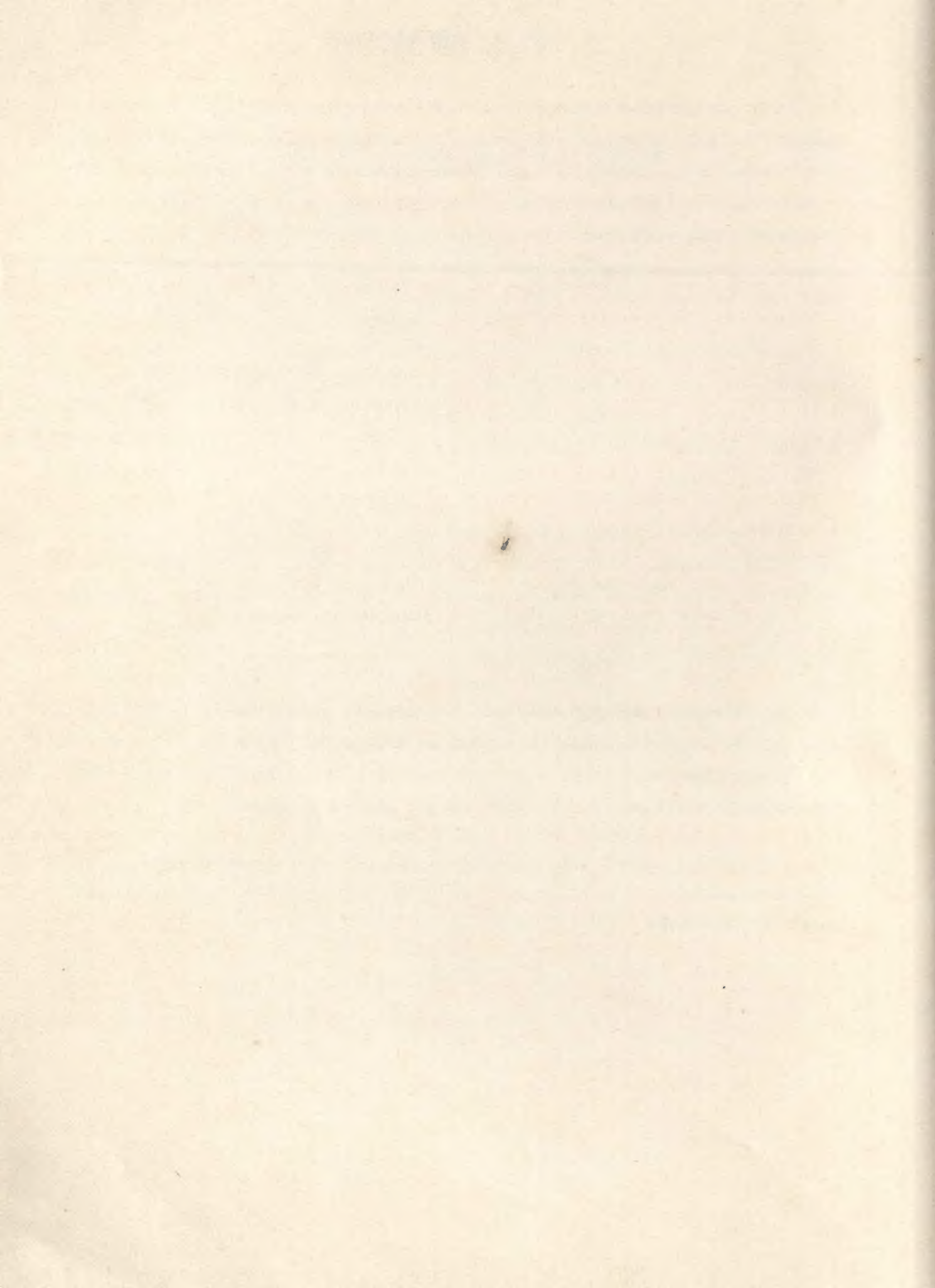
This program introduces the student to the elements of **statistical reasoning** and the manner in which this form of reasoning enters into the process of **behavioral research**. The material is presented in a carefully pre-tested sequence. Fundamental topics such as **variables**, **values**, and **distributions** are broken down into a series of small, sequentially organized steps. The student is led into discussion of such relatively complex concepts as **decision rules** and the probability of **Type One** and **Type Two errors** only after the prerequisites for his comprehension have been developed in detail.

After dealing with data as a collection of observed values of a variable, distributions of values are considered along with the manner in which **descriptive statistics** characterize various features of these distributions. **Formulas** for the **mean** and **variance** are developed in detail, and in a way that clearly indicates the exact feature of a distribution represented by each expression.

The concept of a **sampling distribution** is the central theme of the sections on **statistical inference**. Thus, the effects of sampling procedure and sample size in determining distributions are discussed in detail. In addition, the role of **probability theory** in the calculation of theoretical sampling distributions is considered as a basis for using random sampling procedures.

In **programmed learning**, each student proceeds at his own rate. In addition, the material is constructed so that **the student actively participates in the learning process**, supplying answers which require his understanding of that item. The answers constitute immediate feedback and reinforce the student's correct responses.

Periodic tests covering the material are provided to allow the student to evaluate his understanding as he progresses. Two forms of these exams are included in the instructor's manual.



TO THE STUDENT

Your programmed text is quite different from an ordinary book. A program consists of a large number of "frames," or numbered statements, each of which tells you something and asks questions about the material you have learned. The frames introduce new material a little at a time and review old material as needed to make sure you will remember it.

You do not study a program in the same way you study a book. The program is designed to let you work at your own rate of speed.

Get ready to use your program by covering the answer column at the right with the slider provided for you. Next, read the first frame and write the answer either in the blank or on a separate piece of paper. Then, move your slider down to uncover the answer and see if you are right. Go on in the same way to the following frames, checking your answer to each frame before going on to the next. Practice with the following frames.

1. When a blank has nothing under it, you simply fill in whatever best fits the blank. For example, the day of week that follows Thursday is _____. Friday
2. When a blank has two answers underneath it, you select the one that best fits. For example, a dog _____ an _____ animal. is/ is not is

You will find that the left-hand pages of the program are upside-down and backwards. Pay no attention to them until you have finished all of the right-hand pages.

Turn to Frame 1 and begin.

Section I: Data

1. Suppose you were a psychologist studying how accurately a person judges distance. You might place a target some distance in front of a subject and ask him how far away it was. You would be interested in the difference between the **distance** he reported and the true _____ of the target. distance
2. You might decide to move the target after each of the subject's judgements, asking him to judge each new distance. In your experiment, you would be collecting many different judgments of distance by changing or varying the _____ between the subject and the target. distance
3. We refer to something that does not change during an experiment as a **constant**. Since the distance between the subject and the target is varied or changed, it _____ be a constant in the experiment we just would not
would/would not described.
4. We refer to the opposite of a constant as a **variable**. Something that changes in an experiment is called a variable. In the experiment we just described, the distance between the subject and the target is varied; therefore, we would refer to the varying distance as a _____ variable
constant/variable.

Another name for a standard score from a normal distribution is:

a. an A score.

b. a Z score.

c. a relative score.

d. none of the above

b

8. A normal distribution is often described as being shaped like a:

a. square.

b. circle.

c. triangle.

d. none of the above

d

TRUE OR FALSE

9.

The formula for s^2 is $\frac{\sum (X - \bar{X})^2}{N}$

false

10.

Rejecting an hypotheses which is really true is referred to as a Type 1 error.

true

11.

Failing to reject the hypothesis which is really false is called a Type 2 error.

true

5. The target distance would be called a variable in your experiment because this distance was _____ during the experiment. changed
6. As a psychologist, you might wish to study how accurately a person judges weights. You might give the subject a lead ball and ask him to fill up a bag with sand until the bag felt as heavy as the lead ball. When he was finished trying to match the weight of the sand bag and the lead ball, you could compare the _____ of the bag of sand and the weight of the lead ball. weight
7. You could repeat this procedure several times, each time changing or varying the size of the lead ball. In this experiment, the weight of the lead ball would be referred to as a _____. variable
constant/variable
8. The weight of the lead ball was a variable in this experiment since this weight was _____ during the experiment. changed
9. Suppose you conducted the experiment **differently**. Suppose you gave the subject the **same** lead ball each time. You would not expect the subject to produce a bag of sand weighing exactly the same amount each time, even though the _____ of the lead ball remained the same throughout the experiment. weight
- If you weighed each bag of sand carefully, you would doubtlessly find that the weight differed slightly for each bag. Therefore, in this experiment, the weight of the lead ball was a _____, whereas the weight of the bag of sand the subject produced each time was a _____. constant
constant/variable variable

FILL IN THE BLANKS:

1. A process in which there is uncertainty concerning outcomes can often be described as a _____ process.
random
2. The two outcomes from the toss of a coin, heads or tails, _____ be said to be mutually exclusive.
can
3. All of the probabilities in a probability distribution will add up to _____.
1
4. The largest number you could assign to any member of a sample space would be _____.
1

MULTIPLE CHOICE:

5. In order for a list of outcomes to be a list for a sample space, it must be:
 - a. inclusive and complete.
 - b. exhaustive and mutually exclusive.
 - c. inclusive and exhaustive.
 - d. none of the above
b
6. The probability of rolling a die and obtaining more than one spot would be computed by using the:
 - a. addition rule.
 - b. multiplication rule.
 - c. subtraction rule.
 - d. division rule.
a

10. As a psychologist, you might be interested in how well people can remember things. For example, you might read a list of six letters of the alphabet to a subject, such as **m, t, s, p, g, k**, and then ask him to repeat this list. Suppose the subject said that the letters were **m, t, s, p, g, d**. He would have repeated only the first 4/5 letters correctly.

5

11. Suppose you then read another list of 6 letters — for example **p, f, t, m, s, r**. If subject's response was **p, f, t, g, f, n**, he would have repeated only the first _____ letters correctly.

3

12. You could conduct an experiment in which you gave the subject many different lists of ~~six~~ letters, each time asking him to repeat as many of the letters as he could. While the actual list of letters would be varied in this experiment, the number of letters in each list would always be six.

Since the number of letters in each list would always be six, the number of letters would be a constant/variable in this experiment.

constant

13. Every time you give the subject a list of letters, you could record how many letters he repeated correctly before making an error. You would call this his **score** on each list. Therefore, if he repeated four of the six letters correctly before making an error, his score for that list would be _____.

4

14. If you read the subject the letters **m, p, t, x, z, s**, and his response was **m, p, t, x, z, d**, his score on that list would be _____.

5

245. This simply means the **decision rule** you used to reject the hypothesis was such that if the experiment were repeated a large number of times and the hypothesis were actually true, you would erroneously reject it about _____ times out of every 100 replications of the experiment.

246. This concludes our program. It is the author's hope that the information you have obtained will not only increase your understanding of Statistics, but your appreciation of its power as a tool in behavioral science.

15. Since every list contains 6 letters, whenever the subject repeated the list without making an error, his score would be _____. The worst possible score he could make would be zero, which would mean he had repeated _____ of the letters correctly.
none/all 6 none
16. You know, therefore, that all of the subject's scores will will be no **greater** than _____ and no **less** than _____. 6, 0
17. Because the subject's score can vary or change from list to list, it would be called a _____ in this experiment. variable
18. Considering his score as a variable, list below the possible scores that the subject could make (beginning with the worst score and moving in order to the best).
_____ 0, 1, 2, 3, 4, 5, 6
19. We will refer to each of these possible scores from 0 to 6 as a **possible value** of this variable. Thus, the smallest possible **value** of this variable is _____, and the largest possible **value** is _____. 0 6
- The subject _____ receive a score of 10
could/could not could not
because the list contains only six letters. Therefore,
10 _____ a possible value of this variable. is not
is/is not
20. We have used the word _____ to refer to things that may change or vary during an experiment. variable
We use the word _____, however, to refer to things that do not change during an experiment. constant

at the _____ level.

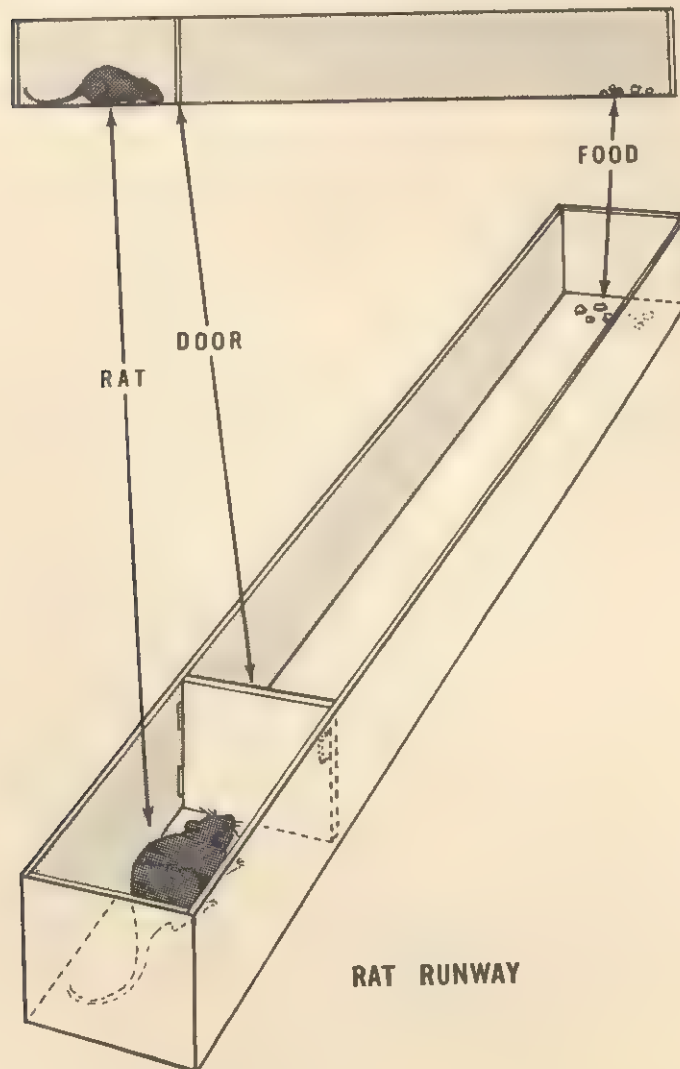
21. One of the variables we considered was distance. A **value** of that variable would be any particular distance. Since 10 feet is a particular distance, it would be a _____ of that variable. value
22. Another variable we considered was weight. Values of that variable could be particular _____ — such as 10 pounds, 2 ounces, 3 pounds, etc. weights
23. There are many different things which could be variables in an experiment. All variables, however, have one thing in common. They are things that may _____ during an experiment. change
24. Since many things may be variables, it is often useful to give each variable a **name**. For example, "distance" is the name we used for one variable we considered, whereas "weight" is the n _____ of another variable we considered. name
25. Any particular **distance** is a value of the variable named "distance." Any p _____ weight is a value of the variable named "weight." particular
26. Two feet _____ be a value of the variable named weight. would not
27. In the experiment in which the subject tried to repeat a list of six letters, we considered a variable named "score." The subject's "score" was the number of letters he repeated correctly before making an error. The different possible _____ of this variable were 0, 1, 2, 3, 4, 5, 6. values

236. In reference to our illustration, suppose the difference in the mean test scores between the group of specially trained and the group of normal students was so unusual that it would occur less than 5 times out of 100. If the hypothesis really were true (if both student groups were really samples from the same population), you might reject the hypothesis that there was **no effect** from the training and say the results were significant at better than the .05 level.
237. Whenever you **reject** an hypothesis on the basis of a decision rule whereby the risk of a Type One error is .05, you can say that the results of the experiment were **significant** at the .05 level. This simply means the decision rule you used to **reject** the hypothesis would cause you to erroneously reject it about _____ times out of 100. In other words, the risk of a Type One error was _____.
238. If you **rejected** an hypothesis on the basis of a **decision rule** whereby the risk of a Type One error was .01, you would say the results of the experiment were _____ at the _____ level.
239. We have considered two types of statistical inference in this section. One was the problem of _____ population statistics on the basis of a sample. The other was the procedure for testing an _____ concerning a population on the basis of a sample.
- estimating
hypothesis
significant, .01

28. Therefore, if the subject's score changed from 3 to 5, we would say that the _____ of the variable _____ had changed. value

29. You have learned that a _____ is something that can change during an experiment. Every time a variable changes, it changes from one _____ to another. variable value

30. Psychologists have studied how fast a rat will run down a narrow alley to secure food. The picture below shows an experimental setup you might have used if you were studying running speeds of rats.



232. However, if they were randomly drawn from the same population, you would/would not expect to find a very large difference between the two means.

233. If the preceding hypothesis were true (the test performance of each group of students could be viewed as a sample from the same population), you would/would not expect one group of students to do a great deal better than the other group.

234. Since the distribution of **differences** between the means of pairs of samples drawn from the same population **can** be derived by probability theory, it is possible to determine how **unusual** any particular size difference would be if the hypothesis were true. There is general agreement among psychologists as to how unusual a particular difference of this sort would have to be in order to reject the hypothesis. A decision rule is often used whereby the hypothesis is **rejected** if an observation is made which would occur fewer than 5 times out of 100 were the hypothesis true. In other words, if the outcome of the experiment has a probability of _____ or less were the hypothesis true, the hypothesis would be rejected.

235. A research worker often says a result is **significant** at the .05 level if the result of the experiment has a probability of .05 or less when the hypothesis is true. Therefore, saying a result is **significant** at the .01 level implies that the result of the experiment would have a probability of _____ or less if the hypothesis being tested were true.

30.

(continued)

The rat is placed in one end of the runway and food is placed in the _____ end of the runway.

same/other

same/ other

other

When the door is opened in front of the rat, he is free to move all the way to the far end of the runway to secure the _____.

food

You would be interested in the time between the opening of the _____ and the time when the animal reached the food.

door

31.

Let's imagine that you are conducting the following experiment. Suppose you took a rat that normally ate four ounces of food a day and that had never been in the runway before. You begin to feed the rat only in the runway. At precisely the same time each day, you place the rat at the starting point of the runway, four ounces of food at the other end. Then, you open the door of the runway. Each day, you would record the _____ between the opening of the door and the rat's reaching the food.

time

32.

In this experiment, you would be interested in a variable/constant named **running time**.

	<u>variable / constant</u>
--	----------------------------

variable

33.

The amount of food placed in the runway each day would be a variable/constant in this experiment.

variable/constant

constant

If it is possible to determine the theoretical sampling distribution when the hypothesis is true, we can determine the probability of a Type _____ error (erroneously **rejecting** an hypothesis) for any particular d _____ rule we might decide to use.

One
decision

This is another example of a procedure for making a statistical _____ about a population on the basis of a sample.

inference

This particular type of **statistical inference** is very useful in evaluating the results of experiments. Very often, the hypothesis being tested is whether or not the experiment demonstrates a real difference between two course could be given to a group of students and the results on a final reading exam could be compared with the results of a regular reading class. You could state the following hypothesis to be tested.

Hypothesis: There was no advantage in taking the special reading course (therefore, the performance of the 2 groups of students can be viewed as 2 samples from the **same** population).

An hypothesis of this sort can be shown to indicate a particular theoretical sampling distribution that is approximately normal and that is called the sampling distribution for the **difference** between two sample means drawn from the **same** population. Even if two random samples were drawn from the same population, you expect them to have exactly the _____ would / would not same mean.

would not

34. Suppose you repeated this experiment for 10 days.
The record of your results might look like this:

<u>DAY</u>	<u>RUNNING TIME</u>
1	200 sec.
2	100 sec.
3	150 sec.
4	80 sec.
5	40 sec.
6	41 sec.
7	15 sec.
8	10 sec.
9	4 sec.
10	3 sec.

On day 1, the rat took 200 sec. from the time the door opened to reach the food. On day 2, he took 100 sec. On day 3, he took _____ sec. 150

35. The rat's running time on day 10 was _____. 3 sec.

36. The rat's running time was _____ on day 7 shorter
longer/shorter
than on day 3.

36. Running time was _____ on day 3 than longer
longer/shorter
on day 2.

37. Instead of referring to "the variable named running time," we shall simply say "running-time" variable. Thus, the variable we call "running time" will be referred to as the "_____ - _____" running-time
variable.

38. **Particular** running times are _____ of the values
"running-time variable".

225. According to the decision rules represented by Graph B, the rule should read as follows:

Decision Rule: If the subject makes more than correct responses, reject the hypothesis that he is only guessing; otherwise it.

5
accept

226. Notice the decision rule represented by Graph A would lead to a greater/smaller risk of erroneously rejecting the hypothesis than would the decision rule represented by Graph B.

smaller

227. The most important feature of the previous illustration was the fact that the hypothesis being tested actually indicated a particular theoretical sampling distribution. This made it possible to evaluate the probability of a Type One error for particular rules.

decision

228. Let's review the major steps in testing an hypothesis on the basis of an observation. Basically, we are trying to determine if a particular hypothesis is consistent with what we have observed. If the observation would have been highly unusual were the hypothesis true, we would decide to accept/reject the hypothesis.

reject

On the other hand, if the observations would not have been unusual were the hypothesis true, we probably would accept/reject the hypothesis.

reject

39. A large part of any scientist's work consists of observing the variables in experiments and making records of these observations.

Psychologists are chiefly interested in **behavior** — either the behavior of human beings or the behavior of animals. Therefore, if you were a psychologist, a large part of your work would probably consist of observing b_____ and making records of these observations.

behavior

40. You might be interested in observing how quickly a person could solve a mathematical problem, how accurately he could estimate the weight of some object, or how much time he spent sleeping. Whatever behavior you were interested in, you would probably make ob_____ of this behavior and records of your ob_____.

observations

observations

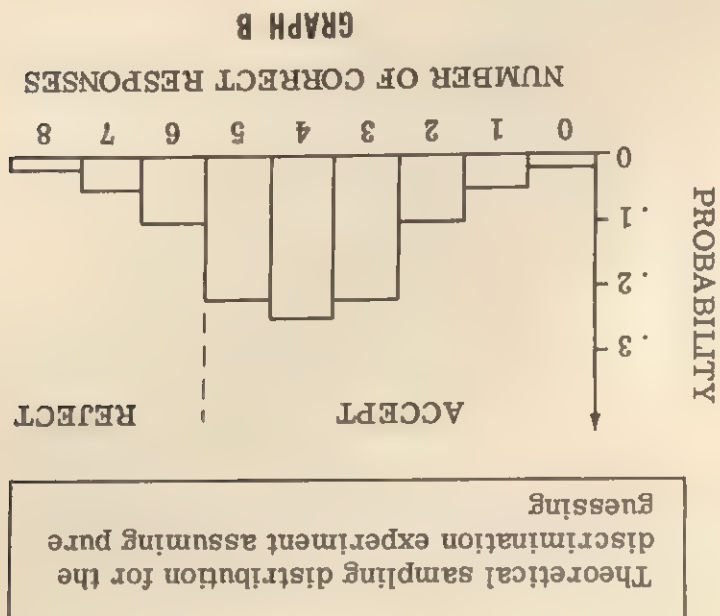
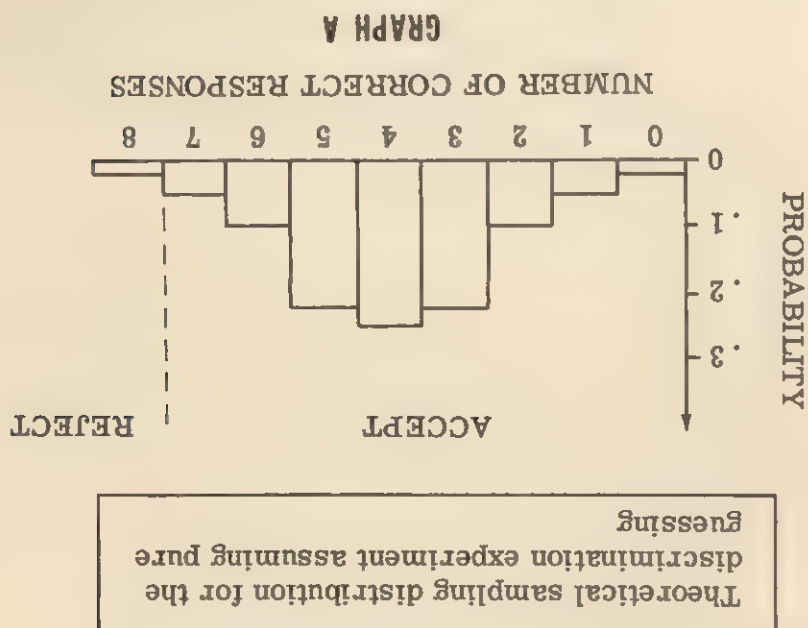
41. Each of the variables we have discussed has had different **possible** values. For example, if you tossed a coin in the air and it fell on one side or the other, the two possible results you could observe would be a "head" facing up or a "_____ " facing up. These two possible outcomes are the possible values of the variable we would call "results of a coin flip," or perhaps "**falls of a coin.**"

tail

42. What if the coin were tossed three times and you **observed** that it fell with the "head" facing upwards all three times? The **observed values** of the variable "falls of a coin" would be "heads," "heads," and "heads." Even though you did not actually observe a "tail," the two _____ values of the variable are still _____ "heads" or "tails."

possible

The following two graphs indicate the same probability distribution described in the previous table. We have indicated, however, a different decision rule on each graph.



The following decision rules is represented by Graph

A / B

Decision Rule: If the subject makes more than 7 correct responses, reject the hypothesis that he is guessing; otherwise, accept it.

43. Suppose you were conducting a telephone survey in which you asked each person you called whether or not he had been watching television. Assuming each person answered the question, the two **possible** answers are "yes" or "no." Thus, the two **possible** values of the "answer" variable are " _____ " and " _____ . " yes, no

44. Suppose four people were called in this telephone survey. Suppose you **observed** that the first person said "yes," the second person said "no," the third person said "yes," and the fourth said "yes." The record of your observations would be:

	<u>Response</u>
1st person	yes
2nd person	no
3rd person	yes
4th person	yes

These four answers are the observed/possible values observed
of the variable under study.

45. Imagine that you were learning how to bowl and that you decided to keep track of how you improved with each lesson. As a test of your skill, you could bowl 5 balls following each lesson. After each ball, you could reset any pins you had knocked down so that exactly 10 pins were standing each of the five times you rolled a ball. The most pins you could knock down with any one ball would be _____ pins. 10
The worst you could do with any one ball would be to knock down _____ pins. 0

220.	We are now in a position to define a decision rule and to evaluate the resulting probability of erroneously rejecting the hypothesis that the subject is simply guessing. For example, consider the following _____ _____	decision rule
221.	According to the previous table, the probability that a subject who is just guessing will obtain 7 correct responses is _____ and the probability of his obtaining 8 correct responses is _____.	.031 .004
222.	Therefore, the probability of erroneously rejecting the hypothesis that he is only guessing (the probability of a Type $\frac{\text{One}}{\text{Two}}$ error) is equal to the probability of a subject who is only guessing obtaining either 7 or 8 correct responses. According to the addition rule of probability, this is equal to _____ + _____, or _____.	One 031, .004, .035
223.	It is a characteristic of normal human hearing that there are some tones a subject can hear sometimes and not at other times. In other words, it is possible for him to hear the tone on some trials and yet not hear the tone on other trials. Thus, a subject who made fewer than 8 correct responses _____ could/ could not have heard the tone on some trials and not be purely guessing.	could

46. If we consider the number of pins you knock down with each ball to be a **variable**, the smallest possible value of the variable would be _____ and the largest possible _____ of the variable would be 10.

0
value

47. List all the possible values of the "pins-knocked-down" variable, starting with the smallest possible value and ending with the largest possible value.

____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

0, 1, 2, 3, 4,
5, 6, 7, 8, 9, 10

48. Suppose after the first lesson you rolled 5 balls with the following results:

PINS KNOCKED DOWN

1st Ball	0
2nd Ball	0
3rd Ball	2
4th Ball	10
5th Ball	5

According to this list of results, you knocked none of the pins down with the first ball, none down with the second ball, _____ down with the 3rd ball, _____ down with the 4th, and _____ down with the 5th ball.

2, 10
5

Thus, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are the _____ values of the variable we are possible/observed

possible

studying (the "pins-knocked-down" variable), and 0, 0, 2, 10, and 5 are the _____ values of that possible/observed variable.

observed

215. If the subject were just guessing, the most probable number of correct responses would be _____, since it has a probability of _____.
216. The probability of a subject obtaining 4 correct responses when he is only guessing is .273. This means that about _____ out of every 1,000 subjects who were purely guessing would be able to make 4 correct responses in 8 trials.
217. While subjects could quite commonly make 4 correct responses if they were just guessing, it would be usual/unusual for a subject who is just guessing to make as many as 8 correct responses, since the probability of obtaining 8 correct by guessing is _____.
218. You would expect only about _____ out of every 100 students who were just guessing to get as many as eight correct, since the probability of getting 8 out of 8 correct (if you are only guessing) is .004.
219. If a subject obtained an unusually small number of correct responses, it would seem reasonable to suppose that he was just unlucky and really couldn't hear the tone. Otherwise, you would have to assume he could hear the tone but had decided to make just the opposite response that he should have made. Therefore, we shall assume that an unusually small number of correct responses indicates that the subject could/could not hear the tone.

.273

4

273

.004

4

unusual

could not

49. Each **observed** value of a variable is referred to as a single **observation**. Therefore, in the illustration we just considered, the five observed values of the "pins-knocked-down" variable would be referred to as five _____ observations
50. If you were studying how fast a rat ran down an alley to secure food and observed that it took him exactly 10 seconds, this time would be an observed value of the "running-time" variable and _____ be referred to as a single observation. would
51. It is possible the rat could take as long as 20 minutes to reach the food. Suppose, however, that the longest observation of running time was 3 minutes. Therefore, whereas $\frac{20}{3}$ minutes would be a possible value of the running-time variable, $\frac{20}{3}$ minutes would be both a possible value and an observed value. 20
3
52. When a particular variable is observed or studied in an experiment, records are made of the observed values of this variable. These records are called **data**. For example, we just considered an experiment in which a rat ran down an alley to get food. The time it took for the rat to reach the food each day was observed and recorded. Our records of these running times are the d_____ from that experiment. data
53. Another experiment we discussed dealt with a subject's repeatedly attempting to match the weight of a lead ball by filling a bag with sand. Records of the weight of each bag of sand the subject produced are the _____ from that experiment. data

212.

If we assume that the subject was simply guessing, the probability of a correct response on each trial would be one-half (just as if he were trying to predict how the fair coin would fall). The following table indicates the theoretical sampling distribution indicated by this hypothesis.

Theoretical sampling distribution for the discrimination experiment assuming pure guessing (no discrimination).

Number of Correct Responses	Probability
0	.004
1	.031
2	.109
3	.219
4	.273
5	.219
6	.109
7	.031
8	.004

213.

This is a binomial distribution indicating the probability of each possible number of successes in 8 trials where the probability of a success on each trial is one-half.

The nine possible outcomes of our experiment indicated in the first column of the table are based on the number of correct responses the subject made in the 8 trials.

214.

You can think of these nine possible outcomes of the experiment as a **sample space**. Thus, the numbers shown in the second column of the table are a probability distribution on the sample space consisting of the 9 possible outcomes of the experiment.

54. Notice that we referred to the possible/observed values of the variable as the data. A list of the **possible** values of a variable would not be considered data. observed
55. Suppose you tossed a coin two times and the observed value of the variable "falls of the coin" was "heads" on both tosses. Then "heads" and "tails" are the two possible/observed values of the variable, whereas possible
 "heads" and "heads" are the two possible/observed values. observed
56. In the preceding frame, the list of values we would call **data** was "heads" and "heads"/"heads" and "tails". "heads" and "heads"
57. "Heads" and "heads" would be considered **data** since they are the possible/observed values of the variable. observed
58. Television stations are naturally interested in which programs are preferred by television viewers. Imagine that you were hired to determine viewing preferences in an area in which there were only three television channels: Channel 5, Channel 7, and Channel 9. Suppose you asked a number of television viewers the following question: "If you had to watch only one of the three television stations for a week, which one would you select?" Acceptable answers to these questions would be Channel or or . 5, 7, 9
59. If you considered the viewer's answer as a variable, the three possible values of the variable would be 5, 7, and . 9

208. If a subject were actually deaf to a tone of this frequency and based his responses on pure guessing, rather than on what he actually heard, he would have the same probability of being correct on each trial as he would if he were _____ the outcome of tossing a coin guessing _____

209. Assuming the experimenter's coin was a fair one, you know the probability of **guessing** how it would fall is _____, since you would expect to be correct just about as often as you would expect to be wrong.

210. If you assumed the subject's performance on these 8 trials was the product of pure guessing, it would be possible to calculate a theoretical sampling distribution for the number of correct responses out of 8 trials, since it would be possible to choose an appropriate b _____ to represent the probability of each number of correct responses in N trials.

211. The following hypothesis actually indicates a particular theoretical sampling distribution for each possible number of correct responses in the 8 trials of the experiment.

Hypothesis: The subject's responses are based on pure guessing (the probability of a correct response on each trial of the experiment is _____).

one-half

60. While you probably would ask many people this question in a real study, let us suppose you asked only three people and the first person selected 7, the second person selected 9, and the third person selected 9. The values 5, 7, and 9 would be the possible/observed possible values of the "answer" variable, whereas the values 7, 9, and 9 would be the observed values of the variable. observed
- Since an observation is any observed value of a variable, the three observations are 7, 9, and 9. 7, 9
- Your **data** would be the values 7, 9, 9, since these were the observed/possible values of the "answer" variable. observed
61. In the television survey, the answer of each of the three people was a constant/variable, but the question they were asked was a constant/variable. variable constant
62. It is often useful to distinguish between **continuous** and **discrete** variables. If you were to count the number of people in a room, it would be possible to have 8 people or 9 people, but it would not be possible to have $8\frac{1}{2}$ people. It is this characteristic of the variable named "number of people in a room" which makes it a **discrete** variable. The variable is discrete since there are no values of the variable between 8 and 9, or between 10 and 11, or between 12 and 13. 13

cycle per second tone (a very high-pitched tone which very few people can hear), you decided to conduct the following experiment. Each student would be a subject in a brief experiment consisting of 8 trials. On each of these trials he would be asked to determine whether or not you had presented a brief burst of the 20,000 cycle per second tone. He was required to say "yes" or "no" to indicate whether or not he thought the tone had been presented. In other words, he would say "_____ " if he thought the tone had been presented and "_____ " if he thought it hadn't (on each of the 8 trials).

yes
no

trials

The data from a particular subject would consist of _____ observations of a variable called "his answer," whose two possible values are "_____ " or "_____ ".

yes, no

To discourage the subject from simply guessing whether or not you had presented the tone on each trial, you informed all of the students that you would determine whether or not to present the tone on each trial by secretly tossing a coin at the beginning of each trial. If the coin landed "heads," you would present the tone; and if it landed "tails," you would not. In other words, if a student could not hear a tone of such a high pitch and attempted to guess whether you actually presented a tone, he would be in the same position as if he tried to

guess

_____ how a coin would land when it was tossed.

On the other hand, the variable we call "length" is an example of a **continuous** variable. No matter which particular pair of lengths you considered, it be possible to imagine a length **between** would/would not these two lengths. For example, $8\frac{1}{2}$ inches is between 8 inches and inches. (By "between", we mean greater than one length but less than the other.)

would

9

Similarly, $2\frac{1}{2}$ feet is a value of the variable called "length," which is the values 2 feet and 3 feet. In other words, $2\frac{1}{2}$ feet is larger than 2 feet but less than 3 feet.

between

63.

A **continuous** variable has an unlimited number of values because no matter how close two values are to each other it always possible to imagine another is/is not value which would lie between them. For example, even though 2.10 inches and 2.20 inches are close together, inches is between them. On the 2.15/2.25 other hand, if the variable you are studying is **discrete**, you can find two values of the variable such that there is no value between them. For example, the number of pennies you have in your pocket would be a value of a variable, since you could only discrete/continuous have in your pocket one penny, two pennies, three pennies, and so on. Of course, you could not have any number **between** these values.

is

2.15

discrete

A.C.E.R.Y. West

Date.....

Acc. No.....



205. In the preceding illustration we obtained the theoretical sampling distribution by estimating it from an experimental sampling distribution. It is possible to test certain hypotheses for which a theoretical sampling distribution is actually defined. In fact, it is highly desirable to phrase the question about an experimental observation in such a manner that the hypothesis to be tested actually specifies a theoretical sampling distribution. The following illustrations will serve to demonstrate the manner in which an hypothesis may actually define a particular theoretical sampling distribution. Suppose you were a psychologist who was interested in the ability of people to hear very high-pitched tones. In order to determine how many people in your psychology class could actually hear a 20,000

204. The most important contribution of a statistical approach of this kind to decision-making is your ability to specify the relation between particular _____ and the risk of making particular kinds of errors.

203. The preceding illustration once again indicates how the particular d _____ r _____ used in hypothesis testing determines the risk of making the two possible kinds of errors.

202. On the other hand, while decision rule B reduces the risk of making a Type Two error (erroneously failing to reject the hypothesis), it does produce the largest _____ of making a Type One error (erroneously rejecting the hypothesis), since it will result in more/fewer non-defective parachutes being shipped back to the factory than will the other rule.

decision
rules

decision rule

more

risk

64. No matter how similar the weights of two objects, it would always be possible to imagine an object whose weight was less than one but greater than the other. Therefore, "weight" is an example of a _____ variable. continuous
discrete/continuous

65. It will not be necessary to distinguish between discrete and continuous variables very often. Most of the illustrations in this text involve discrete variables. Those illustrations that involve continuous variables are treated as if they were discrete variables. For example, if you measured a person's height to the nearest inch, you might say he was 65 inches tall or 66 inches tall, because you "round off" his "height" to the nearest inch, you would never say he is $65\frac{1}{2}$ inches tall. In other words, you are treating the _____ variable called "height" as if it continuous
continuous/discrete
were a _____ variable. In other words, discrete
discrete/continuous
you are pretending that there are no values between 65 inches and 66 inches, or between 67 inches and 68 inches, and so on.

66. To determine the **number** of people in a room, you would count them. To determine the number of eggs in a basket, you would count them. Whenever you _____ things, you are determining how many count
(what number) of things there are.

67. Any collection or group of things can be counted. The procedure we call **counting** tells you the **number** of things that are in the group or collection. For example, we can determine the _____ of windows in number
a building by counting them.

Once again, the shaded columns in each graph represent the probabilities of $\frac{\text{correctly/erroneously}}{\text{rejecting the}}$ hypothesis because you obtained a non-defective parachute with an unusual weight. Since the shaded columns in each graph represent the probability of making a Type One error, you could add these probabilities according to the simple addition rule to find the probability of making a Type _____ error with each decision rule.

199.

The preceding graphs should make apparent the fact that the probability of erroneously rejecting the hypothesis (the probability of a Type One error) is $\frac{\text{smaller/larger}}$ if you use decision rule A than if you use decision rule B.

smaller

200.

The risk of a Type One error (erroneously rejecting the hypothesis) is greatest under decision rule B, since even slightly unusual weights will result in a rejection of the hypothesis. In fact, only parachutes weighing 44, 45, or 46 ounces would lead you to $\frac{\text{accept/reject}}$ the hypothesis and release the parachute for use by an airman.

accept

201.

Because you are much more concerned with avoiding Type Two than you are Type One errors, you would probably use decision rule $\frac{A/B}{\text{rejecting the}}$, since it would result in the smallest risk of making a Type Two error.

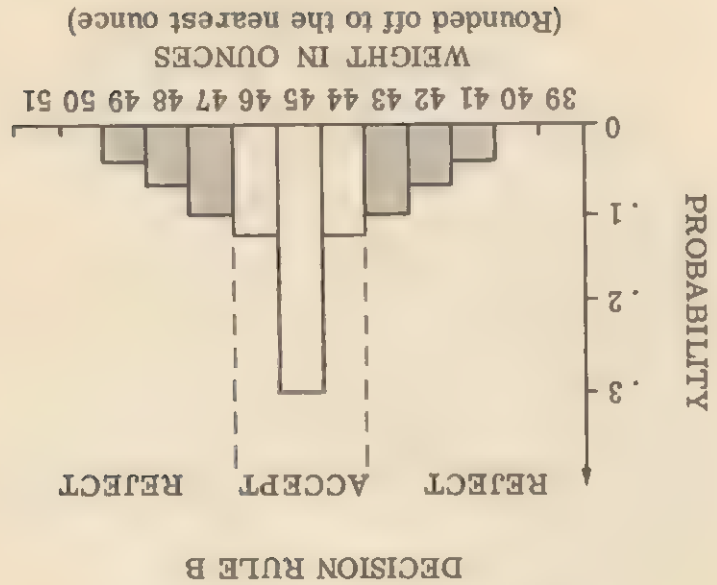
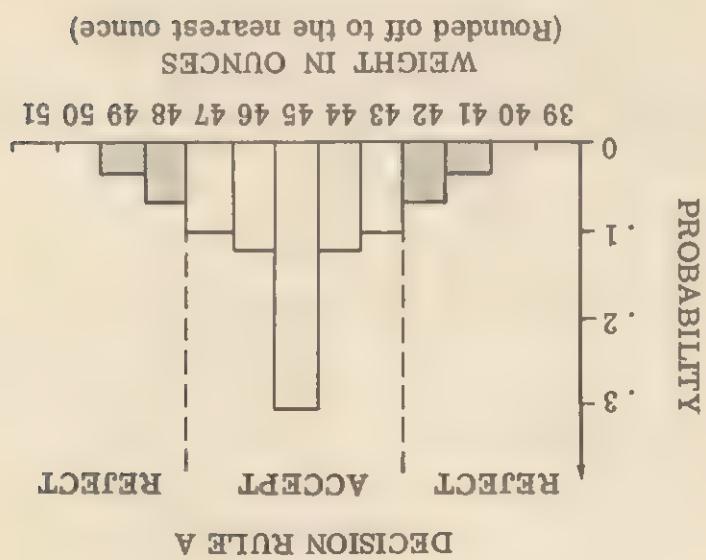
B

68. It is often useful to distinguish between the name of something and the thing itself. A "name" is something you speak, write, or read. For example, "Boston" is the name of a city. You could go to Boston, walk through the streets of Boston, or even live in Boston. On the other hand, the name "Boston" is something you speak, write, or read. Similarly, if you had a dog named "Rover," you might scratch the dog/name behind the ears, dog whereas you might print his dog/name on his dog house. name
69. Suppose a mother didn't decide to name her new baby "Kendall" until three days after the baby was born. The baby/name would be three days old before it was given a baby name.
70. We will use quote, " ", when we are referring to the name of something rather than to the thing itself. Thus, in the previous example we would refer to the baby as Kendall and to the name the baby received as "Kendall." Similarly, when we speak of the city of Boston and of the name "Boston," we will say that you could live in Boston/"Boston" Boston and that you write the name "Boston"/Boston as part of an "Boston" address on an envelope.
71. The number of things in a group or collection is a characteristic of that group of things, just as the color of a person's hair is a characteristic of that person. We refer to a person's hair color with a **name**, such as "red," "brown," and "black." Similarly, we refer to the number of things in a group with a name, such as "five," "twenty," "eighteen," and "forty." Thus, "red" would be the **name** of a particular hair color and "five" would be the name of a particular number.

A Notice that decision rule $\frac{A}{B}$ requires the weight of a parachute to be **more** than unusual before rejecting the hypothesis than does decision rule $\frac{A}{B}$.

B

198. Decision rules A and B can be illustrated in the same manner as the previous decision rule by drawing vertical lines separating the possible weights into three classes, as shown below.



72. Names for other _____ are "two," "6," "four," and "one hundred." numbers
73. There are different ways of representing the same number. For example, the _____ of players on a basketball team could be represented either by the name "five" or by the name "5" or by the name "V." number
74. Number is a characteristic of a group or a collection of things. The names (such as "ten," "four," "6") that are used to represent this characteristic are often called **numerals**. Therefore, the number of players on a basketball team could be represented either by the numeral "5" or the Roman numeral _____. V
75. Since the names "red," "green," "Democrat," and "Boston" are not the names of numbers, they are not numerals. Therefore, of the two names "6" and "blue," " _____ " is a numeral since it is the name of a 6 number
76. We pointed out earlier that there is a difference between the name of something and the thing itself. A numeral is the name we give to a number. You determine the _____ of things in a group number
number/numeral
by counting them; you represent this characteristic of the group by writing or saying a _____. numeral
number/numeral
77. The same number can be represented by different numerals. "Four," "4," and "IV" are three names for the same number; in other words, they are three **numerals** which represent the same _____. number

rejecting

195. We said earlier that a characteristic of this particular testing problem was the fact that we were much more concerned with avoiding Type Two errors than we were with avoiding Type One errors. In other words, we were ~~less~~ concerned with erroneous rejections of the hypothesis than we were with erroneous failures to reject the hypothesis. The reason for this was the fact that failing to reject the hypothesis when we should would result in an airman being issued a defective parachute, whereas _____ the hypothesis when we shouldn't would only result in needlessly returning a parachute to the factory. (Remember, the hypothesis being tested is that the parachute is **not defective**.)

196. You have already seen how it is possible to modify a decision rule so as to alter the risks involved in making the two possible kinds of error. Typically, if you decrease the risk of making one kind of error, you _____ the risk of making the other type of error.

increase

197. Consider the two decision rules described below:

Hypothesis: The parachute is suitable (non-defective).

Decision Rule A: If a parachute weighs less than 43 ounces or more than 47 ounces, **reject** the hypothesis; otherwise, accept it.

Decision Rule B: If the parachute weighs less than 44 ounces or more than 46 ounces, **reject** the hypothesis; otherwise, accept it.

190. The shaded columns in the previous graph represent the probability of a $\frac{\text{defective/non-defective}}{\text{parachute having}}$ a weight which will cause us to reject the hypothesis that it is non-defective, since this is a distribution based on $\frac{\text{defective/non-defective}}{\text{parachutes}}$.
191. In other words, the shaded columns in the previous graph represent the probability of $\frac{\text{correctly/erroneously}}{\text{rejecting a non-defective parachute because it has an unusual weight for a non-defective parachute}}$.
192. Erroneously rejecting an hypothesis when it is actually true is an example of a Type $\frac{\text{One/Two}}{\text{error}}$. (Remember, the **single** key word is rejecting.)
193. Therefore, the shaded columns in the previous graph represent the probability of a error .
194. Once again, we see how it is possible to evaluate a particular **decision rule** in terms of the probability of making a particular kind of error. On the other hand, since we **do not** know the distribution of weights among defective parachutes, we cannot specify the probability of correctly rejecting the hypothesis. In order to specify the probability of correctly rejecting the hypothesis (or of erroneously failing to reject), we would have to know the probability that a $\frac{\text{defective/non-defective}}{\text{parachute}}$ would have a weight which would lead us to reject the hypothesis.

defective

Type One

One

erroneously

non-defective

non-defective

85. When we use the names "twenty inches," "two feet," and "ten feet," we are using the numerals/numbers numerals "twenty," "2," and "10" as **names** for particular values of the variable called length.

86. Football and baseball players often have numerals/numbers written on the backs of their uniforms to help people in the stands to identify the different players. numerals

87. It is important to be careful when numerals are used to represent characteristics other than number. It makes sense to say that twenty things are **twice** as many things as ten things. It is also appropriate to say that something weighing twenty pounds is **twice** as heavy as something weighing ten pounds. It does/does not necessarily make sense to say that a does not baseball player with the numeral "20" written on his jersey is **twice** as good as the baseball player with the numeral "10" on his jersey. The fact that one player was assigned the numeral "10" and another assigned the numeral "20" does **not** necessarily imply anything more about the players themselves than does the difference in their names (John and Charles, for example).

88. The value of a variable can often be represented by numerals. We shall refer to these variables as **numerical variables**. For example, we discussed an experiment earlier in which we were interested in the time it took a rat to run down an alley to reach food.

6747



We have drawn vertical lines, dividing the possible weights into 3 groups. The group on the far left labeled "reject" represents all weights **less than** _____ ounces. (We could describe this group of weights as consisting of all weights of "_____ ounces or less" rather than saying "all weights less than 42 ounces.")

41

187. According to the previous **decision rule**, we would consider any parachute weighing 41 ounces or less to be so usual/unusual that we would **reject** the hypothesis that the parachute was suitable (non-defective).

unusual

188. Similarly, the group of weights at the far right of the graph consisting of all weights of 49 ounces or more would also be considered so unusual for a non-defective parachute that we would accept/reject the hypothesis that the parachute was non-defective when it weighed 49 ounces or more.

reject

189. On the other hand, any parachute weighing **more than** _____ or **less than** _____ ounces would not be

41, 49

considered sufficiently unusual (for a non-defective parachute) to cause us to reject the hypothesis. Therefore, this group of weights is identified by the word accept/reject, which is written between the vertical lines isolating this group of weights.

accept

The values of this running-time variable were represented by numerals such as "20 seconds," "10 seconds," "800 seconds," Therefore, we refer to running time as a numerical would
would/would not
variable.

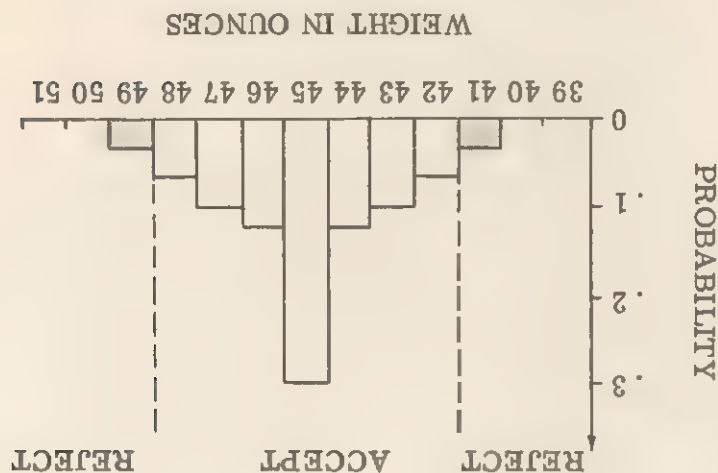
89. Variables whose values are not represented by numerals will be called **non-numerical variables**. If we were interested in hair color, therefore, the values we might observe would be black, red, brown, and so forth. Since these values are not represented by numerals, we will refer to hair color as a variable. non-numerical
numerical/ non-numerical

90. Political party is a variable, and particular values of this variable are Democrat, Republican, Socialist, and so on. This would be an example of a variable. non-numerical
numerical/ non-numerical

91. The age of American presidents when they were elected to office would normally be a numerical
numerical/ non-numerical
variable.

92. We said earlier that the **data** from a study was a record of the observed values of the variables being studied. If these observed values are represented by numerals, the variable under study is a numerical
numerical/ non-numerical
variable. We would refer to the data, therefore, as **numerical data**.

(Rounded off to the nearest ounce)



The decision rule can be illustrated in terms of a previous distribution of weights for non-defective parachutes.

Hypothesis: Parachute is suitable (non-defective).
Decision Rule: Reject the hypothesis if the parachute weights less than 42 ounces or more than 48 ounces. Otherwise, accept the hypothesis.

Imagine that you adopted the decision rule indicated below.

186. suspect that it was defective.
 would be $\frac{\text{typical/unusual}}{\text{and you probably would/would not}}$

185. A parachute weighing 45 ounces would not be suspicious, since this was a typical or highly probable weight for a non-defective parachute. On the other hand, (considering the distribution of non-defective parachutes), if a parachute weighed 41 ounces or 49 ounces, its weight

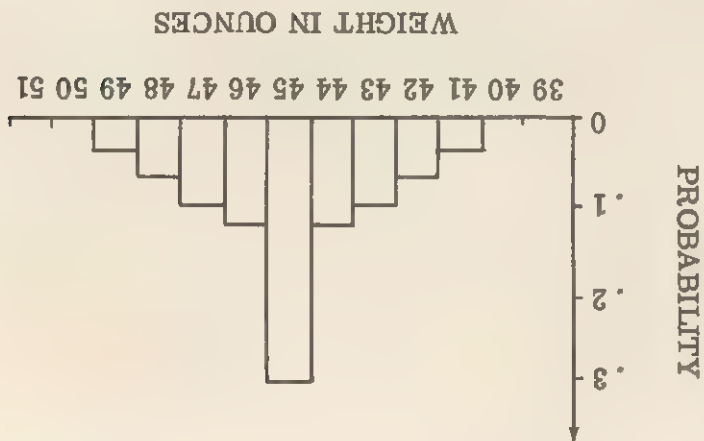
184. On the other hand, if a parachute weighed 45 ounces, you probably $\frac{\text{would/would not}}{\text{suspect that it was defective}}$ on the basis of its weight.

93. Records of the observed running times in the experiment in which we observed the time it took for a rat to reach some food would be _____ data. numerical
numerical/non-numerical
94. Earlier, we considered a study in which people were asked which of three television channels they preferred: 5, 7, or 9. We could name the three possible answers they gave as "5," "7," or "9." The names "5," "7," and "9" are used simply as names for their answers. However, "5," "7," and "9" are also used as **names** for the values of the variable we call number and are therefore called numerals. Even though the variable we call "their answer" is not really a number, its values are represented by _____. numerals
numerals/numbers
95. Because we said that any variable whose values could be represented by numerals would be called a numerical variable, we would say that the person's answer in the television survey was a _____ variable, even though the numerical/non-numerical _____ numerals we use as names for the values of this variable _____ represent different numbers in this do not
do/do not particular case.
96. Because the list of the **observed** values of the answer variable would be a list of numerals, it would be an example of numerical d _____. data
97. A list of the possible values of answers in the television survey _____ be called numerical data however. would not
would/would not

attributed to normal and unimportant variations in the materials that make up the parachutes. According to the distribution, the most typical weight for a non-defective parachute is _____ ounces.

45

Probability distribution describing the distribution of weights to be expected in a large group of non-defective parachutes.



(Rounded off to the nearest ounce)

181. Similarly, you would never expect to find a non-defective parachute that weighed less than _____ ounces or more than _____ ounces.

41
49

182. Note the probability that a non-defective parachute will weigh 45 ounces is _____. Therefore, you would expect about _____ out of every 100 non-defective parachutes to weigh 45 ounces.

.3
30

183. If you found a parachute whose weight was unusually light or unusually _____, you might suspect that the parachute was defective.

heavy

98. Such a list would not be called numerical **data** because, although it is a list of values represented by numerals (which makes it a numerical list), it is not a list of observed values. Only a list of _____ values is referred to as **data**. observed
99. Earlier, we listed how many pins a person knocked down each time he rolled a bowling ball. Each time he rolled the ball he could knock down anywhere from 0 to 10 pins. Thus, any number between 0 and 10 was a(n) _____ value of the variable "pins knocked down." a possible
possible/observed
100. Each value of the variable "pins knocked down" was represented by a numeral. Therefore, "pins knocked down" is an example of a _____ numerical
numerical/non-numerical
variable.
- The numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are the _____ values of that variable. possible
101. The list 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 _____ would not
would/would not
be considered data because it is simply a list of the _____ values of the data. possible
possible/observed
102. The list of **observed** values of the variable used in the earlier illustration was 0, 0, 2, 10, and 5. This list _____ be considered data. would
would/would not

177. If you reject the hypothesis that a particular parachute is suitable when it actually is suitable, you would make a Type _____ error, since you are erroneously **rejecting** the hypothesis (one key word).
178. If you fail to reject the hypothesis that a particular parachute is suitable when it is actually defective, you would be making a Type $\frac{\text{One}}{\text{Two}}$ error.
179. If you made a Type One error (**reject** the hypothesis that the parachute is suitable when it is actually suitable), the company will have the unnecessary expense of sending the parachute back to the factory. On the other hand, if you make a Type Two error (**fail to reject** the hypothesis that a particular parachute is suitable when it is actually defective), you will send a defective parachute out to be worn by an airman, which could result in the loss of his life. Therefore, in this particular situation, you would be much more anxious to avoid a Type _____ error, even if this resulted in a higher $\frac{\text{One}}{\text{Two}}$ risk of making the other kind of error.
180. Imagine that you were only one of several people who ran tests on the parachutes and your particular task was to weigh the parachute in order to determine if it had an unusual weight, which might indicate defective quality. Imagine the following probability distribution describes the distribution of weights you would normally expect to find among a large group of **non-defective** (suitable) parachutes. The fact that all non-defective parachutes do not have the same weight can be

103. This list of observed values _____ be would
would/ would not
considered **numerical** data, since each value of the
variable (the number of pins knocked down) is
represented by a _____. numeral

104. One way in which men differ from one another is
whether or not they have a beard. Suppose you were
interested in how many men wore beards. If you went
to a busy intersection and made a list of whether or not
each man who passed wore a beard, your list might look
like this after 5 people had passed:

<u>PERSON</u>	<u>BEARD</u>
1	_____ yes
2	_____ no
3	_____ yes
4	_____ no
5	_____ yes

105. Since this list is a record of your observations, it
_____ be referred to as data. could
could/ could not

106. Here we simply recorded the values of the variable
we were studying as "____" or "____" rather than
writing out "He wore a beard" or "He didn't wear a
beard." yes, no

Suppose you used a statistical decision-making procedure to test the following hypothesis.

Hypothesis: The parachute is non-defective

(suitable for use). If you $\frac{\text{accepted/rejected}}{\text{this}}$ accepted

hypothesis, the parachute could be sent out for use by a pilot, whereas if you $\frac{\text{accepted/rejected}}{\text{the hypothesis,}}$ rejected

the parachute would be sent back to the factory for

repair.

175.

You should recall that a Type One error occurs when you erroneously $\frac{\text{accept/reject}}{\text{the hypothesis and that a}}$ reject

Type Two error occurs when you erroneously fail to reject the hypothesis.

176.

A convenient way to remember which is a Type One and which a Type Two error, is to recall that wrongly rejecting a hypothesis is a Type One error and the single key word is **rejecting**. On the other hand,

erroneously failing to reject is a Type Two error (two key words: **failing and reject**). Thus, erroneously rejecting an hypothesis is a Type $\frac{\text{One / Two}}{\text{error}}$

since the **single** (one) key word is **reject**. Erroneously failing to reject an hypothesis involves two key words ($\frac{\text{and is,}}{\text{and is,}}$) and is, therefore, a Type $\frac{\text{One / Two}}{\text{error}}$.

107. When we recorded the observations of whether or not a man wore a beard, we could have used the numerals "1" or "0" instead of "yes" or "no." We could have recorded a "1" for each man who had a beard and a "0" for each man who didn't have a beard. Thus, the list of observations we just presented would look like this:

<u>PERSON</u>	<u>BEARD</u>
1	1
2	0
3	1
4	0
5	(?)

Since the fifth person we saw had a beard, we would complete our list by replacing the question mark with a $\frac{1}{0}$.

1

108. Because we represented the values of the variable we are studying with **numerals**, our data is

numerical/non-numerical.

numerical

109. Earlier, we stated that number/numeral is the

number

characteristic of a group or collection of things which we determine by **counting** how many things are in the collection.

110. The numerals "1" and "0" are usually used as names for particular numbers. However, when we use these numerals as names for the two values of the variable "bearded or beardless," they do/do not represent the characteristic we call number.

do not

Type One errors (assuming the animal was sick when he really wasn't) than you would be with avoiding Type Two errors (assuming the animal was well when he was really sick).

172. If you were **more** concerned with the risk of infecting

other animals in the laboratory than of needlessly destroying your own experimental animal, you would probably be willing to assume that the animal was sick if he ate even a slightly less than normal amount of food during the day on which you based your decision.

While this would reduce the risk of erroneously accepting the hypothesis that the animal was well, it would

the risk of deciding the animal was increase/decrease

infected when he actually wasn't.

173.

It is obvious that statistical decision-making cannot

eliminate all possibility of making an error. What it

can do is allow you a choice in determining the risk of

the different kinds of error. By choosing a particular

decision rule, you can **decrease** the risk of one type of

error, even though you may _____ the risk

increase

of making the other kind of error.

174.

In some situations, it is obvious that one kind of error

would be more costly than the other kind of error. For

example, suppose you were in charge of checking the

quality of parachutes at a parachute factory. Each

parachute that passed in front of your test station could

either be defective or non-defective (suitable for use).

111. Thus, we have just seen a case where our data was numerical but the numerals didn't have anything to do with the characteristic we call number. The numerals/numbers were simply used as **names** for the numerals different values of the variable we were studying: "1" if he had a beard or "0" if he didn't.
112. When we talk about **number**, we can say such things as "three people are a **greater number** of people than two/ten people." two
113. We can say the number represented by the numeral "4" is **twice as large** as the number represented by the numeral "_____". 2
114. We can also say that since 7 minus 5 equals 2, and 3 minus 1 equals 2, the difference between 7 and 5 is the same as the difference between 3 and _____. 1
115. Sometimes when numerals are used to represent variables other than number, we can make similar statements. For example, it would make sense to say that a temperature of 100 degrees above zero was greater (or hotter) than a temperature of 50/150 degrees above zero. 50
116. It would also make sense to say that "4" pounds was twice as heavy as "_____" pounds. 2
117. It would also make sense to say that the **difference** in height between a person who was $5\frac{1}{2}$ feet tall and one who was 6 feet tall was the same as the **difference** between someone who was $4\frac{1}{2}$ feet tall and someone who was 5/7 feet tall, since the difference in height was 1/2 foot in both cases. 5

168. A Type One error would result in a costly and unnecessary re-running of the experiment, whereas a Type Two error would only result in the loss of a few additional, inexpensive laboratory animals. Therefore, you would want a **decision rule** that would provide very little risk of a Type $\frac{\text{One}}{\text{Two}}$ error, even if this increased your chances of making the other type of error.

169. Since a Type One error is erroneously rejecting the hypothesis and since a Type Two error is erroneously failing to reject it, you could decrease the probability of a Type $\frac{\text{One}}{\text{Two}}$ error by requiring your observation to be highly unusual (rather than a slightly unusual) before rejecting the hypothesis.

170. On the other hand, requiring the event (observation) to be highly unusual reduces the number of observations that would lead you to **reject** the hypothesis. This means you have $\frac{\text{increased/decreased}}$ the risk of accepting the hypothesis when it is actually false.

171. Imagine that the consequences of the two kinds of errors were different from those we just considered. Suppose it would require little effort to test a new animal were the present monkey destroyed. Suppose also that the other animals in the laboratory were very rare and expensive experimental animals. Therefore, if the infection were allowed to spread, it could be extremely costly for the laboratory. Under these circumstances, you would be $\frac{\text{more/less}}$ concerned with avoiding less

118. We have seen cases, however, where numerals have been used to represent values of a variable so that statements of this sort are not appropriate. For example, when we considered asking people to choose their favorite from among three television channels, we represented the values of the answer variable with the numerals "5," "7," and "9." It make much sense to say that would not
would/would not
choosing Channel 7 was **greater** than choosing Channel 5.
119. Also, it be appropriate to say that the would not
would/would not
difference between answering "5" and answering "7" was the same as the difference between answering "7" and answering "9" because $7 - 5 = 9 - 7$.
120. You should NOT assume that just because a variable is numerical it is similar to "number" in some way. It possible for a variable to be numerical and have is
is/is not
nothing more in common with "number" than the use of numerals as names for its values.
121. A variable is numerical simply because we use as names for values of the variable. numerals
122. Later, we will consider in greater detail the use of numerals to represent variables other than number. For the present, we only wish to emphasize that statements like "greater than," "the same difference as," and "twice as much as" appropriate or make may not be
are always/may not be
sense — even when a variable is represented numerically.
123. Many variables of interest to a scientist **are** similar in some way to the variable we call "number." For example, the variable called "length" is similar to "number," so it would make sense to say something ten inches long is **longer** than something inches long. 5
5/12

165. The two possible kinds of error that can occur when you decide to accept or reject an hypothesis are:

Rejecting when you really shouldn't (which is

called a Type $\frac{\text{One/Two}}{\text{error}}$).

Failing to reject when you should reject (which

is called a Type $\frac{\text{One/Two}}{\text{error}}$).

The more **unusual** you require your observation to be

before rejecting the hypothesis, the greater will be the

probability of your making a Type $\frac{\text{One/Two}}{\text{error}}$.

166.

Only rejecting the hypothesis when a highly unusual event

occurs lowers your chances of wrongly rejecting the

hypothesis. On the other hand, since this also reduces

the number of observations that would lead you to reject the

hypothesis, it usually **increases** your chances of making

a Type $\frac{\text{One/Two}}{\text{error}}$.

Two

167.

In order to determine the appropriate decision rule to

apply in any particular case, you must consider the

relative **cost** of the two kinds of errors. For example,

suppose the consequences of making a Type One error in

the previous illustration (deciding that the animal **was**

sick when he **really wasn't**) would result in the loss of

this very important experimental animal and a costly

re-running of the experiment. On the other hand,

suppose that a Type Two error (deciding that the animal

was well when he was really sick) would, at the very

worst, result in the contamination of a few more

relatively inexpensive animals. Under these conditions,

you would probably be more concerned about avoiding a

Type $\frac{\text{One/Two}}{\text{error}}$ than you would the other type

One

of error.

124. We determine the **number** of a collection of objects by counting them. We determine the **length** of something in a variety of ways, although the most common procedure is to use a ruler. Both counting and the use of a ruler are procedures by which we determine the appropriate **numeral** to represent a value of a particular variable. Procedures of this sort are called **measurement** procedures. Another example of a m _____ procedure would be the use of scales to determine a person's weight.

measurement

125. The numeral 10 represents a _____ number
larger/smaller
than does the numeral 5, and ten ounces represents a
_____ weight than five ounces. Therefore, it
larger/smaller
is possible to make statements about values of the variable "weight" similar to the statement you made about the variable "number." For example, you could say the number 20 is twice as large as the number 10, just as a weight of 20 pounds is _____ as heavy as a weight of 10 pounds.

larger

larger

twice

On the other hand, some variables are **not** at all similar to the variable called "number," even though you could use numerals to represent values of these variables. For example, the fact that the license number on your automobile is larger than the license number on your neighbor's automobile probably _____ indicate any difference between the two
does/does not
automobiles (although it might indicate you obtained your license number on an earlier date than did your neighbor).

does not

161.

The new **decision rule** requires the animal's food consumption on the test day to be even more unusual for a healthy animal than was required by the old rule, in order to reject the hypothesis that the animal was well, the probability of wrongly rejecting the hypothesis is smaller/larger in Graph A (which illustrates the new

smaller

162.

By requiring an even more unusual observation before you reject the hypothesis that the animal was well, you have increased/lowered the probability of making a Type One error (wrongly rejecting the hypothesis that the animal was well), since a healthy animal will consume 0 pellets **less** frequently than "0 or 1" pellets.

lowered

163.

On the other hand, it is perfectly clear that you could eliminate all possibility of making a Type One error by using a **decision rule**, according to which you would never accept/reject the hypothesis that the animal was well, since you then would never reject the hypothesis when it was actually true.

reject

164.

While you would never erroneously reject the hypothesis that the animal was well if you never rejected the hypothesis, you would have a probability of 1 of failing to reject the hypothesis that he was well when he was actually sick. Stated more formally, while you had reduced the probability of a Type One error to 0, you would have increased the probability of making a error to 1 (you would **always** fail to reject the hypothesis when you should reject it).

Type Two

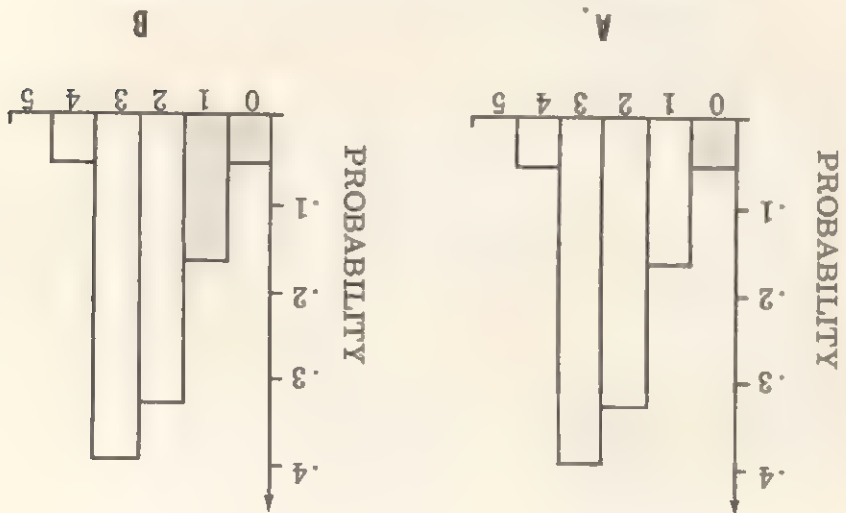
126. Whenever a variable is similar to the variable we call "number," it is possible to assign numerals to represent values of that variable in such a way that you maintain the similarity between that variable and the variable "number." Determining how similar a particular variable is to the variable we call "number," and assigning numerals in the appropriate manner is called **measurement**. We will not have time to consider the topic of **measurement** in this program. The major point we wish to emphasize is that you should be careful **not** to suppose a particular variable is similar to "number" just because that particular variable is numerical. Numerals are often used to represent values of a variable simply because numerals are convenient or familiar names. Just because one value of a variable is represented by the numeral 8 and another value of the variable is represented by the numeral 4 necessarily imply that one value is in does/does not any sense larger or greater than the other one.

127. In this section, we have distinguished between "**numeral**" and "**number**" in order to indicate how easy it is to convert a non-numerical variable into a numerical one by simply substituting numerals for the non-numerical names of the values. On the other hand, in most of your reading you will find no distinction is made between "number" and "numeral." Therefore, throughout the remainder of this program we will use the word "number" without attempting to distinguish between "number" and "numeral." Remember, however, that using "numerals" to represent the values of a variable **does not** insure any similarity between that variable and the variable called "number." The procedure of deciding how similar a variable is to number and assigning numerals in an appropriate fashion to represent this similarity is called m .

measurement

represented by Graph $\frac{A/B}{A/B}$ (shown below) while the old decision rule (reject if fewer than 2 pellets are consumed) could be represented by Graph $\frac{A/B}{A/B}$.

Experimental sampling distribution of a healthy monkey's behavior (represented as a probability distribution).



160.

Once again, the shaded columns in each graph represent the probability of erroneously rejecting the hypothesis that the animal was well, since they represent the probability of a healthy animal consuming a number of pellets which would lead you to $\frac{\text{accept/reject}}{\text{accept/reject}}$ the hypothesis that the animal was well.

128. It is often useful to list things underneath one another, in what is called a **column**. The following list of numerals is arranged in a column.

8
6
5
2
9

The following list of colors is also arranged in a
c _____.

column

red
green
blue
green

129. Another way of listing things is side by side, in what is called a **row**. The same list of numerals we just arranged in a column could also be arranged in a row, as follows:

8, 6, 5, 2, 9

The same list of colors we just arranged in a column could be arranged in the following _____.

row

red, green, blue, green

To summarize, then, we have chosen a **decision rule** according to which the probability of making a Type One error is .2. A healthy monkey would eat less than 2 pellets on about 2 out of every 10 days. If you applied the decision rule each day, you would have wrongly decided that the animal was sick on about _____ out of those 10 days.

One of the most important contributions of statistical **decision-making** is that it is often possible to **specify** the probability of making different kinds of errors. While it is impossible to eliminate the possibility of making errors in many situations, it is very useful to be able to specify the risk — just as it was useful to not only estimate a population parameter but to also specify how accurate the estimate is likely to be.

Furthermore, it is possible to change the decision rule so that the probability of making a particular kind of error is either increased or decreased. This can be illustrated as follows: Suppose you had chosen another decision rule, according to which you would only **reject** the hypothesis that the animal was well if he ate 0 pellets of food on the test day. Thus, this new decision rule could be

130. Since a **column** is a list of things one underneath the other and a **row** is a list of things arranged side by side, these numerals are listed in a and row

2, 8, 9, 6, 3

these days are listed in a . column

Monday
Tuesday
Wednesday
Thursday
Friday

131. Earlier, we considered an experiment in which we observed how fast a rat ran down an alley to reach food on ten successive days. The following data was presented as an example of what we might have observed:

<u>DAY</u>	<u>RUNNING TIME</u>
1	200 sec.
2	100 sec.
3	150 sec.
4	80 sec.
5	40 sec.
6	41 sec.
7	15 sec.
8	10 sec.
9	4 sec.
10	3 sec.

The different observed values of the running time variable were listed in a . column

behavior during the previous 100 days was

representative of his **normal** eating habits, the

probability distribution shown above represents the

eating habits of a monkey who was actually

well/sick

well

155. Since this probability distribution represents the eating

behavior of a healthy monkey, the shaded columns

represent the probability of accepting/rejecting the

hypothesis that the animal is well when he actually is

well (when the hypothesis is actually true).

156. Since the shaded columns represent the probability of

rejecting the assumption that the animal is well when he

actually is well, they represent the probability of making

a Type One

error

157. You should recall that a Type One error is erroneously
rejecting a hypothesis because you had observed an
event that was unusual when that hypothesis is actually
true. Even the healthy monkey occasionally ate only
0 or 1 pellets in a day, although this was **unusual**.

According to the simple addition rule of Probability Theory,
you can find the probability of his consuming either 0 or
1 pellet by **adding** the probabilities of each of these

outcomes. Our estimate of the probability of eating

0 pellets is .05, and of eating 1 pellet, .15. Thus, the

probability of a healthy monkey eating 0 or 1 pellets is

_____ + _____, or _____.

.05, .15, .20

132. We can also list things in a pair of columns placed side by side. For example, consider the following:

COLUMN ONE	COLUMN TWO
red	brown
green	orange
blue	pink

Thus, the colors appearing in the first column are red, green, and blue and those appearing in the second column are _____, _____ and _____.

brown, orange,
pink

133. We could also think of the same arrangement of colors as being made up of three rows. For example:

ROW ONE	red	brown
ROW TWO	green	orange
ROW THREE	blue	pink

Thus, the colors appearing in the **first** row are red and brown, while the colors appearing in the _____ row are blue and pink.

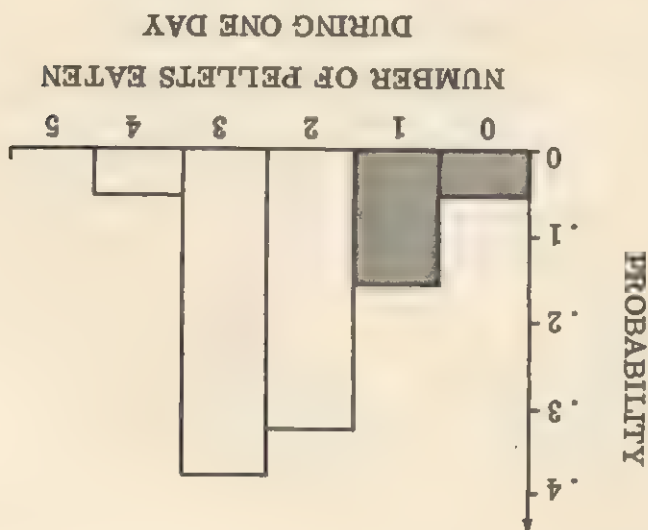
third

134. A table is an arrangement of things in rows and columns. The arrangement of colors we just considered is a table because it was formed by $\frac{2}{3}$ rows and $\frac{2}{3}$ columns.

3, 2

This character of the preceding **decision rule** can be illustrated by the following probability distribution.

Experimental sampling distribution of a healthy monkey's eating behavior (represented as a probability distribution).



Notice that we have shaded columns representing the probability of each type of observation that would cause you to accept/reject the assumption that the animal is well. However, these probabilities describe the behavior of the healthy animal.

The unshaded columns represent the probabilities of those observations that would lead us to accept the hypothesis that the animal was healthy. (Once again, these probabilities describe the eating behavior of a healthy animal.)

Notice the preceding probability distribution indicates the probability that the animal will consume any particular number of pellets (as shown along the base of the graph). Since we assumed the monkey's eating

135. The **table** of numerals shown below consists of $\frac{2}{3}$ columns and $\frac{2}{3}$ rows.

3

2

	COLUMN ONE	COLUMN TWO	COLUMN THREE
ROW ONE	8	4	9
ROW TWO	6	5	4

Notice in the table of numerals that the numeral 8 is located in the first row AND the first column. The numeral 6 is also located in the first column but in the $\frac{1}{2}$ row.
first/second

second

136. Every numeral in the table falls in a particular combination of row and column. For example, the numeral 9 is located at the intersection of the third column and the first row.

9

The numeral 5 is located in the second row and the second column.

second

second

Notice that the numeral 4 is located in the first row, second column and also in the second row, third column.

second

third

137. In each of the tables we have seen, "row one" was the row on the $\frac{1}{2}$ and "column one" was the $\frac{1}{2}$ column on the $\frac{1}{2}$. This is the way we will always number the rows and columns in a table.

top

left

151. The error you would be making in the previous frame would be a Type $\frac{\text{One/Two}}$ error.
152. If you **reject** an hypothesis when it is really true, you make a Type $\frac{\text{One/Two}}$ error. If you **fail to reject** an hypothesis when it is really false, you make a Type $\frac{\text{One/Two}}$ error.
153. Notice that the Type One error in the previous frame has the **single** key word **reject** in bold type. The Type Two error in the previous frame has the two key words **fail to reject** in bold face type. Thus, to **reject** when you shouldn't is a Type One error since the **single** key word is **fail to reject** when you should reject is a Type Two error since the two key words are **fail to reject** and _____.
154. One of the most important characteristics of a **statistical decision rule** is that you can often determine the probability of making a Type One error. In other words, you can determine the proportion of times you would expect to **erroneously** reject an hypothesis because an unusual event had been observed which you viewed as inconsistent with this hypothesis.

138. It is often useful to present or record data in the form of a **table**. Suppose you were interested in how a person who took a three-day trip spent his money for food. You could record your observations of this "food cost" variable in a table like the one that follows.

	THURSDAY	FRIDAY	SATURDAY
Breakfast	\$1. 00	\$1. 15	\$1. 50
Lunch	\$2. 50	\$3. 00	\$1. 50
Dinner	\$4. 00	\$4. 15	\$5. 00

Notice that instead of numbering rows and columns, we have identified the _____ with the various meals

rows

of the day, while the _____ are identified with

columns

particular days of the trip. The names of the meals and the names of the days identify the names of the data in the columns and the rows. They are not themselves part of the data, since you _____ have written these row and column headings before you actually made the observations.

could

139. The row and column headings **identify** the data; they _____ part of the data. Since "breakfast" is the _____, are/ are not .
- row heading of the first row and "Friday" is the column heading of the second column, the entry in the first row and second column of the data (\$1. 15) is the amount spent for _____ on _____.
- The amount spent for dinner on Friday was \$ _____, while the amount spent for _____ on _____ was \$5. 00.

are not

breakfast, Friday

4. 15

dinner, Saturday

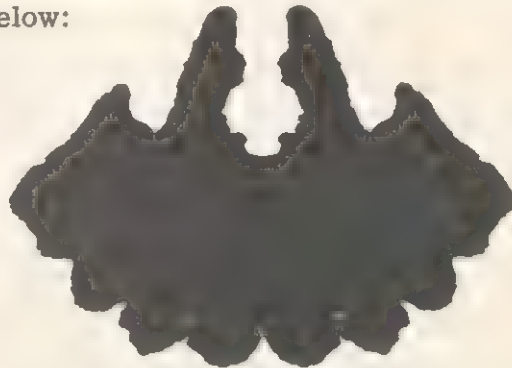
147. We said that a Type One error was **rejecting** an hypothesis which was really false/true.
148. Accepting an hypothesis that was really true or rejecting an hypothesis that was really false would/would not be an error.
149. Suppose you tried to use a cigarette lighter and snapped the lever on the lighter several times without lighting the wick. You might decide there was something wrong with the lighter. Even when a lighter is working perfectly, you don't necessarily obtain a flame every time you snap the lever. Consider the following hypothesis.
- Hypothesis:** There is nothing wrong with the lighter.
- If the hypothesis is true, then you were simply unlucky the first few times you tried to operate the lighter. If you decided the lighter was broken (or needed lighter fluid) simply because you hadn't ignited the wick, you would be accepting/rejecting the previous hypothesis when it was actually true/false.
150. If you decided the lighter wasn't working properly when it actually was (simply because you were unlucky and hadn't got a flame the first few times you snapped the lever), you would be incorrectly/correctly rejecting the hypothesis that there was nothing wrong with the lighter.

140. How much money was spent for all three meals on Saturday? \$ _____ 8.00

How much was spent for all three lunches? \$ _____ 7.00

The day he spent the most money on breakfast, lunch and dinner was _____. The most expensive meal on each of the three days was _____. Friday dinner

141. You may have heard of a psychology test called the **Rorschach test**, a test in which people are asked to report their impressions of ink blots, like the one shown below:



This ink blot was formed by spilling ink onto the center of a piece of paper and then folding the paper in half so that the ink is squeezed or blotted between the folds of the paper. When Dr. Rorschach invented this test, he believed the shape of the blot was so vague that a person's answers would reflect as much about the person as about the ink blot. Suppose you were developing a test of this sort. You might make four different ink blots and show all of them to ten people. You could ask each of the people to report whether they thought a particular ink blot gave them an impression of something that was "pleasant," "neutral," or "unpleasant." Thus, their answer would be a

_____ variable whose three
numerical/non-numerical
possible values were " _____ "
" _____ " and " _____ ."

non-numerical

pleasant

neutral,
unpleasant

143. If you fail to reject an hypothesis when the hypothesis is actually false, you would be making a Type Two error.

Suppose you decided your friend's phone was working perfectly and you had simply been unlucky enough to call him when he was using the phone. If his phone were actually working perfectly, you would be accepting/rejecting the hypothesis that it was working perfectly when it actually was/was not. Therefore, you would not be making a Type Two error.

144. If you decided that you had just been unlucky and that your friend's phone was really busy when, in fact, it was out-of-order, you would have accepted/rejected the hypothesis (the hypothesis that his phone was working perfectly) when you should have accepted/rejected it in order to have been correct.

145. If you assumed his phone was working properly when it was actually out-of-order, you would have failed to reject the hypothesis when it was actually false. This would be an example of a Type One/Two error.

(Remember, a Type One error is rejecting when you shouldn't reject, while a Type Two error is failing to reject when you really should reject.)

146. We said that a Type Two error was failing to reject an hypothesis when the hypothesis was really true/false.

142. You could record the answers of the subjects in a table which might look like this:

SUBJECT	INK BLOT A	INK BLOT B	INK BLOT C	INK BLOT D
1	pleasant	pleasant	neutral	pleasant
2	neutral	neutral	neutral	pleasant
3	neutral	pleasant	neutral	pleasant
4	neutral	unpleasant	neutral	pleasant
5	pleasant	neutral	pleasant	pleasant
6	unpleasant	unpleasant	unpleasant	unpleasant
7	neutral	neutral	neutral	pleasant
8	pleasant	neutral	neutral	pleasant
9	neutral	neutral	neutral	pleasant
10	pleasant	pleasant	pleasant	pleasant

143. Notice that "INK BLOT A" is a _____ heading and _____ column
 "SUBJECT 1" is a _____ heading. _____ row
 row/ column
144. The row and column headings _____ data, whereas _____ are not
 are/ are not
 the terms "pleasant," "neutral," "unpleasant" in the
 table _____ data (since they are observed values _____ are
 are/ are not
 of the "answer" variable).
145. The _____ headings identify who made the answer, _____ row
 row/ column
 while the _____ headings identify the ink blot _____ column
 row/ column
 being shown when the answer was made.

On the other hand, if the animal were really sick but happened to eat two or more pellets, you would fail to reject the hypothesis that he was well and you would be making a Type $\frac{\text{One/Two}}{\text{error}}$.

Two

141.

Suppose you tried to telephone a friend several times during the day and you received a busy signal each time. Consider the following hypothesis.

Hypothesis: Your friend's telephone is working properly and the phone was actually busy each time you attempted to call.

If you decided that it was so unusual for his phone to be busy every time you tried to call that you would assume his phone was actually out-of-order, you would be accepting/rejecting the previously stated hypothesis.

rejecting

142.

If you **reject** an hypothesis when the hypothesis is actually **true**, you would be making a Type One error. Suppose you decided your friend's phone was out-of-order when it was actually working perfectly (because you were unlucky enough to call him several times when he was using his phone). You would be accepting/rejecting the hypothesis (stated in the previous frame) when the hypothesis was true/false. Therefore, you would be making a Type One error.

would

true

rejecting

146. All the answers in the first column of the table were made in response to ink blot $\frac{A}{D}$, whereas all the answers in row 1 were made by subject $\frac{1}{10}$. Subject 1 thought blot A was pleasant, blot B was pleasant, blot C was _____, and blot D was _____. Subject 4 thought blot B was unpleasant, but he thought blot _____ was pleasant. Subject 7 thought blot D was _____.
147. Since the answers from all ten subject to ink blot A are in the first column, we could determine how many subject reported ink blot A was "pleasant" by counting all the occurrences of " _____ " in the _____ column.
148. Four subjects reported ink blot A seemed "pleasant," whereas _____ subjects reported that ink blot B seemed "pleasant."
149. Subject 1 felt two of the blots were pleasant, whereas subject 2 saw only _____ of the blots as pleasant.
150. What if you were told that one of your subjects was severely depressed and had been under treatment by a psychotherapist. You might expect someone who was very depressed or sad to find _____ ink blots "unpleasant" than would a typical subject.
151. You would compare different _____ of the table in order to compare how different subjects reacted to the ink blots.

A

1

neutral
pleasant
D
pleasant

pleasant, first

three

one

more

rows

136. Notice that two of these four consequences would represent mistakes (errors), whereas the other two consequences would represent **correct decisions**. For example, outcome 1 would represent a correct/incorrect decision, since we accepted the hypothesis that the monkey was well when it actually was well/sick.

137. The other possible way of being **correct** was to reject the hypothesis that the animal was well when he actually was/was not well.

138. The two outcomes of our decision making, which represent **errors** would be to accept/reject the hypothesis that he was well when he was actually sick, or to accept/reject the hypothesis that he was well when he was actually well.

139. Whenever you attempt to decide whether to accept or reject a particular hypothesis, you run the risk of making two kinds of errors. You can _____ the hypothesis when it is really false, or you can _____

140. Rejecting an hypothesis which is really true because you were unlucky enough to observe a highly unusual event (highly unusual when the hypothesis is true) is often referred to as a **Type One** error. The other sort of error, failing to reject a hypothesis which is really false, is called a **Type Two** error. Thus, if the monkey were actually well but ate such an unusually small number of pellets that you decided he was sick, you would have made a **Type _____ One/Two** error.

152. If you were looking for a subject who seemed to find an **unusual** number of the blots unpleasant, you would probably pick subject _____ since this subject found _____ of the blots unpleasant. 6
all (4)
153. There might be something about particular ink blots which really does make them seem more or less pleasant. If you had to pick out the ink blot which might really appear more pleasant than the rest, you would probably pick ink blot _____, since more subjects found this ink blot pleasant in terms of the other three ink blots. D
154. Notice in the table of data from the ink blot study how the column headings refer to particular ink blots. We could think of each of these four ink blots as a particular value of a variable, which we might call the "ink-blot" variable. Thus, each column of the table would be identified with a particular _____ of the value
"ink-blot" variable.
155. Similarly, each row is identified with a particular subject. We could think of the subject number as a particular value of a variable we might call the "subject" variable. Thus, a particular row would be identified with a particular _____ of the value
"subject" variable.

132. Let's suppose that you determine to use the following

rule for making your decision:

Decision Rule

If the animal eats fewer than 2 pellets you will reject the assumption that he is well and destroy him.

In other words, if the number of pellets consumed by

the animal today is _____ or _____, you will consider this amount sufficiently unusual for a healthy animal to reject the hypothesis that he is well.

133.

Let's consider the possible consequences of this type of decision rule. First of all, the hypothesis can either be true or not true. In other words, the animal can either

be infected or _____ infected.

not

134.

The two possible decisions we might make on the basis

of our observation are to either _____ or to _____ the hypothesis that the animal is healthy.

accept
reject

135.

Therefore, there are four possible consequences of our decision:

1. The animal could be well and we could accept the hypothesis that he was well.

2. The animal could be well and we could _____ the hypothesis that he was well.

reject

3. The animal could be sick and we could accept the hypothesis that he was well.

sick

4. The animal could be _____ and we could reject the hypothesis that he was well.

156. When we draw the table, we might include the name of the variable whose particular values are associated with each row. For example, consider the following table:

Subject	INK BLOT			
	A	B	C	D
1	pleasant	pleasant	unpleasant	unpleasant
2	pleasant	neutral	unpleasant	unpleasant
3	unpleasant	neutral	pleasant	neutral
4	neutral	neutral	neutral	unpleasant

In this table, the ink blots can be thought of as a variable, whose particular values are A, B, _____, and _____.

C
D

Each value of the ink-blot variable is identified by a particular _____.

column

157. You could also think of the subject to whom the ink blot was presented as a variable. Therefore, each particular subject is a particular _____ of that variable. The word "subject" in the table we have just seen refers to a variable, and the numerals 1, 2, 3, and 4 identify particular _____ of that variable.

value

values

158. The table is useful in organizing the data in terms of the subject variable and the ink-blot variable. The _____ are associated with particular ink blots and the _____ are associated with the _____ particular subjects.

columns

rows

Notice that the height of each column in the preceding graph represents the _____ that the animal would consume a particular number of pellets on any randomly chosen day.

129. The number of pellets with the largest probability would be a typical number of pellets for the animal to consume on any randomly chosen day. This number of pellets is _____.

130. On the other hand, the preceding probability distribution indicates that it would be quite _____ for the $\frac{\text{usual}}{\text{unusual}}$ animal to consume 0 pellets, since the probability of its eating that many pellets is only $\frac{.05}{.50}$. In other words, you would expect it to consume 0 pellets of food on only about _____ out of every 100 days.

131. We expressed the experimental sampling distribution as an estimated probability distribution on a sample space consisting of each possible number of pellets the animal could consume. The problem confronting us is to decide whether the number of pellets he consumes today is so unusual (in light of his previous eating habits) that we should $\frac{\text{reject/accept}}$ the hypothesis that he is well and destroy him in order to protect the other laboratory animals.

159. If you were interested in comparing one subject's responses with another subject's responses, you would compare rows/columns. On the other hand, if you were interested in comparing the responses to a particular ink blot with the responses to another ink blot, you would compare rows/columns.
160. For example, in the table we just saw, it is a simple matter to determine **which ink blot** was found unpleasant by the most subjects. Since you are comparing different ink blots, you would compare different columns. The column that contains the most "unpleasant" responses is D. On the other hand, to find **which subject** made the most "neutral" responses, you would compare different rows. You would find that subject rows, 4 was the one who had made the most "neutral" responses.

161. Suppose you were interested in comparing the grades made by four students in three different courses. We could arrange their grades in a table, as follows:

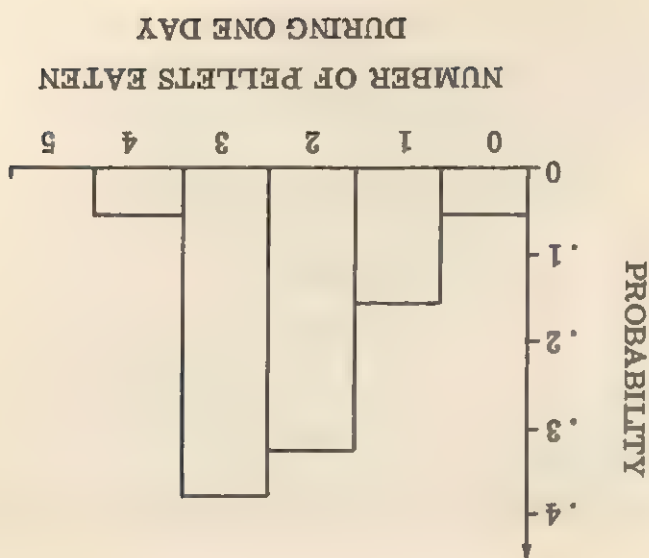
STUDENT	COURSE		
	Math	English	Gym
A. M.	A	A	B
J. T.	B	C	D
A. C.	A	A	D
R. K.	D	B	C

DATA ON GRADES IN VARIOUS COURSES

Notice the rows are identified by the initials of particular subjects/courses, whereas the columns are identified with particular subjects/courses.

Let's assume that the animal's daily food consumption on the previous 100 days **could** be regarded as a simple random process. We could use the proportions in the third column of the previous table as **estimates** of the proportion of days on which the animal would eat each possible number of pellets if we observed him for an unlimited number of days. In other words, we could use each proportion shown in the previous table as an **estimate** of a probability — the probability that the healthy animal would consume a particular number of pellets on any one day. Thus, we could use the following **probability distribution** to represent the animal's eating behavior based upon observations made on the preceding 100 days.

Experimental sampling distribution
of a healthy monkey's eating behavior
(represented as a probability
distribution).



162. The word "course" is written at the top of the table and might be thought of as the name of a variable whose particular values correspond to the particular _____ in the table. You could think of _____, _____, and _____ as particular values of the "course" variable. columns
Math, English, Gym
163. We identify each student by an initial. Thus, if we think of students as a variable, the particular values of that variable are _____, _____, _____, and _____. A. M., J. T., A. C., R. K.
164. You would find the course in which the students did most poorly by comparing different _____. You would find the student who had done the best in his courses by comparing different _____. columns
rows
165. In the preceding table, the variable we are interested in is "grade." The different values of that variable are A, B, C, and D. The data presented in this table are _____ observed values of the grade variable. 12
12/4
166. Earlier we considered a way in which you might keep track of your improvement as you took bowling lessons. After each lesson, you could bowl 5 balls and record the number of pins you knock down with each ball. For the purposes of this test, you always set up any pins you knock down before you bowl the next ball. Therefore, the fewest pins you could knock down with any one ball are _____, while the greatest number of pins you could knock down with one ball is _____. 0
10

121.	The entries in the second column of the previous table indicate on how many of the 100 previous days the animal had eaten each of the possible number of pellets shown in the first column. For example, the first row of the table indicates that on 5 of the previous 100 days, the animal had eaten _____ pellets.	0
122.	On 35 of the previous 100 days, the animal had eaten _____ pellets.	2
123.	The modal number of pellets consumed daily was _____, since this many pellets had been consumed on _____ days.	3
124.	Three pellets is the modal number of pellets consumed daily, since this was the most frequently occurring daily pellet consumption within the last _____ days.	100
125.	In the third column of the above table, we have indicated the _____ of the previous 100 days on which the animal consumed each of the number of pellets shown in the first column of the table.	proportion
126.	Since the animal had eaten 0 pellets on 5 of the previous 100 days, we can say the proportion of days on which he ate 0 pellets was _____ (as indicated in the third column of the table).	.05
127.	Since the animal ate 3 pellets on 40 of the previous 100 days, we could represent the proportion of days on which he consumed 3 pellets as _____, as the fraction _____/100, or as the percentage _____%.	.40 40/100, 40

167. Suppose you took 4 lessons and bowled 5 of these test balls after each lesson. Imagine you recorded the data from these tests in the following table.

BALL	LESSON			
	1	2	3	4
1	0	0	5	8
2	0	8	4	2
3	3	0	8	5
4	2	2	10	10
5	1	6	6	10

DATA FROM TESTS OF IMPROVEMENT

Since your data are observed values of the "pins-knocked-down" variable, each value is identified with a particular ball you rolled following a particular lesson. In this table, the 5 test balls you rolled are associated with 5 different _____, while the
rows/columns

4 lessons that you took are associated with 4 particular
_____.
rows/columns

ROWS

columns

168. The total number of pins knocked down following the first lesson was $\frac{\quad}{6/10}$.

6

169. The total number of pins knocked down following the second lesson was _____. The most pins were knocked down following lesson _____, since there were _____ pins knocked down with the 5 balls rolled on that day.

10

4

35

Your decision-making strategy could be summarized as follows: in order to decide whether or not to accept the hypothesis, you will make an **observation** and then decide whether this observation would be such an unusual event if the hypothesis were true, that you would accept/reject the hypothesis.

In order to determine what would be an **unusual** amount of food for the monkey to eat in one day, you referred to your records of his previous eating behavior.

Let's suppose your experimental animal ate large pellets of food, which were made available to him each day in the experimental testing chamber. In each of the previous 100 days, he had eaten somewhere between 0 and 5 pellets. The first column of the following table lists the _____ of pellets the monkey could have eaten on each of these 100 days.

Eating Behavior of a Healthy Experimental Monkey over a 100 day Period		
Number of Pellets Eaten	Number of Days	Proportion of Days
0	5	.05
1	15	.15
2	35	.35
3	40	.40
4	55	.05
5	0	.00

170. We have now looked at several tables and can summarize certain things about tables. First of all, the definition of a table is "any arrangement of things in _____ and _____." When the things you are arranging in rows and columns are data, the row and column headings help you to identify each particular piece of data, but they _____ part of the data. rows, columns are not
171. It is often useful to think of the row and column headings as particular values of a _____. variable
172. One of the first things you should learn to look at when you see a table of data are the row and column headings. These headings identify where the observed values that make up the _____ were obtained. data
173. For example, the table shown below contains data from two different subjects: subject _____ and subject _____. These two subjects were observed on **four** different _____. A, B days

DAY	SUBJECTS	
	A	B
1	10	5
2	6	8
3	9	2
4	4	1

other hand, if you accepted/rejected the hypothesis, rejected
 you should destroy your experimental animal in order to prevent the spreading of the disease to other animals in the laboratory.

116. The problem of deciding whether the animal was ill was a difficult one, since the only noticeable symptom of the illness during the first two days of infection was a lowering of the animals' normal food consumption. In other words, if your experimental animal suddenly began to eat less than he normally did, you would probably feel inclined to accept/reject the hypothesis that he was healthy.

117. Since the experimental animal was already in the testing chamber, you did not have to worry about his infecting any of the other animals until he was removed at the end of the day. Therefore, you could wait until the end of the day and observe how much food he ate that day before deciding whether or not he should be destroyed. If the animal ate an **unusually small amount** of food that day, you could accept/reject the hypothesis and destroy the animal.

118. On the other hand, if the amount of food consumed by the experimental animal that day was **not unusual** (i.e., if he ate a normal amount of food), you might be willing to accept/reject the hypothesis and continue with the experiment.

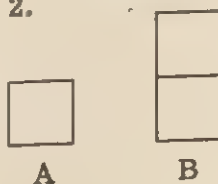
174. The data in the table below is the same as/different from the preceding table, since the data below was obtained from four _____ on two different _____. different from

subjects, days

SUBJECTS	DAYS	
	A	B
1	10	5
2	6	8
3	9	2
4	4	1

This illustrates how carefully you should examine the row and column headings on a table. Many errors in interpreting data can be traced to a simple failure to pay sufficient attention to the headings on the table.

175. You have seen that one way of presenting or recording a list of values for some variable is to arrange the names of these values in _____ and _____. rows, columns
176. Next, we will consider a way of representing values of a variable other than simply arranging the names of the values in rows and columns to form a _____. table
177. If you represented the numeral 1 with a square, you could represent the numeral 2 by putting another square on top of it. Thus, drawing $\begin{array}{c} \square \\ A, B \end{array}$ shown below would A
represent numeral 1, whereas drawing $\begin{array}{c} \square \\ A/B \end{array}$ would B
represent numeral 2.



is sufficiently unusual to allow you to accept/reject the assumption (hypothesis) that the sample came from this particular population.

114. Let's consider another example of the kind of reasoning

which takes place in a **statistical hypothesis test**.

Suppose you were a psychologist conducting research on monkeys and had used a particular animal in an

experiment for over 100 days. One day, while the

monkey was in the experimental testing chamber, the

animal keeper for the laboratory approached you with a

problem. Since several monkeys in the animal colony

had contracted a highly contagious disease, it would be

necessary to destroy any animals who appeared to have

contracted the disease. The problem you faced as an

experimenter was to decide whether to destroy/save

your experimental monkey (which would disrupt your

experiment and waste the data you had already collected)

or to save/destroy the monkey and run the risk of having

save

destroy

the disease spread to the other laboratory animals.

115.

In other words, you are faced with the problem of deciding whether to accept or reject the following

hypothesis:

The experimental monkey is healthy (he has

not contracted the disease).

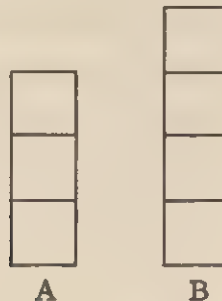
If you accepted/rejected the hypothesis, you could

accepted

continue using the animal in the experiment. On the

178. By adding a third square on top, you could represent the numeral 3. In fact, you could keep adding squares, one on top of the other, for each new numeral you wished to represent. For example, of the two pictures shown below, Picture A would represent the numeral 3, whereas Picture B would represent the numeral _____.

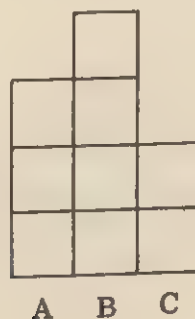
4



179. You can think of these squares placed atop each other as forming columns, and you could place these columns side by side as shown below. Each column represents a numeral in the same way as the columns of squares you just considered. In the picture shown below, we have identified each column with a letter placed directly underneath the column. Column A represents the numeral "3" since this column is three squares high. Column B represents the numeral "____" since this column is four squares high, and column C represents the numeral "____".

4

2



unlikely

While it would be possible to obtain 100 heads in 100 tosses of a fair coin, this sequence of events is so unlikely that you would probably reject the idea that the coin was fair. Instead, you would accept the alternative idea that the coin was biased.

observed

111. In each of the previous illustrations, someone was faced with the following type of problem. Would what has been observed be a highly **unusual** event if a particular assumption were true? In other words, if a particular assumption were true, how unusual would it be for what had been _____ to have occurred?

112. If the observation would have been a highly **unusual** event were assumption true, you would tend to accept/reject the assumption.

113. This reasoning procedure can be contrasted with the type of statistical inference we made earlier when we attempted to **estimate** the value of a population statistic on the basis of a sample. Here we make an assumption called an **hypothesis** concerning the population and then ask whether or not the sample we obtained would be so **unusual** a sample from this type of population that it is possible to calculate theoretical sampling distributions for random samples obtained from particular types of populations. Since a sampling distribution indicates how often each particular type of sample can be expected to occur, it indicates how **unusual** it would be to obtain a particular type of sample. The sampling distribution indicates whether or not a particular sample

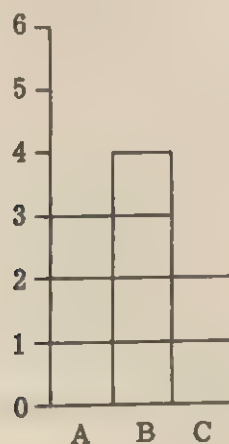
179. (Continued)

The width of each column is the same, but the height differs depending upon which numeral the column represents. In other words, the $\frac{\text{width}}{\text{height}}$ of the column determines which numeral it represents.

height

180. The same three columns are shown below. Now, however, we have drawn a line with marks on it to represent the different possible heights of the column. For example, the mark next to the numeral 1 is at the same height as a column one square high. The mark next to the numeral 2 is at the same height as a column two squares high, and the mark next to the numeral 3 is as high as a column ____ squares high.

3



181. Notice Column A is 3 squares high. This is indicated by the fact that it is the same height as the mark next to the numeral ____.

3

107. This problem of choosing between two or more alternative interpretations of a collection of data is a fundamental problem of statistical reasoning. The chief contribution of statistical procedures in reaching decisions of this sort is the ability to specify precisely how unlikely a particular observation would be if a certain interpretation were, in fact, correct. For example, you have seen that it is possible to calculate the probability for obtaining any particular number of heads in a particular number of tosses of a *fair* coin. Suppose you were faced with the problem of deciding whether a particular coin were fair or biased — in other words in deciding whether or not the probability of obtaining heads on each toss of the coin was _____ or not. Suppose you tossed the coin twice and observed that both times it came up heads. The proportion of heads in your sample would be one, which could lead you to suppose that the coin was biased. On the other hand, the probability of obtaining exactly two heads in two tosses of a coin is one-fourth. Thus, you would expect to obtain two heads in about one out of every _____ pairs of tosses of a *fair* coin.
108. You would be _____ likely/unlikely to decide that the coin was biased, since two heads in two tosses of a fair coin was a common event.
109. However, suppose you tossed the coin 100 times and observed that the coin landed heads each time. This kind of a sample _____ be possible if the coin were fair, even though it would be very unlikely,

would

unlikely

four

one-half

182. Although Column B is not touching the line with the marks on it, you can see that Column B is 4 squares high and, therefore, at the same height as the mark next to which the numeral ____ is written. 4

183. The mark next to the numeral 2 is at the same height as the top of Column A/B/C since that column is two squares high. C

184. Suppose you were tossing a die (one of two dice). You might be interested in the number of dots shown on its top when it came to rest. For example, the following die has ____ dots showing on its top surface. 2



The number of dots appearing on the top of the die after each new toss would/would not be a variable since this would

number could change or vary from toss to toss. The possible values of this variable could be represented by the numerals "5," "4," "2," "3," "1," and "____." 6

We could represent each possible number of dots showing on the top of the die by a column of squares.

very unusual event. In other words, while it was possible that the television set had failed, you would view this as such a(n) usual/unusual event, in light of your previous experience, that you would **reject** this interpretation in favor of the alternative interpretation that you had been sleeping for a long time and that the television station had gone off the air at its normal time of 2 a.m.

105. Let's consider a third example of the same kind of reasoning process. Suppose you replaced a bad

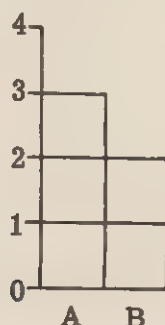
light bulb on your porch. Shortly after you put the new bulb in, it too apparently burned out. Shortly after you replaced the second bulb, a third bulb also burned out. At this point, you decided that it probably was the light fixture itself which was at fault, since it didn't seem reasonable that three light bulbs would all be faulty. In other words, even though there is some possibility that you could have been unlucky enough to have selected three faulty bulbs, you view this possibility as being likely/unlikely. You **rejected** this idea in favor of the alternative assumption that something was wrong with the light fixture itself.

106.

In each of the preceding examples, you were faced with the problem of choosing between two alternative interpretations or assumptions. Both of the alternative interpretations were consistent with what you had observed in each illustration. However, you were able to reject one of the two interpretations in each case, on the grounds that what you had would have been highly unlikely if that particular interpretation had been true.

For example, we could represent the die falling so that 3 dots were on top by column A/B shown below:

A



185. Since the largest number of dots we could observe on the top of the die would be _____, the highest possible column of squares we would need would be _____ squares high. 6 6
186. Since the fewest dots we could observe on the top of the die would be a single dot, we could represent this outcome by a column _____ square high. 1
187. Suppose you tossed a die four times and recorded the number of dots showing after each toss, your results might be like those shown in the following table.

TOSS	DOTS SHOWING
First Toss	5
Second Toss	3
Third Toss	6
Fourth Toss	1

On the other hand, even if they had fixed the tire, there is some possibility that between the time the tire was fixed and the time you returned that evening, a new injury to the tire had taken place. This second interpretation would be perfectly consistent with what you had seen. However, you probably would reject this interpretation because the number of times that a newly repaired tire would suffer a second flat in such a short while is so frequent/inrequent that it would be more reasonable to assume that the tire had not been repaired.

As another illustration of this kind of reasoning process, imagine that it was the night before your final Statistics examination and that you decided to watch television briefly before studying for the exam. Suppose you suddenly awoke later in the evening, having fallen asleep in front of the television set. Glancing at the screen, you notice that although the set was still turned on, the screen was filled with snow, much as it would if the station had gone off the air. If you had no idea how long you had slept and if there were no clock available, you might immediately draw the conclusion that it was at least 2 a.m., since the television station normally goes off the air at 2 a.m. after the late, late show. However, an alternative interpretation of the same evidence would be that you had not slept that long but rather that something had happened to your set which caused the picture to disappear. **Both** of these interpretations would be **consistent** with the evidence (your observation of the set). However, you would probably **reject** the idea that the television set had failed since this is a

187. (Continued)

On the first toss, there were 5 dots showing, on the second toss there were 3 dots showing, on the third toss there were _____ dots showing, and on the fourth toss there was _____ dot showing.

6
1

188. Notice how the data in the previous table are represented by a of numerals indicating the different column
column/row
observed values of the variable.

189. The column heading "dots showing" can be thought of as the name of the variable we are observing and the numerals in that column as the observed
values of that variable. observed/possible

190. Notice that we have used numerals to represent the different observed values of the variable. These numerals are simply names for the values. Another way of representing these values is with columns of squares that form a sort of **picture** of the data.

Four columns are shown on the following page. Each column represents one of the observed values in the previous table. We have identified the particular toss represented by each column by writing its name directly below that column.

When an experimental sampling distribution is expressed in terms of the relative frequency (proportion) of each type of sample, you can think of each proportion as an estimate of the probability of obtaining each particular type of sample. Thus, if you obtained 100 samples by a particular sampling procedure and if you observed that eight-tenths of these samples had the same mean, your best **estimate** of the probability of obtaining that particular type of sample in the future would be

So far, we have considered the problem of **estimating** population statistics on the basis of samples. We have considered which sample statistics provide the most appropriate estimate of the population statistic and also the manner in which you can specify your confidence in the accuracy of the estimate. Next, we shall consider a type of statistical reasoning called **hypothesis testing**, which plays a very important role in research. While some of the more advanced techniques of hypothesis

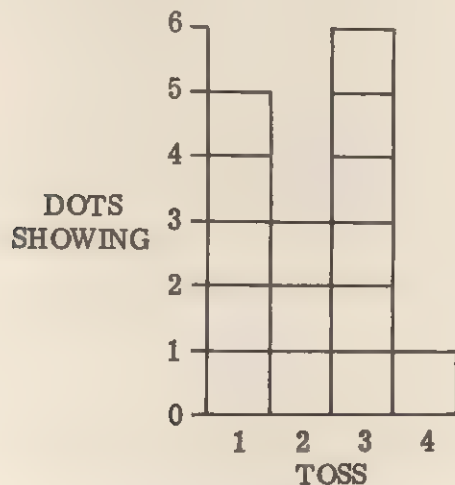
testing can become quite complicated, it is important to realize that the basic procedure of hypothesis testing is almost identical to a very simple reasoning process that you probably use every day.

An illustration of this very common reasoning process is provided by the following example. Suppose you left your automobile at a service station in order to have a flat tire repaired. Upon your return later in the day, you notice the car parked in front of the station with the same tire flat. Your obvious conclusion would be that they had/had not fixed the tire.

The column for the 3rd/4th toss is 6 blocks high since
6 dots were showing on the _____ toss.

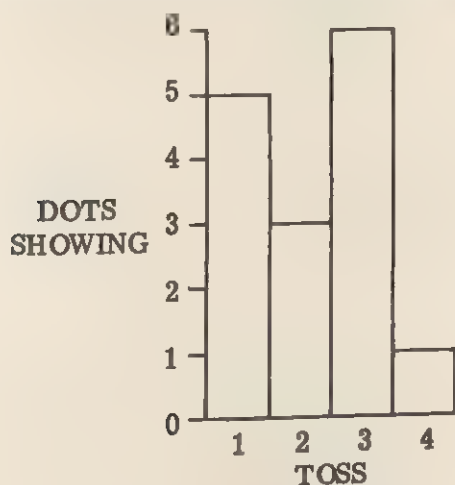
3rd

3rd



191. Since the line next to the column on the left has marks to indicate the different possible heights of the columns, we really don't need the lines between each square in the column.

We have redrawn the columns below, leaving out the lines separating each square in a particular column.



We can still see that the third column represents the numeral 6 since the third column is the same height as the mark next to the numeral _____.

equivalent to asking what proportion of times would the batter obtain a hit in an unlimited number of times at bat. In this sense, we can view any actual series of times at bat as a _____ drawn by a random procedure from an infinite population consisting of an unlimited number of times at bat.

98. We can view the proportion of hits in any series of times at bat as an **estimate** of the proportion of hits that would have been observed in an unlimited number of times at bat. In other words, we can view the sample proportion as an estimate of a _____ proportion.

population

99. According to the law of large numbers, the more times the player is at bat, the better estimate we will have of the population proportion — since the sample proportion will become more and more similar to the true population proportion. In this sense, a baseball player's batting average (which is simply the proportion of times he has received a hit in previous times at bat) could be viewed as an _____ of the **probability** of his obtaining a hit each time he came to bat.

estimate

100. This is similar to the kind of reasoning behind a statement such as, "The **probability** that it will rain today is .8." This statement really means that it rained on eight-tenths of all the previous days on which conditions were the same as they are today. This means that we view the process which determines whether or not it will rain today as a random process. Our previous observations of this process lead us to estimate the probability of rain as .8 since _____ of all the previous outcomes under these conditions have been "rain."

.8

192. A figure of this sort where the values of a variable are represented by the height of each column is a type of **graph**. In this graph, the value of each variable is represented by the _____ of each column rather than by a name or numeral, as in the previous table.

height

193. When you roll a **die**, there will be a certain number of dots showing on its top face when the die comes to rest. For example, the die shown below has _____ dots showing on its top face.

6



194. Suppose you tossed a die four times and recorded the observations shown in the following table:

TOSS	DOTS SHOWING
1st	3
2nd	2
3rd	1
4th	3

According to this data, the **fewest** number of dots were showing on the _____ toss.

third

94. Similarly, when we considered drawing a single opinion from the population of student opinions at a particular college, you saw that the probability of drawing a favorable opinion equaled the _____ of favorable opinions in the population.

95. The proportion of observations having a particular value in a random sample is your best estimate of the proportion of times that value occurs in the population. Therefore, the _____ of times that a single observation drawn randomly from the population will have a particular value is equal to the proportion of times that value occurs in the population. Therefore, the _____ of times a particular value occurs in a random sample is your best estimate of the probability that a single observation drawn randomly from the population will have a particular value is equal to the proportion of times that value occurs in the population.

96. In other words, the sample proportion is your best estimate of the probability of sampling that particular value from the population. For example, we considered how any series of tosses of a coin could be regarded as a sample from a population consisting of an unlimited number of tosses of a coin. We agreed that if the coin were fair, the proportion of times it would fall heads in an unlimited number of tosses would be _____ . Therefore, we would say the probability of obtaining a head on any single toss was equal to _____ .

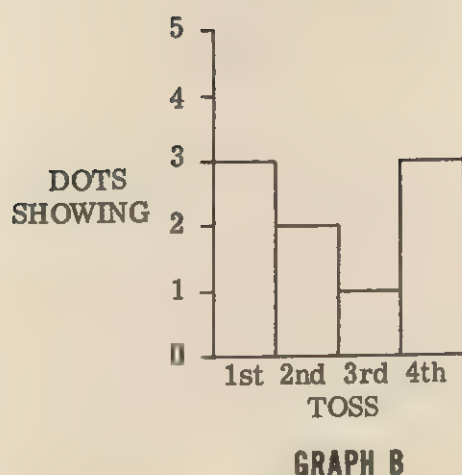
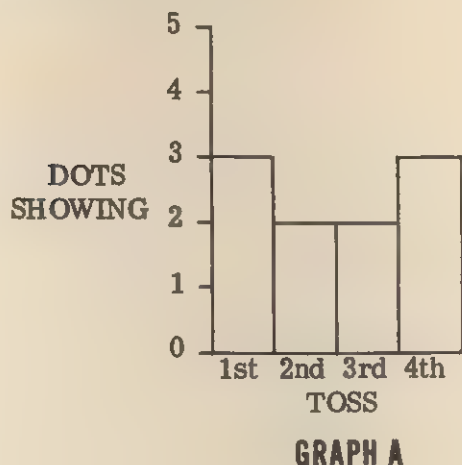
one-half

one-half

97. Similarly, we considered how it might be possible to represent a baseball player's turns at bat as a series of successive outcomes of a random process. If we only considered 2 outcomes (a hit or no hit), we could ask the question, "What is the probability that the batter will obtain a hit on any particular time at bat." This is

195.

Two graphs are shown below.



In the previous table it was indicated that there were 3 dots showing on the first toss. Both Graph A and Graph B indicate there were _____ dots showing on the first toss because the column representing the first toss in both graphs is _____ squares high.

3

3

196.

Compare the data shown in each of the previous graphs with the data shown in the previous table. Graph A/B agrees perfectly with the table, whereas graph A/B does not.

B

A

90. Even if we supposed that the sampling procedure was random, we still don't know the size of the sample. According to the law of large numbers, you would expect a $\frac{\text{large/small}}{2}$ random sample to be more representative of the population than you would a $\frac{\text{large/small}}{\text{random}}$ sample.
91. Variability in sample proportions would tend to be less with large samples than with small samples.
- In other words, if the sampling procedure were random, you would expect the variance of the theoretical sampling distribution to be smallest for $\frac{\text{large/small}}$ samples.
92. If you knew enough about the sampling procedure and the size of the sample to know that the theoretical sampling distribution had a very small variance with most of the sample proportions clustered around the true population proportion, you would have $\frac{\text{more/less}}{\text{confidence in the estimate than you would if you determined that the theoretical sampling distribution had a large variance.}}$
93. Earlier, we concluded that if a single observation were drawn randomly from a population, the probability of that observation having a particular value is equal to the proportion of times that value occurs in the population. Thus, if you imagined a population in which half of the observations had the value 8, and if you repeatedly drew a single observation from that population by a random sampling procedure, you would expect about $\frac{\text{one-half}}$ of all the observations sampled to have the value 8.

Both the table and Graph B indicate one dot was showing on the third toss, whereas Graph A indicates _____ dots were showing on the third toss.

two

197. Notice how easy it is to compare the values in the two graphs because the values are shown as a **picture**, rather than as numerals in a table. Because it shows a picture of the values rather than simply listing their names, a _____ is useful.
graph/ table

graph

198. Earlier, you considered an experiment in which we recorded the time it took a rat to run down an alley to reach food. The data for that experiment were observed values of the "running-time" variable. Let's consider how we could represent values of the running-time variable in the form of a graph. Four values we might have observed in that experiment are shown below in the form of a _____.
graph/ table

table

DAY	RUNNING TIME
1	30 Seconds
2	20 Seconds
3	10 Seconds
4	5 Seconds

Whenever you encounter an estimate of a population proportion in your reading, it will be important to consider the probable **accuracy** of the estimate. For example, consider the following statement:

"Two out of three college students prefer

programmed instruction to regular

textbooks."

One interpretation of this statement is that two-thirds of all college students in the country prefer programmed instruction to regular texts. It is/is not clear, however,

whether "2 out of 3 students" refers to the **population** proportion or to a **sample** proportion.

88.

Another possible interpretation of the statement would be that two-thirds of the students in a sample were in favor of programmed instruction. If this were the case, it would be important to consider the sampling

distribution of this sample statistic in order to determine the degree of confidence you should have when you use the sample statistic as an estimate of the population statistic. Furthermore, it is/is not clear from the

preceding statement whether or not you could use

probability theory to calculate a theoretical sampling

distribution.

89.

One reason it is not clear whether or not you can

calculate a theoretical sampling distribution is that you

don't know the **procedure** used in obtaining a sample.

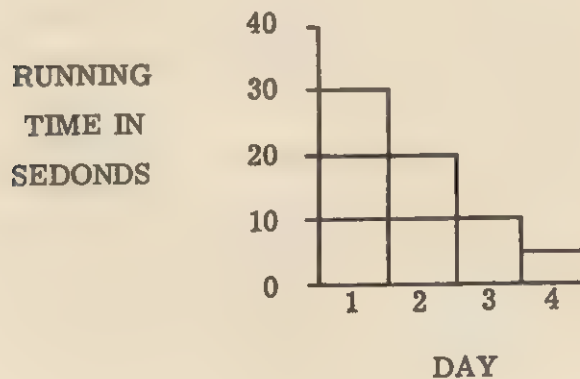
You could only use Probability Theory to calculate a

theoretical sample distribution if the sample procedure

were

199. The **longest** observed running time was _____ seconds. 30
 The shortest observed running time was _____ seconds. 5

200. The graph shown below represents the same data as the previous table. Notice that we have once again represented a value of the variable by the height of each column. The column representing Day 1 is the same height as the mark next to the numeral 30. The column representing Day 2 is the same height as the mark next to the numeral _____. At the left of the graph we have written what the numerals represent — that is, "running time" in seconds. Since the column for Day 1 is next to a mark with the numeral 30, you know that the rat took _____ seconds to run down the alley to reach the food on the first day. 20 30



201. On Day 4, the rat took 5 seconds to reach the food (as was indicated in the previous table). The column for Day 4 is only half as high as the mark next to the numeral 10. In other words, it represents a running speed of _____ seconds. 5

probability of obtaining a sample leading to an error of estimation greater than one-tenth is equal to the sum of the probabilities represented by the shaded/unshaded columns.

shaded

83. The probability of making an error of estimation of

one-tenth or less is .771. The probability of obtaining any other kind of sample is 1 minus .771, or _____.

.229

84. Because of our knowledge of the theoretical sampling

distribution, we can make the following confidence statement concerning the accuracy of an estimate. "If the population proportion is one-fifth, the probability of obtaining a sample whose proportion was within one-tenth of the population value is equal to _____."

.771

85. You could say the same thing in a different way. You

could say:

"Only about _____ out of every 1,000 random samples of size 10, drawn from a population in which p equals one-fifth, will differ from the true population proportion by more than one-tenth."

229

86. The main point of the preceding illustration is that the theoretical sampling distribution indicates the degree of **accuracy** you can expect with a particular size sample randomly drawn from a particular type of population. This allows the person making an estimate to state not only his best bet concerning the population parameter but also his degree of confidence in the a _____ of this estimate.

accuracy

202.

A table and a graph are shown below. The figure marked A is the graph/ table, and the figure marked B is the graph/ table.

table

graph

DAY	SCORE
1	5
2	10
3	7

FIGURE A

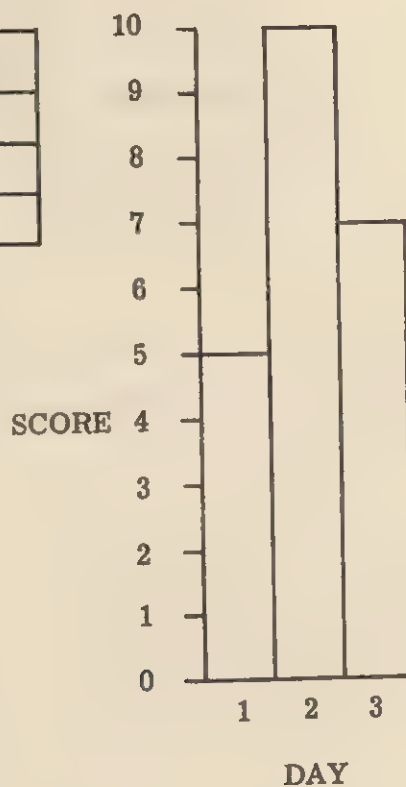


FIGURE B

203.

The numerals along the side of the graph are different values of the "day"/"score" variable.

"score"

204.

The largest observed "score" was 10/15 on Day 2/3.

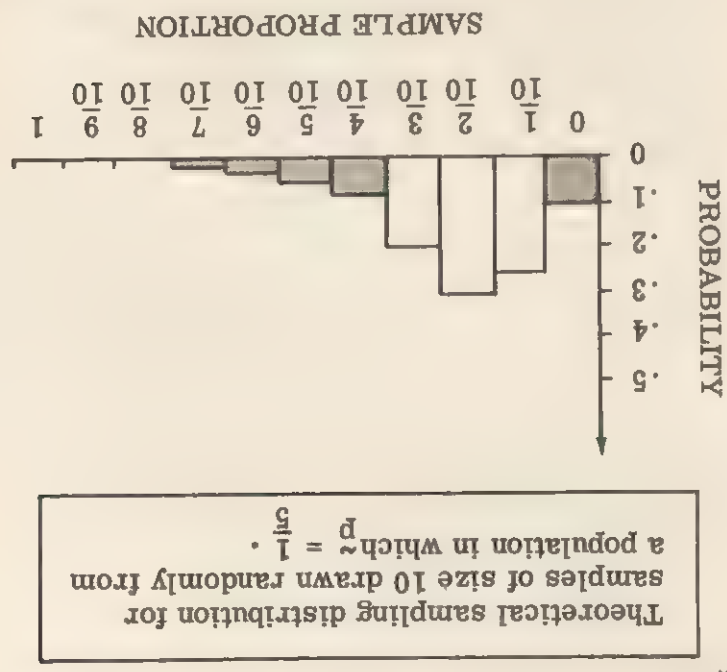
10, 2

Notice how clearly this is shown by the graph: Day 2/3 has the highest column.

2

80. There are only 2 possible outcomes in this new sample space. Thus, the probability of obtaining a sample in the other group is simply equal to 1 minus .771, since the sum of the probabilities for the two groups must equal _____.

81. We could illustrate the previous theoretical sampling distribution with a graph instead of a table, as shown below.



82. We have shaded the columns representing the probability of obtaining a sample in which the sample proportion would lead you to make an error of estimation greater than _____. The unshaded columns represent the probability of each of the 3 types of samples that would lead to an error of estimation of one-tenth or less. Therefore, the probability of obtaining any sample in that group is equal to the sum of these three probabilities. Similarly, the

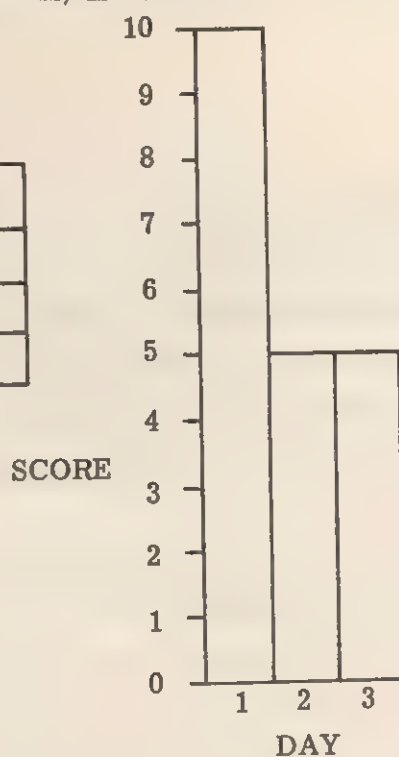
205.

Consider the following graph and table. The "score" value shown for Day 1 is/is not the same on the graph

is

as on the table.

DAY	SCORE
1	10
2	5
3	?



A score of $\frac{5}{8}$ is shown for Day 2, both on the graph and on the table.

5

The observed value for Day 3 is indicated in the graph/table but not in the graph/table.

graph, table

To make the table identical with the graph, you should put a score of 5 in place of the question mark shown in the table.

5

206.

It is clear that the scores were the same on Day 2 and 3 because the columns are the same height for both days on the graph.

height

75. Since the only samples leading to an error of estimation of one-tenth or less are those in which 1, 2, or 3 opinions were favorable, the probability of obtaining a sample in this group is equal to _____ + _____ + _____ .

76. In other words, the probability of obtaining 1, 2, or 3 favorable opinions in a sample of size 10 is equal to the sum of the probabilities of each type of sample. This sum is equal to .268, + .302 + .201 or _____ .

77. This means that about 771 out of every 1,000 random samples of size 10 obtained from this population would be expected to have a sample proportion that differed from the true population proportion by _____ or less.

78. The use of s^2 as an estimate of the population variance is so common that some authors refer to s^2 as the "sample variance." Naturally, whenever you think of a collection of data as a sample, you are almost always interested in the _____ statistics rather than the sample statistics.

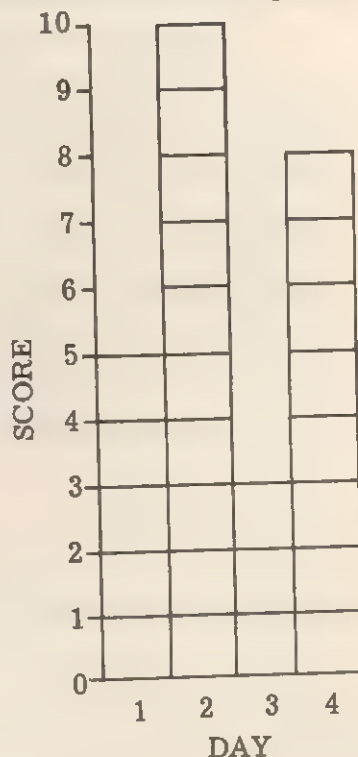
Therefore, you are almost always interested in using the sample to _____ the population statistics estimate rather than in describing the sample itself.

79. Strictly speaking, the variance of the sample (σ^2) is equal to the "sum of the squared deviations from the sample mean" divided by the "number of observations in the sample." You will normally be more interested in s^2 , which is equal to the "sum of the squared deviations from the sample mean" divided by the "number of observations minus _____."

207.

Another graph and table are shown below. Fill in the table so that it is identical to the data shown on the graph. Now compare your work with the table given as the answer.

DAY	SCORE
1	
2	
3	
4	

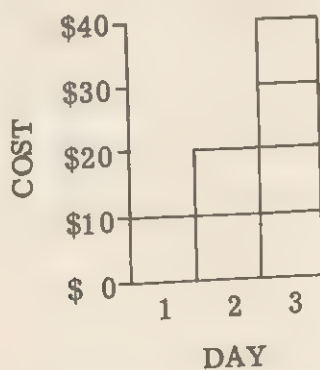


DAY	SCORE
1	5
2	10
3	3
4	8

208.

The data in the following graph are values of a variable named "_____". A value of this variable was observed on each of three _____.

cost
days



70. If the sample contained less than favorable opinion or more than favorable opinions, the absolute error of your estimate would be greater than one-tenth.

1
3

71. The theoretical sampling distribution shown in the preceding table can be regarded as a **probability distribution** on a **sample space** consisting of 11 outcomes. These 11 outcomes are simply the 11 possible types of samples. Each type of sample is assigned a number, or probability, such that the sum of these numbers equals _____.

1

72. In other words, this probability distribution assigns a probability to each of the possible outcomes of the random sampling process. The particular shape of this distribution is a product of not only the random **sampling procedure** but characterizes the **population** as well. Notice that the most frequently occurring types of samples (i.e., the samples with the highest probability) are those in which the proportion of favorable opinions is the same as the population proportion ($p = \frac{1}{5}$). is/ is not

is

73. Notice that samples containing 8, 9, or 10 favorable opinions were so infrequent that when the probability value was rounded off to the closest thousandth it equals _____.

.000

74. Suppose we divided the possible outcomes of the sampling process into two groups: (1) samples that would lead to an estimate of the population proportion which was an error by no more than one-tenth; and (2) all other types of samples. According to the simple **addition rule** of probability theory considered earlier, the probability of obtaining a sample from either group is equal to the _____ of the probabilities of those samples included in the group shown in the previous table.

sum

208. (Continued)

The largest cost was observed on Day _____, since the column for that day is _____ than any other column.

3
higher

209. To fill in the column and row headings in the following table so that it represented the same things as the graph we just considered, you would label the column containing the values with the word "_____" and the three rows with the numerals "_____", "_____", and "_____."

cost
1, 2
3

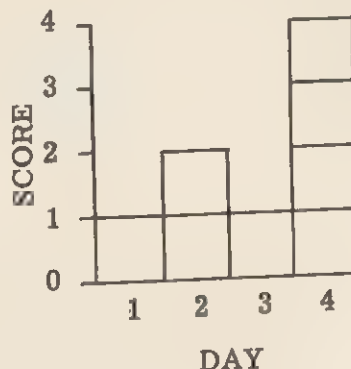
	\$10
	\$20
	\$40

210. Since each row represents a different _____ on which a cost was observed, the square above the numbers 1, 2, and 3 should have the word "_____" written in it.

day

day

211. Fill in the table below so that it represents the same things as the graph does.



DAY	SCORE
1	1
2	2
3	1
4	4

66. Similarly, according to this distribution, you would expect about _____ out of every 1,000 samples obtained in this manner to contain exactly 5 favorable opinions.

67. These probabilities represent the **limiting relative frequencies** with which each of the 11 possible types of samples would be expected to occur in an unlimited number of samples obtained in this manner. In other words, as you collected more and more random samples of size 10, you would expect the proportion of samples containing 1 favorable opinion to settle around a value of _____. By "settle around," we mean that while there would be considerable variability in the proportion of such samples when only a few samples had been collected. As the number of samples collected became very large there would be progressively less variation in the proportion of such samples.

68. Suppose we **grouped** all types of samples into two categories — first, samples that led you to make an estimate of the population proportion which would differ from the true value by no more than one-tenth; second, a group of samples which would lead to an error of estimation **greater than** one-tenth. Since the true population proportion was one-fifth ($\frac{1}{5}$), a sample containing three unfavorable opinions would represent a sample proportion of _____. Using this as an estimate would cause you to make an **absolute error** of estimation of _____.

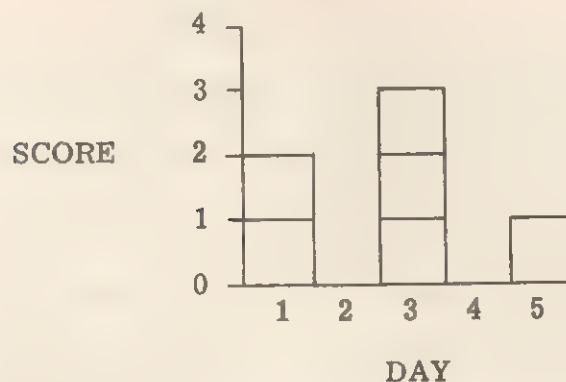
69. An estimate of the population proportion based upon a sample containing one-tenth favorable opinions would be equally as accurate, since the difference between the estimate of one-tenth and the true population proportion (two-tenths) would also represent an absolute error of _____.

212. What if you had a score of zero and you wished to represent it in a graph. If a column three squares high represented a score of 3, and a column two squares high represented a score of 2, and a column one square high represented a score of 1, then a column no squares high would represent a score of _____.

zero (0)

213. Therefore, on the following graph a score of zero was recorded on Day 2 and Day _____.

4



214. Notice the line on the left of the graph has a mark to indicate the height of each column and a numeral indicating the "score" value of each height. The numeral _____ indicates that if a column had no height at all it would represent a score of _____.

"0"

0

215. Often you will see graphs in which the top of a column is at a height **between** two of the numerals or marks indicating values.

This table indicates a theoretical sampling distribution for samples of size 10 drawn randomly from a population in which each member has one of two possible values. For example, it could represent a population consisting of the opinions of all of the students at a university concerning a particular issue, where each student had either a favorable or an unfavorable opinion. We will suppose, furthermore, that the proportion of favorable opinions in the population equals one-fifth or, as it is indicated in the table heading, $\frac{1}{5} = \frac{p}{1}$.

62. Every random sample of 10 opinions will lead to one of possible sample proportions. (These are the possible values of p where p equals the proportion of favorable opinions in a sample.)

63. Thus, $p = 0$ implies that _____ of the people in the sample had favorable opinions, and $p = 1$ implies that _____ of the people in the sample had favorable opinions.

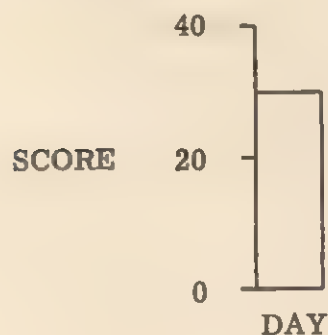
64. We have listed all possible values of the sample proportion (p) in the first column of the table. In the second column we have indicated the probability of obtaining each of the 11 possible types of samples. For example, the probability of obtaining a sample containing 3 favorable opinions is _____.

65. Saying the probability of a sample containing 3 favorable opinions is .201 implies that if a very large number of samples were collected, you would expect about 201 out of every 1,000 samples to contain _____ favorable opinions.

215. (Continued)

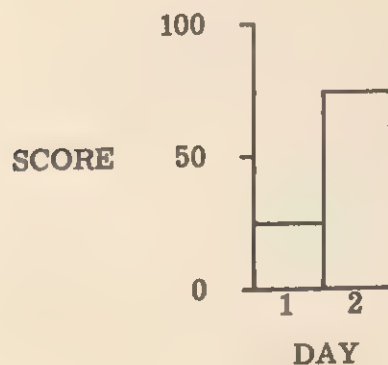
For example, the following **column** would probably indicate the value 30 because it is lower than 40 but higher than 20, and approximately half-way between their marks.

30



216. According to the following graph, the score on Day 1 was 25 and the score on Day 2 was 75.

25, 75



By the **accuracy** of an estimate, we mean the **absolute** difference between the estimate and the true population statistic. Thus, while a sample mean of 8 would lead to an underestimation of a population mean of 10, and while a sample mean of 12 would lead to an overestimation, both estimates would be equally _____, since the absolute difference between 10 and 8 is the same as the absolute difference between 10 and 12.

60.

While 12 would represent a positive deviation from the true mean ($\mu = 10$), and while 8 would represent a negative deviation, the absolute size of the deviation would equal _____ in both cases.

2

61.

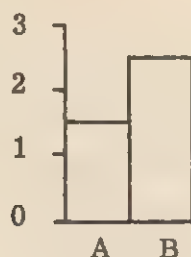
If you use a sample statistic as an estimate of the population statistic, the theoretical sampling distribution of the sample statistic indicates how accurate you can expect your estimate to be. For example, consider the theoretical sampling distribution described by the following table.

Theoretical sampling distribution for samples of size 10 drawn randomly from a population where $p = \frac{1}{5}$.		
Sample Proportion	Probability (to nearest $\frac{1}{1000}$)	
0	.107	
1/10	.268	
2/10	.302	
3/10	.201	
4/10	.088	
5/10	.026	
6/10	.006	
7/10	.001	
8/10	.000	
9/10	.000	
1	.000	

217. In the same way, you could represent the value $1\frac{1}{2}$ with a column one and one-half squares high. For example, Column A would represent a score of $1\frac{1}{2}$ and Column B would represent a score of $\frac{2\frac{1}{2}}{2\frac{1}{2}/3}$ because it is

$$2\frac{1}{2}$$

between the height of 2 squares and the height of 3 squares.



218. Suppose you were recording how much time it took a person to solve a puzzle. You might have collected the data shown in the table below:

SUBJECT	TIME TO SOLVE
1	20 Seconds
2	10 Seconds
3	15 Seconds
4	25 Seconds

Thus, there is data from $\frac{20}{4}$ subjects, and each observed value is a particular $\frac{\text{time}}{\text{subject}}$.

4

time

55. In some cases, naturally, the descriptive sample statistic is also used as an estimate of the population statistic. Thus, $\frac{\bar{x}}{\mu}$ is the mean of a sample and it is also used to estimate $\frac{\bar{x}}{\mu}$.
56. Similarly, the proportion of times a particular value occurs in a random sample (p) is a(n) $\frac{\text{biased/unbiased}}{\text{estimate of the proportion of times that value occurs in the population (p)}}$.
57. The sample proportion p is an unbiased estimate of \tilde{p} , since the **average** value of p in a very large number of samples will tend to equal \tilde{p} . On the other hand, σ^2 is a biased estimate of σ^2 , since the average value of σ^2 in a large number of samples will tend to be $\frac{\text{larger/smaller}}{\text{than } \sigma^2}$.
58. Knowing that a particular estimate is an **unbiased** estimate of a population statistic $\frac{\text{does/does not}}{\text{indicate how}} \frac{\text{accurate an estimate it will be.}}$
- By the **accuracy** of an estimate, we simply mean the size of the discrepancy between the estimate and the true population statistic. Therefore, if μ equaled 10, an estimate of 9 would be more $\frac{\text{accurate}}{\text{than would}}$ an estimate of 8.

Section II: Distributions

1. We have referred to records of the observed values of a variable as **data**.

It is often useful to summarize or describe data, rather than simply listing all the observed _____ which make up the data.

values

2. Suppose there were eight possible values of a variable and you observed only three of these possible values. Your data _____ be summarized by the statement: All the possible values of the variable were observed.

could not

3. Of the following two tables, Table _____ contains data which might be summarized or characterized by the statement: The value of the variable was the same for all observations.

B

TABLE A

Day	Score
1	20 sec.
2	10 sec.
3	20 sec.

TABLE B

Day	Score
1	20 sec.
2	20 sec.
3	20 sec.

Table B indicates the value of the "score" variable was _____ seconds on each of the _____ days.

20, 3

larger

51.

However, the fact that s^2 is always larger/smaller than σ^2 can be shown to result in a theoretical sampling distribution of s^2 that has a mean equal to σ^2 (the population variance).

52.

The **theoretical sampling distribution** of the sample statistic s^2 has a mean equal to the population variance. In other words, the average value of s^2 computed from a very large number of random samples taken from the same population will tend to equal $\frac{\mu}{\sigma^2}$.

53.

The sample variance tends to be an **underestimate** of the population variance, whereas s^2 is just as likely to be an overestimate as it is to be an underestimate. This is why σ^2 is said to be a biased/random estimate of σ^2 , whereas s^2 is said to be a(n) biased/unbiased estimate of σ^2 .

54.

It is important to clearly distinguish between sample statistics that simply **describe** the sample and sample statistics that are used as **estimates** of population statistics. For example, $\frac{\sigma^2}{s^2}$ is a sample statistic whose primary purpose is to serve as an **estimate** of the variance of the population (i.e., an estimate of $\frac{\sigma^2}{s^2}$).

σ^2

σ^2

unbiased

biased

σ^2

4.

TABLE A

Student	Grade
1	D
2	B
3	C

TABLE B

Student	Grade
1	D
2	A
3	A

You could describe or characterize the data in

Table above with the statement: A different value

A/B

A

was recorded each time the "grade" variable was observed.

5.

Suppose the grades shown in the previous tables were based on the usual grading system of A, B, C, D, and F, where A is the "best" grade and F is the "worst." You could describe the data shown in Table with

A/B

B

the statement: An A was the best grade observed.

You could describe the data in Table(s)

A/B/A and B

A and B

by saying the "worst" grade observed was a D.

6.

The reason we say these statements **characterize** or **summarize** the data is that they tell us something, but not everything, about the data. In the previous example, we could describe Table by saying: Only two of

A/B

B

the possible values of the "grade" variable were observed. This statement describes how many of the possible values are represented in the data. The statement tell us which particular

does/does not

does not

values were observed.

46. Since you are dividing the sum of the squared deviations by a **smaller** number when you calculate s^2 than when you calculate σ^2 , it will always be the case that s^2 is larger than σ^2 . For example, suppose the sum of the squared deviations from the mean in a sample of size 3 equalled 10. This would mean that $\frac{s^2 / \sigma^2}{\sigma^2}$ equalled 10 divided by 3, while $\frac{s^2 / \sigma^2}{s^2}$ equalled 10 divided by 2, since N equals $3N - 1 = 2$.
47. It is clear that $\sigma^2 = \frac{3}{10}$ is $\frac{larger/smaller}{\text{than}}$ $s^2 = \frac{2}{10}$, since $\frac{3}{10} = \frac{6}{20}$ and $\frac{2}{10} = \frac{6}{30}$.
48. Notice that the difference between σ^2 and s^2 is trivial when the sample size is very large.
49. For example, suppose the sum of squared deviations, $\sum (X - \bar{x})^2$ in a sample of size 1,000 equalled 100. The formula for σ^2 would equal 100 divided by 1000, while the formula for s^2 would equal 100 divided by 999. The difference between 100 divided by 1,000 and 100 divided by 999 is trivial compared to the difference between $\frac{3}{10}$ and $\frac{2}{10}$. This illustrates the fact that the difference between σ^2 and s^2 becomes $\frac{large/small}{\text{trivial}}$ when the sample size is very $\frac{large}{small}$.
50. The main reason for using s^2 instead of σ^2 as your estimate of the population variance is that the mean of the theoretical sampling distribution of s^2 is $\frac{\sigma^2}{2}$ than the true population. In other words, σ^2 typically will be smaller than the true population variance.

7. Consider the table of data shown below.

Meal	Cost
Breakfast	\$1. 50
Lunch	\$2. 00
Dinner	\$4. 10

The data consist of three values of a variable named

"cost"/"meal".

cost

8. You could characterize (describe) the data shown in the preceding table with the statement: The largest observed value of the cost variable was \$_____.

4. 10

9. Saying \$4. 10 was the largest observed value, _____ does not tell you which particular meal cost the most.

does not

If you were only interested in the smallest amount paid for any one meal, the statement "\$1. 50 was the smallest amount paid for any one meal" _____ describe the data in sufficient detail for your interests.

would

10. Therefore, if you were only interested in a particular characteristic of the data, a summarizing statement often answers your question most simply. However, a summarizing statement _____ tell you as much about the data as a complete list or table of data.

does not

(Instead of dividing it by the "number of observations in the sample" as you did to compute σ^2 .) In other words,

$$\frac{\sum (X - \bar{x})^2}{N} = \text{_____}$$

and

$$\frac{\sum (X - \bar{x})^2}{N - 1} = \text{_____}$$

43. Thus, the formula for s^2 differs from the formula for σ^2 simply in that you divide the sum of the squared deviations by N to calculate $\frac{\sigma^2}{2}$, whereas you divide the sum of the squared deviation by $N - 1$ to calculate $\frac{s^2}{2}$.

44. Remember, to compute σ^2 , you divide $\sum (X - \bar{x})^2$ by N , but to compute s^2 , you divide by $N - 1$ instead. Thus, if $\sum (X - \bar{x})^2 = 50$ and $N = 5$, you would find σ^2 by dividing 50 by 5, and you would find s^2 by dividing 50 by $\frac{5}{4}$.

45. Of the following two formulas, formula $\frac{A}{B}$ represents σ^2 , while formula $\frac{A/B}{B}$ represents s^2 .

A. $\frac{\sum (X - \bar{x})^2}{N}$

B. $\frac{\sum (X - \bar{x})^2}{N - 1}$

11. One of the first things you might ask about a collection of data is how often were particular values of the variable recorded. For example, the following table indicates a "score" of "8" was observed _____ times out of the 5 observations making up the data.

3

Day	Score
1	8
2	8
3	2
4	5
5	8

The score value 5 occurred the same number of times as the value 2, since they each occurred _____.

once

12. You could **characterize** (describe) the data in the previous table by saying the value "_____" appeared three times and the values "_____" and "_____" each appeared once.

8

2, 5

This summary of the data _____ be sufficient
would/would not

would

if you were only interested in which score value occurred **most frequently**. The statement would tell you that the value "_____" was observed more often than any other value.

8

The previous summary statement _____ tell
does/does not

does not

you enough about the data to determine on what particular day a score of "2" was observed.

13. Whether or not a certain way of summarizing the data is suitable _____ depend on what particular
does/does not

does

characteristic of the data you are interested in.

39. We mentioned earlier that statisticians have shown that the variance of a sample is more likely to be **smaller** than the population variance than it is to be larger. In other words, σ^2 is a biased/unbiased estimate of σ^2 .

40. Remember, σ^2 is the population/sample variance, while σ^2 is the population/sample variance.

41. Statisticians use a sample statistic represented by the symbol s^2 as an estimate of σ^2 . Although s^2 is very similar to the population/sample variance, σ^2 , it differs just enough to make it a better estimate of σ^2 than is σ^2 .

The chief feature of s^2 making it preferable to σ^2 as an estimate of σ^2 is that s^2 is just as likely to represent as large an overestimate of the population variance as an underestimate. In other words, while the average value of σ^2 based on a large number of samples will tend to be equal to/less than the population variance, the average value of s^2 based on a large number of random samples will tend to be less than/equal to the population variance.

42. The difference in the formulas for σ^2 and s^2 is very slight. In fact, when the sample size is very large, the difference is unimportant. To calculate s^2 , you simply divide the sum of the squared deviations from the sample mean by the number of observations in the sample, minus 1.

14.

Suppose you asked ten people to judge whether a particular painting was "good" or "bad." You might obtain data of the sort shown in the following table.

Person	Judgement
1	good
2	bad
3	good
4	good
5	bad
6	good
7	bad
8	bad
9	good
10	good

You could summarize this table of data by counting the number of times each of the $\frac{2}{3}$ possible values of the

2

"judgment" variable were observed.

The two possible values of the variable named "judgment" are _____ and _____.

good, bad

According to the table of data, the value "good" was observed _____ times and the value "bad" was observed _____ times.

6

4

15.

Another way of saying the value "good" occurred 6 times is to say the **frequency** of "good" was 6. Thus, instead of saying the value "bad" occurred 4 times, you would say the frequency of "bad" was _____.

4

sampling distributions is that the **average** sample proportion equals the population proportion. In other words, if you obtained many random samples from the same population and calculated the proportion for each sample, the average of all these sample proportions would tend to equal the

_____ .
The tendency for the average sample proportion to equal the population proportion is not a characteristic of all sample statistics.

population proportion

It can be shown, for example, that the **variance of a sample** will tend to be slightly **smaller** than the **true population variance**. This does not mean that the variance of every random sample will be smaller than the population variance. It simply means that the sample variance will be

_____ than the population variance will be

smaller

variance more often than it will be _____ .

larger

37. The variance of a sample is often said to be a **biased** estimate of the population variance, since it tends to be _____ than the population variance.

smaller

38. You will recall that the variance of a sample is given by the formula shown below:

$$\sigma^2 = \frac{\sum (X - \bar{x})^2}{N}$$

Note that the formula for the variance is simply the sum of the _____ deviations from the mean divided by the number of observations (N) in the sample.

squared

16. If you were to say the frequency of a certain value was ten, you would simply mean that you counted how often that value had occurred in the data and found it had occurred _____ times. 10
17. If you said the frequency of a certain value was 30, you would mean that you had counted the number of times that value had occurred in the data and found it had occurred _____ times. 30
18. If you had a set of data in which a particular value occurred _____ times, you would say the frequency of that value was 25. 25
19. If 8, 6, 5, 2, 6, and 1 were a collection of data, you would say the frequency of the value 6 was $\frac{2}{3}$ and 2
the frequency of the value 8 was _____, since "6" 1
occurred twice in the data, whereas "8" occurred only
_____ once
20. If a collection of data contained the values 8, 8, 2, 9, 2
6, 6, and 5, the frequency of the value 8 would be _____ 2
and the frequency of the value six would be _____ 2
21. To say a value has a frequency of zero means that value occurred in the data _____ times. zero (0)
- In a particular set of data, therefore, we might find the frequency of the value 20 was **zero**. This would mean the value 20 never occurred/occurred 20 times in the data. never occurred

33. Thus, a confidence statement is a useful way of specifying the _____ of making an estimation error of any particular size when estimating a population statistic from a sample.

34. In order to specify the risk of obtaining an unrepresentative sample that would lead to a large error of estimation, it is necessary to know how often each type of sample would tend to occur from a particular population with a particular sampling procedure. In other words, it is necessary to know the _____ distribution.

sampling

35. Very often you will be interested in estimating the proportion of times a particular value occurs in a population on the basis of a sample from that population. For example, you might wish to estimate the proportion of color-blind people in the United States on the basis of a sample of 100 people drawn randomly from that population. Statisticians show that the proportion of color-blind people in your sample is your best estimate of the population proportion. In other words, if 2 of the 100 people in your sample were color-blind, your best estimate of the proportion of color-blind people in the population would be _____-hundredths.

two

36. The main argument for using a sample proportion as your estimate of the population proportion is that this estimation procedure is just as likely to result in an over-estimate of the population proportion as it would an under-estimate. You saw earlier how it was possible to calculate a theoretical sampling distribution of a sample proportion. A characteristic of these theoretical

22. Suppose a variable had three values which could be represented by the three letters A, B, and C. Suppose your data consisted of the following observed values: A, A, B, A, B, B, A, and A. The frequency of the value A is ____ and the frequency of the value B is ____.
- Since there were no observed values of the possible value C, the frequency of C is ____.
23. If the data consisted of ten observations, we would have a list of ten observed values. If all ten of these values were the same, we could say the frequency of that value was ____.
24. If the data are 100 observations, it ____ would/would not be possible to have a frequency of some value which was greater than 100.
25. Suppose you tossed a coin a hundred times and let it fall on one side or the other each time. The frequency of "heads" in your data could not possibly be greater than ____.
26. The fewest number of times "heads" could occur would be none at all. Therefore, the smallest possible frequency of heads would be ____.

5, 3

0

ten

would not

100

0



28. If samples are obtained by a random sampling procedure, statisticians can show that the theoretical sampling distribution of sample means will have a mean equal to the population mean. In other words, they can show that the average or typical value of the sample means will tend to be the same as the _____.
29. Therefore, if you used the mean of a sample as an estimate of the population mean, you will sometimes overestimate or underestimate the mean. However you will tend to over-estimate just about as often as you _____.
30. Therefore, one reason for using the mean of a sample to estimate the population mean is that you run the same risk of over-_____ the population mean as of under-_____ the population mean.
31. It is important to bear in mind that **random** sampling procedures do not guarantee a representative sample. They do allow you to say something about the **risk** of obtaining an unrepresentative sample, since it is usually possible to calculate a theoretical sampling distribution when you use a _____ sampling procedure.
32. For example, while you know that the mean of your sample is your best guess concerning the mean of the population, it would be very useful to know how good a guess it was likely to be. To put it differently, it would be nice to know what risk you ran of making errors of any particular size. A statement that specifies the risk of making an error of estimation of a particular size (or larger) would be similar to the type of c_____ statements we have already considered.
- confidence
- random
- under-estimating
- over-estimating
- under-estimate
- population mean

27.

Whenever we count things, we obtain a number. Since we count the values in the data to find their frequency, each frequency _____ a number. One way of _____ is/ is not

is

summarizing or characterizing data is to count how often each of the possible values of the variable occur. You could determine a frequency of occurrence for each of the possible values of the variable.

Consider the following table of data.

Student	Grade
1	A
2	A
3	B
4	A
5	D
6	F
7	A
8	B
9	B
10	B

The variable represented in the data is named "_____" and a value of this variable was recorded for _____ students. Since the grade A occurred 4 times, the frequency of the value A is ____.

grade

10

4

28.

The frequency of B grades is _____, and the frequency of C grades is _____ because no C's were recorded. D and F both occurred just once. Thus the frequency of D is the same as the frequency of F and equals ____.

4

0

1

23. If samples are obtained by a _____ sampling procedure, it is often possible to calculate a theoretical sampling distribution using the mathematical theory of _____.
- probability
24. Probability theory was originally developed to represent processes such as games of chance (dice, cards, etc.). It is the similarity between the random process involved in a random sampling procedure and the random process involved in games of chance that allows us to use _____ in calculating _____.
- probability theory
25. Perhaps the chief advantage of using a _____ sampling procedure is that we can calculate the theoretical sampling distributions which indicate how often you would expect to obtain different types of samples from particular kinds of populations.
- random
26. Let's consider how useful theoretical sampling distributions can be in drawing inferences about a population on the basis of a sample from that population. One of the most important procedures in _____ statistics is estimating a inferential/descriptive population statistic on the basis of a sample statistic.
- inferential
27. For example, you might wish to estimate the value of the population mean on the basis of the sample mean. In other words, you might wish to estimate _____ on the basis of _____.
- μ
- \bar{x}

29. We could summarize this frequency information about the data in the previous table as follows:

Possible Values Of Grades	Frequency
A	4
B	4
C	0
D	1
F	1

The data in this table is represented by the numbers 4, 4, 0, 1, 1. These numbers _____ represent
do/do not

values of the "grade" variable; they do, however, represent the _____ of times each possible value was observed (occurred in the data).

do not

number (frequency)

30. Because grade A occurred four times, the numeral 4 occurs in the same row of the table as grade _____.

A

31. Because grade B occurred four times, the numeral _____ occurs in the same row as grade B. Since a grade of _____ was never observed, a _____ occurs next to that letter.

4

C, 0

32. The last two rows in the frequency column contain 1's, since both grade _____ and grade _____ were observed only once.

D, F

33. The table in Frame 27, which contained a grade for each student, is often referred to as a table of the **raw data**. The word "raw" means that we have not summarized the data in any way; we have merely listed all the observations. The data are represented in the

19. One advantage of obtaining samples by a **random**

size

of the sample is increased, the variability of the sampling distribution tends to become smaller.

20. The tendency of a random sample to become more and more representative of a population as the size of the sample is increased is summarized in the law of

$$\frac{1}{n}$$

21. According to the law of large numbers, the frequency of highly unusual or unrepresentative samples will be

smaller

if the sample size is large than it will larger/smaller

be if the sample size is small.

22. Experimental sampling distributions indicate something

about the risk involved in making an inference about a population on the basis of a sample. If you only had a single sample from a population, however, you could not calculate an experimental sampling distribution.

Furthermore, it would often be more difficult to obtain the number of samples necessary to calculate an

experimental sampling distribution than it would be

to collect all the observations in the population. You

have seen, however, that if a sample is obtained in a

particular manner, it is often possible to calculate a

sampling distribution based on

logical considerations rather than on a large number of

actual samples.

theoretical

table in Frame 29, by the frequency of each value. This table would/ would not be considered a table of raw data.

would not

34. If you added together the frequencies of the possible values shown in the table in Frame 29, you would find that the sum or total of these frequencies equal ____.

10

35. The total of all the frequencies in a frequency table is/ is not equal to the total number of observations in the corresponding table of raw data.

is

36. This is what we would expect, of course, since each value in the table of raw data contributes to only one of the frequencies in the frequency table. For example, the four observed grades of A in the table of raw data were only counted when the frequency for grade _____ was being determined.

A

37. If a coin were tossed a hundred times and you were told the observed **frequency** of "heads" was ninety-nine, you would know that the frequency of "tails" was _____ because the frequency of "heads" plus the frequency of "tails" must equal _____.

1

100

38. If your table of raw data contained 1, 000 observations and a particular value had a frequency of 1, 000, you would know all the other possible values of the variable had frequencies of _____.

0

16. While most of the samples were quite representative of the population, a few samples were less so. Samples with means of 8, for example, would be unrepresentative of the population. Yet, according to the previous distribution, about $\frac{.1}{.5}$ of all the samples obtained had means of 8.

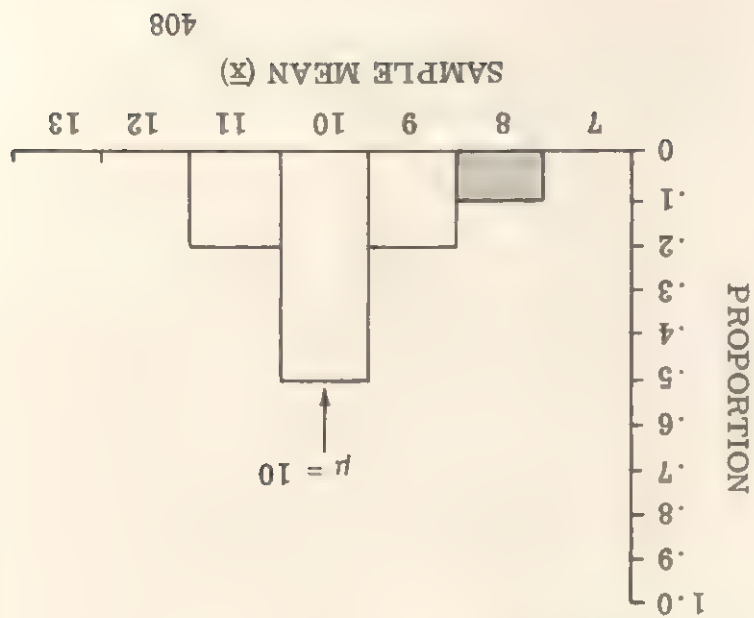
.1

17. This experimental sampling distribution indicates something about the **risk** you would be taking when you used a sample mean obtained in this manner to estimate the mean of the population. One way of describing this risk would be by the following **confidence statement**:

"If the mean of the population were actually 10, I would expect to make an error of estimation **greater than 1** on about only $\frac{\text{---}}{10}$ of the samples obtained in this manner."

1

18. The fact that about only $\frac{1}{10}$ of the samples obtained in this manner had means differing from the population mean by **more than 1** is illustrated by the following graph. We have shaded the column representing the proportion of times sample means differed from the true population mean by more than $\frac{1}{10}$.



408

39. Summarizing or characterizing data in the form of a frequency table is suitable if you are only interested in the _____ of times each value occurred in the data. If you were interested in the sequence or order in which each value was observed, a table of raw data _____ be suitable for your purposes.
- number (frequency)

would
- would/ would not
40. Whether a frequency table or a table of raw data is required _____ depend upon what particular aspect of the data you are interested in. Each of the frequencies in the frequency table is a kind of summary of your data obtained by counting how often each value occurred. We _____ think of this frequency as a number that describes or summarizes the data.
- does

can
- does/does not

can/ cannot
41. Any number or term that summarizes or describes a collection of data is called a **statistic**. Each of the frequencies, therefore, would be called a _____.
- statistic
42. Frequencies are often called **enumerative statistics**, because the word enumerate means to count and because we _____ the number of times a value occurs in the data in order to determine its frequency. Thus, we refer to frequencies as enumerative statistics because the word "enumerate" means to _____.
- count

count
43. Each of the frequencies in a **frequency table** is a number which summarizes the data. Each of these frequencies is called an _____ statistic because the word "enumerate" means to count.
- enumerative

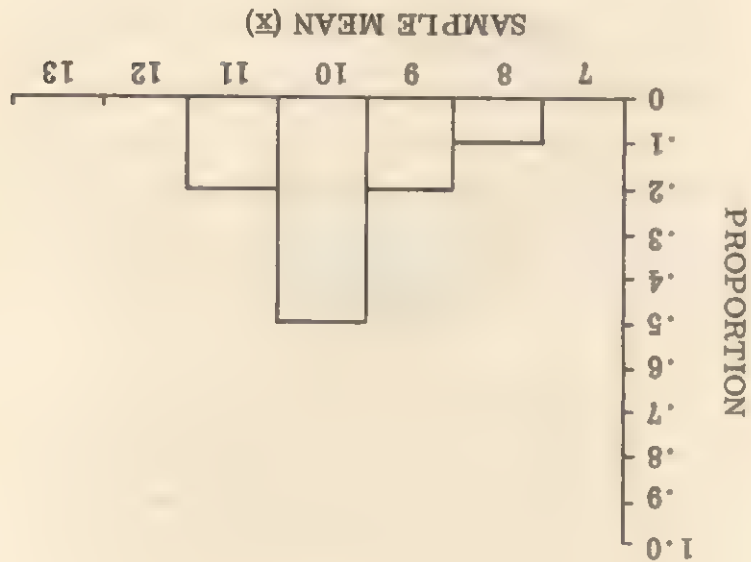
13.

An experimental sampling distribution characterizes the sampling procedure, since it suggests how often you should expect to obtain each particular type of sample using a _____ procedure of that sort to draw samples from that particular population.

sampling

14.

For example, suppose the proportional frequency distribution shown below were an experimental sampling distribution of sample means obtained by a particular sampling procedure from a population with a mean of 10. (Assume the sample means have been rounded off to the closest whole number.)



15.

Notice that the most frequently occurring type of sample had a mean that _____ was/was not _____ mean.

was

44.

Suppose you were conducting the following experiment. You present a subject with one pair of tones and ask him which of the two tones appeared louder — the first tone or the second tone. Suppose you had presented the subject with ten pairs of tones and asked him make a response after each presentation. You might have recorded the ten responses in the following table of _____.

raw data

Tone Pair	Answer
1	1
2	2
3	1
4	1
5	2
6	1
7	1
8	2
9	1
10	1

In this table, an answer of "1" represents a response indicating that the first of the two tones was the louder, whereas the answer "2" represents a response indicating that the second of the two tones was the louder. The data in this table could be called, therefore, _____ data.

numerical

numerical/non-numerical

45.

The two possible values of the "answer" variable in the previous table are "____" and "____". The number of observations represented in this table is ____.

1, 2

10

9. There is always some risk involved in drawing conclusions

about a population on the basis of a sample, since the sample may or may not be **representative** of the

population. The procedure you use for obtaining the

sample from a population (along with the character of the population itself) determines how likely you are to

obtain a representative sample. You have seen how an

experimental **distribution**

characterizes a particular sampling procedure because

it indicates how often each type of sample tends to occur.

10.

Suppose you obtained a large number of samples from a

particular population. If you calculated the mean of

each of these samples, you could make a frequency

distribution of these sample means. This distribution of

sample means would be an example of an

$$\frac{\text{experimental/theoretical}}{s}$$

distribution of sample means.

11.

Similarly, you could have calculated some other

descriptive statistic, such as the range of each of these

samples. The frequency distribution of these sample

ranges would be an e

s d of

$$\frac{\text{sample means/ranges}}{\text{sample means/ranges}}$$

12.

Suppose you collected 200 random samples of size 5 from

a population. You could calculate the median of each

sample. The distribution of these medians

$$200/5$$

200

would be an experimental sampling distribution of

sample .

medians

46.

The previous table of raw data could be summarized in the following _____ table.

frequency

Answer	Frequency
1	7
2	3

This frequency table contains the two enumerative statistics "____" and "_____."

7, 3

47.

Since the total of the frequencies in the frequency table must equal the total number of observations in the table of raw data, we did not have to count the number of times answer 2 occurred if we knew how many times answer 1 had occurred. For example, if the data in the table of raw data had been different and the frequency of the answer 1 had been 6, we would have known immediately that the frequency of the answer 2 was _____, since 10 minus 6 equals 4.

4

48.

We have referred to a table containing a list of each observed value as a table of raw data. A table listing the frequency of occurrence of each value is called a _____ table.

frequency

49.

Each frequency in a frequency table is the number of times a particular _____ has been recorded in the data.

value

50.

Since the frequency of each value is determined by counting, each frequency is a _____.

number

6. **Descriptive statistics** represent one of the two major uses of statistics. The second major role of statistics in research concerns the distinction between samples and populations. As a psychologist, you will often find that while you are interested in the characteristics of a particular collection of data, you only have part of that complete collection available to you. In other words, although you are interested in a $\frac{\text{population}}{\text{sample}}$, you only have a $\frac{\text{population}}{\text{sample}}$ available.

sample
population

7. A statistical procedure for drawing conclusions about a population on the basis of a sample from that population is called an **inferential statistic**. Statistics that simply describe a particular collection of data are called **statistics**. Another major use of statistics concerns drawing inferences or conclusions about a population on the basis of a sample from that population. This type of statistic is called a(n) inferential/descriptive statistic.

inferential
descriptive

8. The most important consideration in drawing conclusions about a population based on a sample is to what degree the sample is **representative** of the population. If you knew all about the population, you could determine to what degree the sample was representative of that population. However, if you knew all about the population, it would/would not be necessary to make an inference about it based on a sample.

51. A **statistic** is a term or number which describes some characteristic of a collection of data. Thus, each frequency in a frequency table is a _____. statistic

52. We refer to the frequency of a particular value as an enumerative statistic because the word "enumerate" means to _____. count

53. Because it lists the value of each observation, the following table could be called a table of _____. raw
data. In other words, the table presents a **complete** list of the data.

Observations	Values
1	A
2	B
3	A
4	A
5	A

54. When the observed data contains only two different values, we _____ be sure there are only two possible values of the variable. cannot
can/cannot

55. The following table could be used as a _____. frequency
table for the previous table of raw data by filling in the frequencies for each value in the appropriate row of column _____. 2
 $\frac{1}{2}$

Value	Frequency
A	
B	

- describing

mean, median, mode

proportions

distribution

The frequency of the value A in the table in Frame 53 is _____. The frequency of the value B in the same table is _____. Notice how the total of the frequencies in the frequency table _____ the total number of observations in the table of raw data.

4
1
is

56.

Suppose the variable represented in the table of raw data in Frame 53 had four possible values: A, B, C, and D. Since there were _____ observations of the values C and D, the frequencies of C and D would both be _____.

no
0

57.

A table of raw data and two frequency tables labeled Table A and Table B are shown below. The frequency table that corresponds to the table of raw data is Table _____.

B

Day	Weight
1	5
2	10
3	5
4	10
5	10
6	5
7	5
8	10

Weight	Frequency
5	5
10	3

TABLE A

Weight	Frequency
5	4
10	4
15	0

TABLE B

Thus, these probability distributions conveniently represent the relative frequency with which we would expect to obtain various kinds of samples if we repeatedly obtained these samples by a random procedure. We are only able to calculate the form of these theoretical sampling distributions if the samples are obtained by a process.

random

58. Notice that the value 5/15 does not appear in the 15
preceding table of raw data and that its frequency is
 presented in Table B.
is/ is not
59. Table A could not represent the data in the table of raw 4
data because the value 10 is represented as having a
frequency of 3, whereas it should have a frequency of 0
 , as it does in Table B. Any value of a variable
not observed (recorded) in the table of raw data has a
frequency of .
60. If there were 8 observations in the table of raw data, 0
we would know that the frequency of the value 15 was ,
so long as we knew that the frequency of the value 5
was four and that the frequency of value 10 was four.
61. Every **possible** value of a variable has some frequency 0
— whether or not it is recorded in the data — since a
variable has a frequency of if it is not recorded
in the data.
62. In summary, we can say a table of raw data lists the value
 of each observation, whereas a frequency frequency (number)
table lists the of times each value
occurred in the table of raw data.
63. Each of the entries in a frequency table is a number enumerative
and each of these numerals is called an
statistic.
- The **total** (sum) of these enumerative statistics equals data
the total number of observations in the table of raw
 .

248.

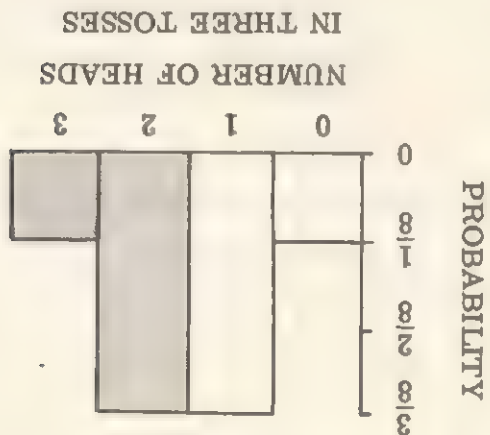
Thus, if you wanted to know the probability of obtaining either 2 or 3 heads, you would add the probability of obtaining 2 heads and the probability of obtaining 3 heads. In other words:

$$\Pr(2 \text{ or } 3 \text{ heads}) = \Pr(\quad) + \Pr(\quad)$$

249.

We have redrawn (below) the previous theoretical sampling distribution, except that we have shaded the columns corresponding to the probability of obtaining _____ or more heads in three tosses of a coin.

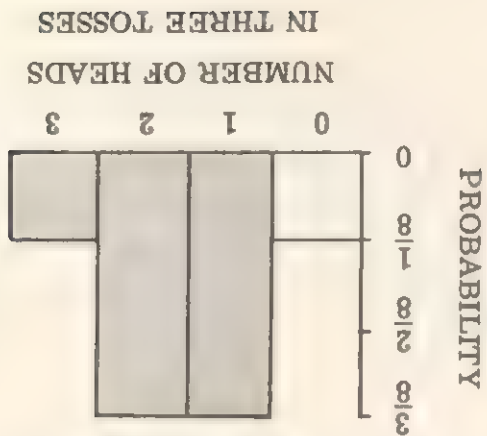
2



250.

On the following graph we have shaded the columns corresponding to the probability of obtaining _____ or more heads.

1



64. A collection of frequencies in a frequency table is called a **frequency distribution** of that variable. A _____ distribution indicates, therefore, how often each value of a variable occurred in the data. In other words, the collection of enumerative statistics in a frequency table indicating how often each value of a variable occurred is called a **f** _____ d _____.

frequency

frequency
distribution

65. Suppose you asked a subject to sort a collection of ten drawings into four boxes, each box labeled with one of the four adjectives **excellent, good, fair, and poor**. The subject would be distributing the ten drawings among the four possible judgments he could make concerning the merit of each drawing. If you recorded his performance, your table of raw data would contain _____ observations of a variable called "judgment,"
 $\frac{10}{4}$
This variable has $\frac{10}{4}$ possible values.

10

4

66. When the subject was finished distributing the ten drawings among the four boxes, each box would have some particular number of drawings in it. The number of drawings in each box could be thought of as the _____ of each value of the "judgment" variable. If the subject put five drawings in the box labeled "good" and five drawings in the box labeled "fair," the frequency of "good" judgments would be the same as the frequency of "fair" judgments, and would equal _____.

frequency

5

241. This theoretical sampling distribution indicates the probability of obtaining 0, 1, 2, or 3 heads in _____ tosses of a coin.
242. According to this distribution, the probability of tossing a coin three times and getting no heads is _____.
243. Similarly, the probability of obtaining exactly _____ heads in three tosses is also $1/8$.
244. The probability of obtaining 1 head is the same as the probability of obtaining _____ heads, since it equals _____ in both cases.
245. One way of interpreting this theoretical sampling distribution is that you would expect to toss a coin three times and get no heads only about _____ out of every 8 groups of three tosses.
246. Suppose you were interested in the probability of getting at least 2 heads in the three tosses. In other words, you would be asking: "What was the probability of getting either _____ heads or 3 heads in the three tosses?"
247. According to the _____ rule of probability theory, the probability of obtaining either 1 outcome or another outcome is simply equal to the _____ of their sum individual probabilities.

The frequency of "excellent" judgments and of "poor" judgments would both equal _____.

0

The numerals 5, 5, 0, and 0 _____ be called
could/ could not
a collection of enumerative statistics describing the
subject's judgments.

could

If _____ be appropriate to say the numerals
would/ would not
5, 5, 0, and 0 in the frequency table define a frequency
distribution.

would

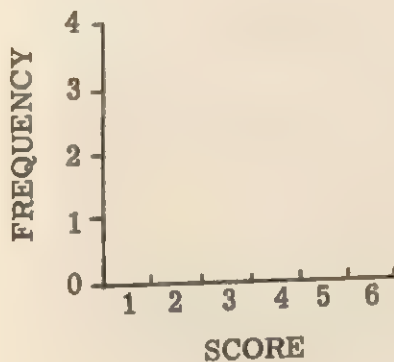
67.

It is often useful to present enumerative statistics by
means of a graph. We could represent the frequency
of each value with the height of a column, just as we
represented the _____ of a particular
observation by the height of a column in earlier graphs.
Of the two types of graphs shown below, Graph _____

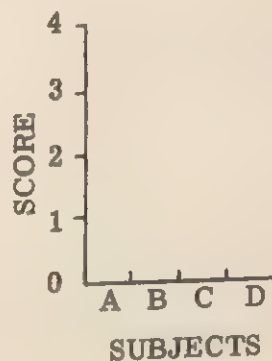
value

A

would be appropriate for showing the frequency
distribution of a variable named "score."



Graph A



Graph B

237.

Statisticians can show that the mean of a distribution of sample means tends to be the same as the mean of the population. In other words, if μ represents the average height of all of the people in the United States (i.e., the population), then the typical value (average) of the sample means would tend to equal _____.

μ

238.

Statisticians can also use probability theory to determine a theoretical sampling distribution of sample means. It is important, however, to bear in mind that this theoretical sampling distribution is only appropriate for samples drawn in a _____ or unbiased manner.

random

Probability theory only provides a means for calculating theoretical sampling distributions for samples obtained by a _____ procedure.

random

239.

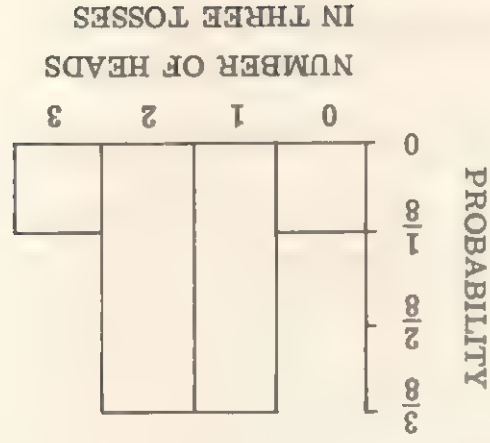
The chief advantage of being able to calculate a theoretical sampling distribution is that it tells you how often you should expect to obtain different types of _____ drawn randomly from a particular kind of population.

samples

240.

For example, consider the following theoretical sampling distribution. It is an example of a b _____ of the sort we have already considered.

binomial
distribution



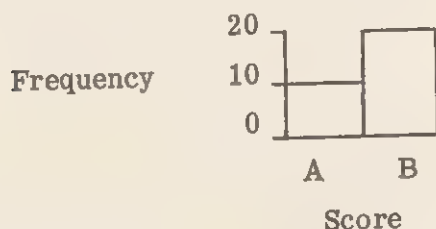
68. If there were only six possible values of the "score" variable, you would know that 6 **columns** would be required in the frequency distribution graph in order to represent all six values. However, since there were only four observations, the largest possible **frequency** for any score would be $\frac{4}{6}$.

4

69. The following frequency distribution graph contains two columns. Score A has a frequency of _____ and Score B has a frequency of _____ (as indicated by the heights of column A and B respectively).

10

20



70. The table of raw data for the previous frequency distribution graph would have contained a total of _____ observations.

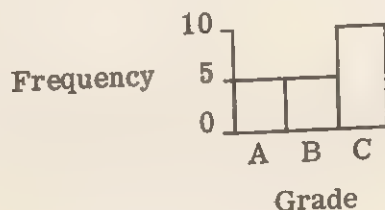
30

71. The number of observations represented by the following frequency distribution graph is _____, since A occurred _____ times, B occurred _____ times and C occurred _____ times.

20

5, 5,

10



The type of continuous curve shown in Figure B (above) is a product of mathematical reasoning, rather than of an actual distribution of data. The important feature of this theoretical distribution is its shape. This can be illustrated quite clearly with a smooth curve, rather than with the series of columns we previously used. Therefore, whenever you encounter a continuous distribution represented by a smooth curve rather than a series of columns, you can almost always think of it as if it were made up of the tops of a large number of closely packed _____.

columns

We said earlier that **normal distributions** are often used as theoretical sampling distributions, just as binomial distributions are used as _____ sampling distributions.

theoretical

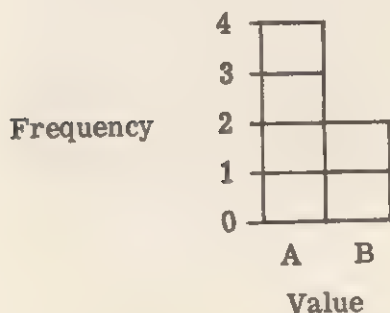
Mathematicians have been able to show that if samples are **randomly** drawn from a population with an approximately normal distribution, the distribution of the means of these samples (the sample distribution of sample means) will also have a normal distribution. As an illustration, let's suppose you were to draw many random samples of size 10 from a population consisting of heights of all the people in the United States. Each of these samples of size 10 would have a mean. Therefore, if you drew many samples, you would have many sample means. You can now think of two distributions — first, the distribution of all the heights in the population; second, the distribution of sample _____.

means

72. You know that the most frequently occurring grade in the previous frequency distribution is C, because this grade has the _____ column.

highest

73. You can think of each column in the frequency distribution graph as composed of a series of blocks — one block for each observation of that particular value. For example, consider the graph shown below.



This graph indicates a frequency of _____ for Grade A and a frequency of _____ for Grade B. Therefore, the column for A is four blocks high and the column for B is _____ blocks high.

4

2

two

Notice the number of blocks in column A forms a column twice as high as column B, indicating that the frequency of A is _____ as great as the frequency of B.

twice

74. Enumerative or frequency statistics are very important in elections. You are undoubtedly familiar with interpreting the data from elections even if you had not thought of these data as statistics. To illustrate this

It is often useful to think of a continuous distribution, (represented by the solid curved line as shown in Figure A below), as if it were made up of many tightly packed columns (as shown in Figure B), so that the _____ of the columns formed the shape of the continuous curve shown in Figure A.

tops

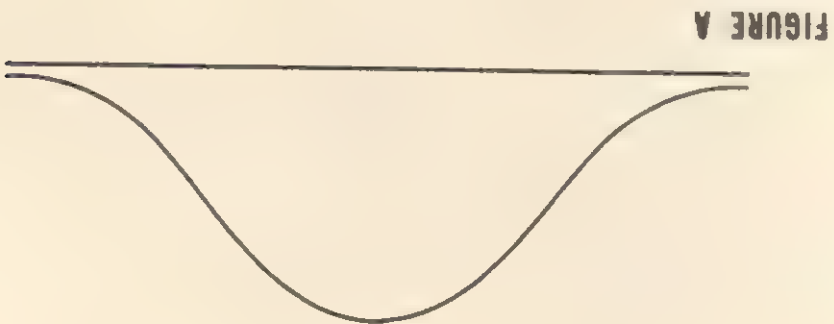


FIGURE A

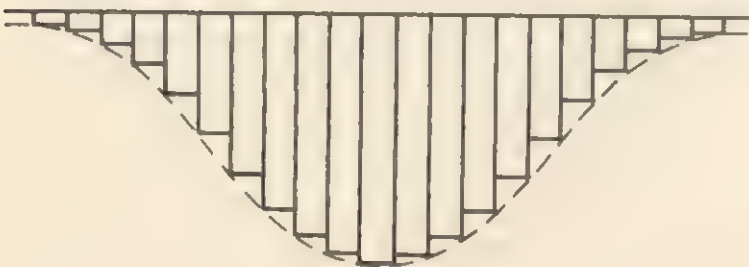


FIGURE B

It is not too important to distinguish between continuous and discrete distributions at this time. We have already shown that a continuous variable is normally treated as if it were _____. Distributions of a continuous variable are almost always formed by grouping or rounding off the values of the continuous variable.

discrete

74. (Continued)

point, suppose that you were conducting an election in which there were three candidates: R. K., A. C. and R. A. We could think of each person's vote as an observation having one of three possible values. These values would be a vote for _____, _____, or _____.

R. K., A. C.
R. A.

75. We could summarize the data from this election in a table such as the one shown below.

Candidate	Number of Votes
R. K.	25
A. C.	75
R. A.	50

Considering "vote" as a variable, this table would be a frequency table indicating that the value "R. K." had a frequency of _____, the value "A. C." had a frequency of _____, and the value "R. A." had a frequency of _____.

25,
75, 50

76. The number of votes cast for candidate R. K. is represented by the number 25 and the number of votes cast for candidate A. C. by the number 75. Both of these numbers are en_____ statistics.

enumerative

77. The group of three numbers, 25, 75, and 50, are the distribution of the 150 votes among the three candidates. In other words, these numbers define a frequency _____.

distribution

On the other hand, using the same data, you could round off the heights to the nearest 5 inches, which would give you $\frac{\text{more/less}}{\text{groups}}$ than when you rounded off to the nearest 10 inches.

You have actually divided each of the previous groups into 2 new groups, as shown in Figure 7.

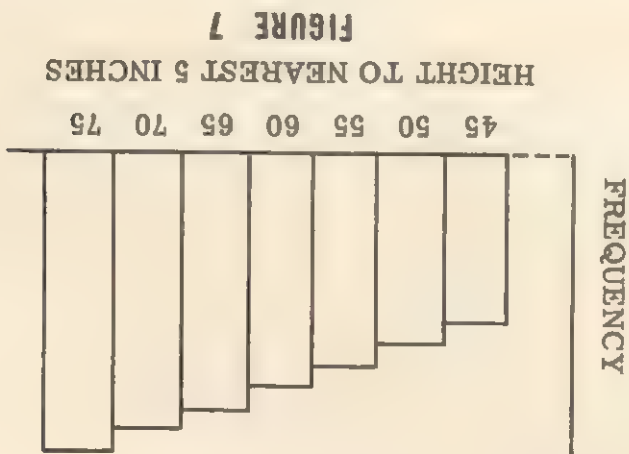


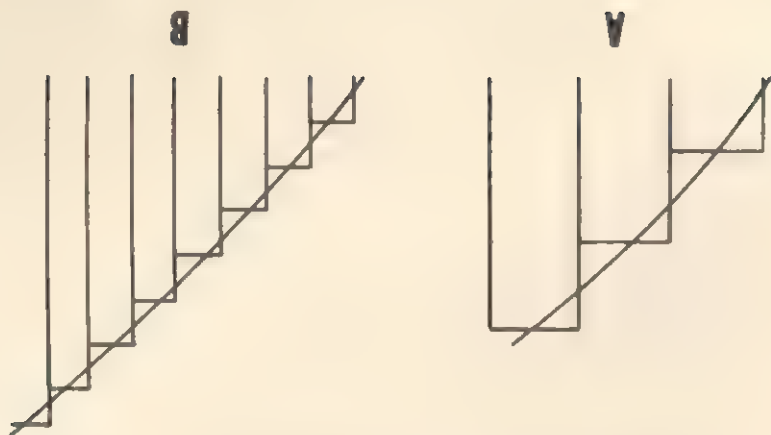
FIGURE 7

Notice that the **shape** of the distribution, as indicated by the **tops** of the columns, is roughly the same, but it has become smoother as you increased the number of columns. For example, notice how the tops of the columns shown in Figure $\frac{A}{B}$ (below) are closer to

the shape of the smooth curve drawn through them than is so in Figure $\frac{A}{B}$.

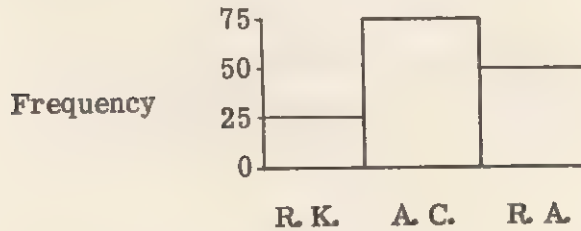
A

B



78. This frequency distribution could also be represented by a graph. The graph shown below would/would not represent the frequency distribution shown in the previous frequency table.

would



79. Notice the total number of voters in the election could be determined simply by adding all the _____ in the frequency table.

frequencies
(enumerative
statistics)

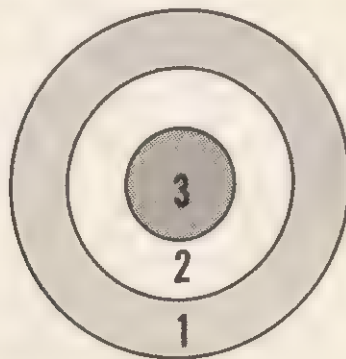
80. It is clear, therefore, that _____ people voted in the election.

150

81. The winner of the election is indicated by the candidate who has the _____ column in the frequency distribution graph. This was candidate named _____, who received _____ votes.

highest
A. C.
75

82. Suppose you were a psychologist interested in studying how accurately a person could throw darts at the dart board shown below:



the values "63" or "64", because we are not attempting to distinguish any of the values between these two heights. While many variables can be viewed as **continuous**, we normally treat them as if they were _____.

discrete

229.

If you have the heights of 10, 000 students, you could make a frequency distribution in which you rounded off the heights to the nearest 10 inches. For example, in the figure shown on the following page, we have indicated how all of the heights between 46 and 75 inches could be divided into three groups — those between 46 and 55 inches regarded as "50" inches, those between 56 and 65 inches regarded as "60" inches, and those between 66 and 75 inches regarded as "70" inches. In other words, if a person's height were closer to 60 inches than it were to 50 inches, you would call it " _____ inches.

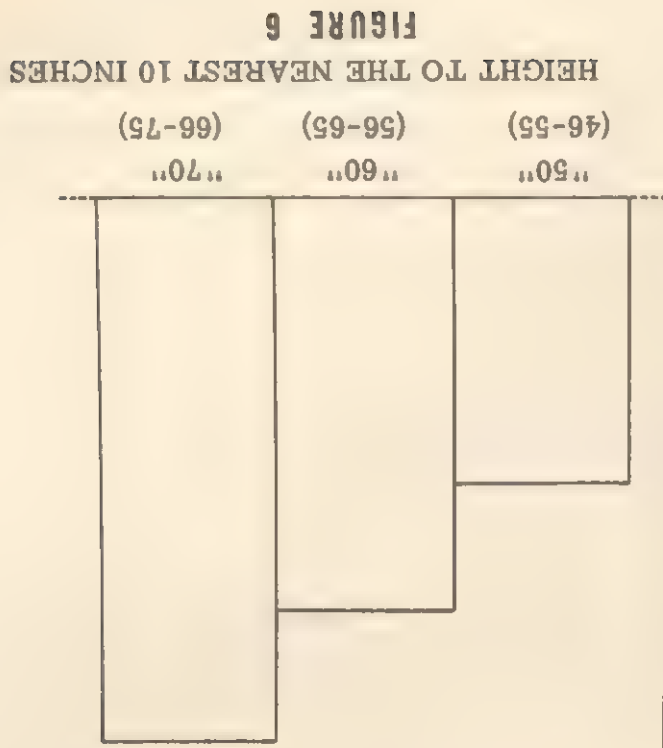


FIGURE 6
HEIGHT TO THE NEAREST 10 INCHES

You might conduct the following experiment. The subject would be asked to stand ten feet away from the target and attempt to hit the innermost ring. Suppose you told him that he would be scored as follows: 3 for the inner ring, 2 for the middle ring, 1 for the outer ring, and 0 for a complete miss. You would then ask him to toss the dart at the board twenty times, recording his score for each toss.

In this experiment, the distance of the subject from the board would be a _____, whereas the score

constant

the subject received for each toss would be a

variable

constant/variable

83.

Suppose you obtained the data shown in the following table:

TOSS	SCORE	TOSS	SCORE
1	0	11	1
2	1	12	2
3	1	13	2
4	2	14	2
5	1	15	3
6	0	16	1
7	2	17	3
8	1	18	2
9	3	19	3
10	2	20	3

This table indicates that the subject _____ miss
the target completely on his first toss. The first bulls-
eye he made was on the _____ toss.

did

9th

226.

A normal distribution is an example of a **continuous** distribution, whereas most of the distributions we have considered up to this point are examples of **discrete** distributions. Earlier, in our discussion of the difference between a **continuous** and a **discrete** variable, we indicated that a **discrete** variable was one for which we could locate two values having no other values **between** them. For example, the number of people in a room is a discrete variable since there can be either two people in the room or three people in the room, but there cannot be any number between the values two and three. Thus, "number of people in the room" is a

discrete/continuous
variable.

discrete

227.

On the other hand, a variable such as **height** is a continuous variable. No matter what two pairs of heights we choose, we could always imagine a height **between** them. For example, one person could be 65.001 inches tall and another could be 65.002 inches tall. It is possible, however, for someone else to have a height of exactly $\frac{65.0015 + 65.0025}{2}$, which would be a value **between** the two previous values.

228.

In virtually every case, we treat continuous values **as if** they were discrete by rounding off or grouping values of a continuous variable.

For example, if you rounded off a person's height to the nearest inch, you would simply be **grouping** all the heights that were closer to 63 inches, say, than to 62 or 64 inches and calling those heights "63 inches." Therefore, the variable "height to the nearest inch" only has

83.

(Continued)

The subject completely missed the target _____
times out of the twenty tosses.

2

84.

To form a frequency distribution, we must count the
number of times each _____ of the score
variable occurred in the data.

value

85.

We already noted that the value 0 occurred only twice
in the data. Therefore, the frequency of a score of 0
would be _____.

2

86.

The frequency of a score of 1 is _____. The
frequency of 2 is _____ and the frequency of 3 is _____.

6

7, 5

87.

This group of _____ statistics defines
a frequency _____ of the variable called
"score."

enumerative
distribution

88.

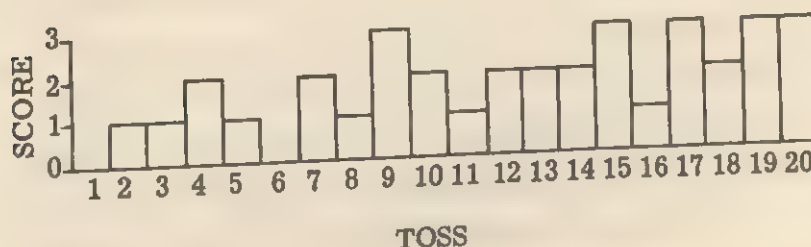
Suppose the data for twenty tosses indicated the
frequency of 1's was 5 and the frequency of 2's was 15.
The frequencies of both 0's and 3's must therefore be
_____.

0

89.

A graph of the _____ is shown
raw data/frequency distribution
below:

raw data



The shape indicated by the dotted line is the characteristic shape of a normal distribution.

normal

224. This shape, however, may differ considerably, depending upon the standard deviation or variance (since the

variance equals σ^2) of the particular normal distribution. The three shapes shown on the following graph are all normal distributions with the same mean (indicated by μ on the base of the graph). These three normal distributions differ only in the size of the variance or standard deviation. Thus, the distribution in which the values are grouped most closely about the mean has a standard deviation indicated by σ . On the

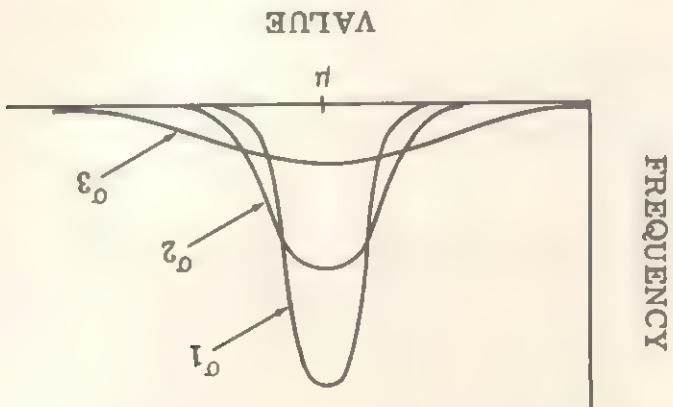
$$1 / 2 / 3$$

other hand, the very flat shape identified by σ_3 is the shape of a normal distribution with a

$$\text{large/small}$$

large

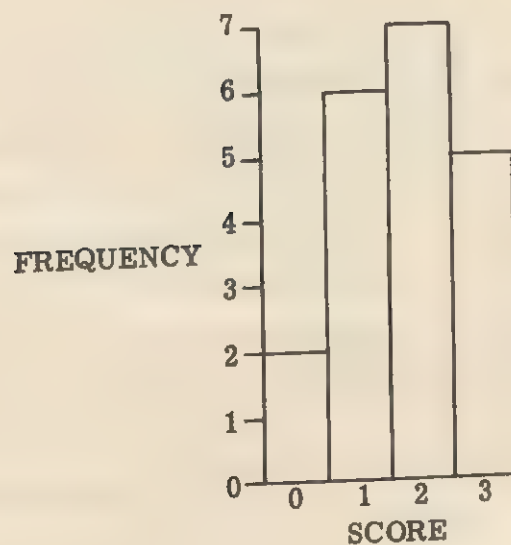
standard deviation.



225. Therefore, depending upon whether the standard deviation of the normal distribution is large or small, the actual shape will differ. It is always true, however, that the number of standard deviations from the mean will be the same in any normal distribution, regardless of its particular mean or variance.

90. Graphing the raw data in this way reveals an interesting characteristic of the subject's performance. In general, the more tosses he took, the better/worse better was his performance.
91. His performance appears to have improved as he continued his tosses because the columns tend to be higher/lower higher towards the right-hand side of the graph.
92. A table and a graph are shown below. The table contains the frequency distribution we just considered. This same frequency distribution is/is not is represented by the graph.

SCORE	FREQUENCY
0	2
1	6
2	7
3	5

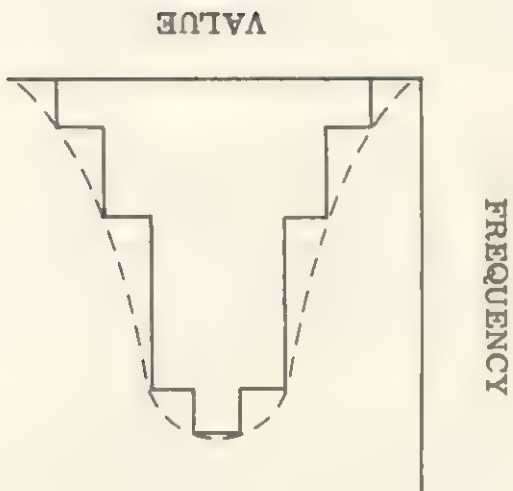


It is important to note that a normal distribution can have any mean or standard deviation. The important feature of a normal distribution is that the appropriate proportion of values fall within each particular number of standard deviations from the mean. For example, consider two normal distributions having a mean of 10.

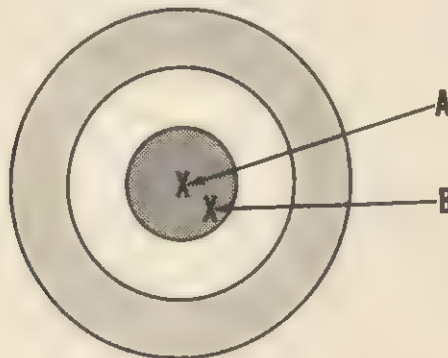
If one distribution has a standard deviation of 2, then .68 of all the values in that distribution would be between 8 and 12, since any value between 8 and 12 would be less than 1 standard deviation away from the mean. Similarly, if a normal distribution had a mean of 10 but a **standard deviation** of 3, then .68 of all the values in that distribution would fall between the values 7 and 13, since all of these values would be within 1 standard deviation of the mean.

223.

Distributions similar to a normal distribution have a characteristic shape often described as **bell-shaped**, such as is shown in the drawing below.



93. The most frequently occurring score is immediately obvious, since the highest column in the frequency distribution graph is for a score of _____. 2
94. While the frequency distribution graph clearly indicates the frequency of each score value, it does/does not does not indicate the gradual improvement in performance during the course of the experiment.
95. A graph of raw data/frequency distribution raw data would be the best way to indicate how the subject's performance improved during the course of the experiment, whereas a graph of raw data/frequency distribution frequency distribution is the simplest way of showing how the subject's tosses were distributed among the different scores.
96. In the previous experiment, one toss of the dart might actually be closer to the center of the target than another and yet receive the same score. For example, consider the target shown below:



If the X marks labeled "A" and "B" represent two places where a dart could have hit the target, the point labeled A/B would represent a more accurate toss of the dart.

A

217.

We also noted that standard scores are often referred to as **Z-scores** if the distribution under discussion is a normal distribution. In other words, a value in a normal distribution exactly two standard deviations from its mean would correspond to a Z-score of _____.

2

218.

A Z-score is simply the number of _____ between a particular value and the _____ in a _____ distribution.

standard
deviations
mean, normal
proportion

219.

We have seen how the previous graph can be used to indicate the _____ of values within any number of standard deviations from the mean of a normal distribution.

220.

We say, for example, that .68 of all the values in a normal distribution were within one standard deviation from the mean. This implies that both values less than one standard deviation above and below the mean represent _____ of all the values in the distribution.

.68

221.

We can state this same fact in terms of Z-scores by saying that .68 of all the values in a normal distribution correspond to an absolute Z-score of 1 or less. We say **absolute Z-scores** since we are including values with both negative and positive deviations from the mean, so long as the deviation is equal to or smaller than _____ standard deviation.

one

If all darts falling within the center circle of the target receive a score of 3, the dart hitting at Point A would receive a score of _____ and the dart hitting at Point B would also receive a score of _____.

3

3

97. Therefore, although one dart was actually thrown more accurately than another, it received the same score. In evaluating the subject's performance, it is easier to **group** all tosses of the dart that hit within the center circle into the same category of accuracy. In a similar manner, we do not consider exactly how far away from the target a dart was if it missed the target completely, since all such darts received a score of _____.

0

98. Another way we might assign a score to each toss of a dart would be to measure exactly how far from the center of the target each dart struck. If we followed this procedure, there _____ be four possible values of the "score" variable.

would not

99. The number of possible values of the "score" variable would depend upon how precisely we wanted to measure the distance between the dart and the actual center of the target.

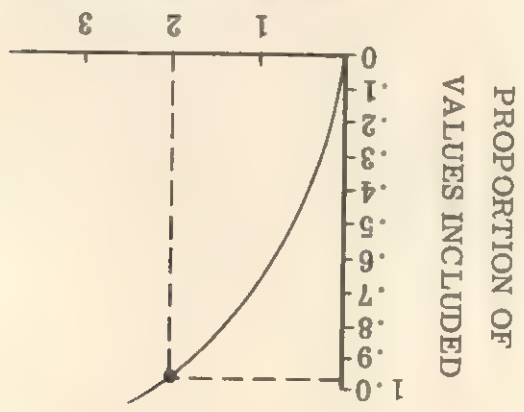
If we measured this distance to the closest one-thousandth of an inch, there would be _____ possible values

more

of the score variable than if we measured down to the closest one-hundredth of an inch. The more precisely we measured the distance, the more possible values of the score variable there would be.

213.

The graph shown below indicates a point on the curve directly above the number _____ on the base of the graph.



214.

Notice that the point directly above the number 2 on the base of the graph is at the same height as the number .98/.91 on the side of the graph.

215.

The fact that the point on the curve directly above the number 2 is at the same height as the number .98 indicates that the proportion of values within two standard deviations of the mean of a normal distribution is _____. You should also recall that a value could be represented as a **standard score**. A standard score was simply the number of standard deviations between a particular value and its mean. We could say that a value two standard deviations away from the mean corresponded to a **standard score** of _____.

216.

Similarly, a value three standard deviations from the mean corresponded to a _____ s. If a value corresponded to a standard score of 1, the difference between that value and its mean corresponds to one _____.

100.

The more precisely we measured the exact distance between the center of the target and the place where the dart struck, the often would we find two
more/less

less

tosses of exactly the same score. If we measured closely enough, in fact, we would probably find that the subject never received exactly the same score on any two tosses.

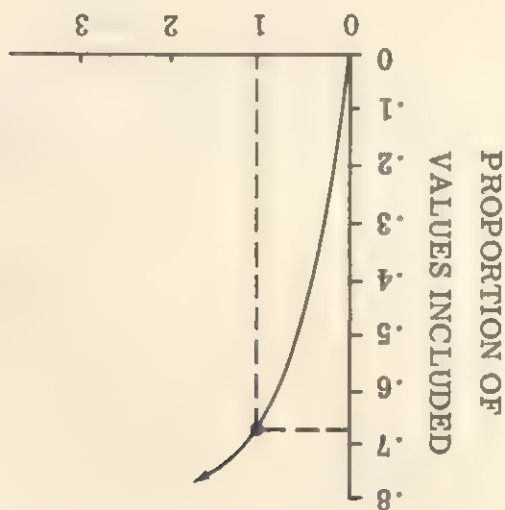
101.

It is often useful to consider two observations that are really slightly different as having the same value. In other words, to **group** together observations that are sufficiently similar and to consider them as having the same value is often a useful procedure. The following illustration will indicate how we can group observations in this way.

Suppose you were studying how long it took a person to solve a certain mathematical problem. If you made observations on **twenty** people, recording the time it took each person to solve the problem, your data might be represented by the following table:

Subject	Time in Seconds	Subject	Time in Seconds
1	60	11	20
2	128	12	72
3	68	13	71
4	69	14	120
5	80	15	85
6	75	16	116
7	32	17	99
8	149	18	15
9	16	19	36
10	110	20	86

To determine the proportion of values in a normal distribution within **one** standard deviation of the mean, you simply locate the number 1 on the bottom of the graph and then locate a point on the curve directly **above** the number 1, as indicated on the graph shown below.



Notice that the point on the curve directly **above** the number 1 on the base line is at the same height as the number $\frac{.68}{.78}$ on the side of the graph (as indicated by the horizontal dotted line).

212.

The fact that .68 is at the same height as a point on the curve directly above the number 1 on the base of the graph indicates that exactly .68 of all the values in a normal distribution are within one standard deviation of the mean. To find the number of values that are within two standard deviations of the mean in a normal distribution, you first locate a point on the curve directly above the number _____ on the base of the graph.

2

101. (Continued)

Notice there _____ a value of the time variable is not
is/ is not

which occurs more than once in the table of data. (You might wish to insert a bookmark indicating this page since we will refer to the table shown above in the next several frames.)

102. Since every observed value of the data occurs once and only once, each of these observed values would have a frequency of _____. 1

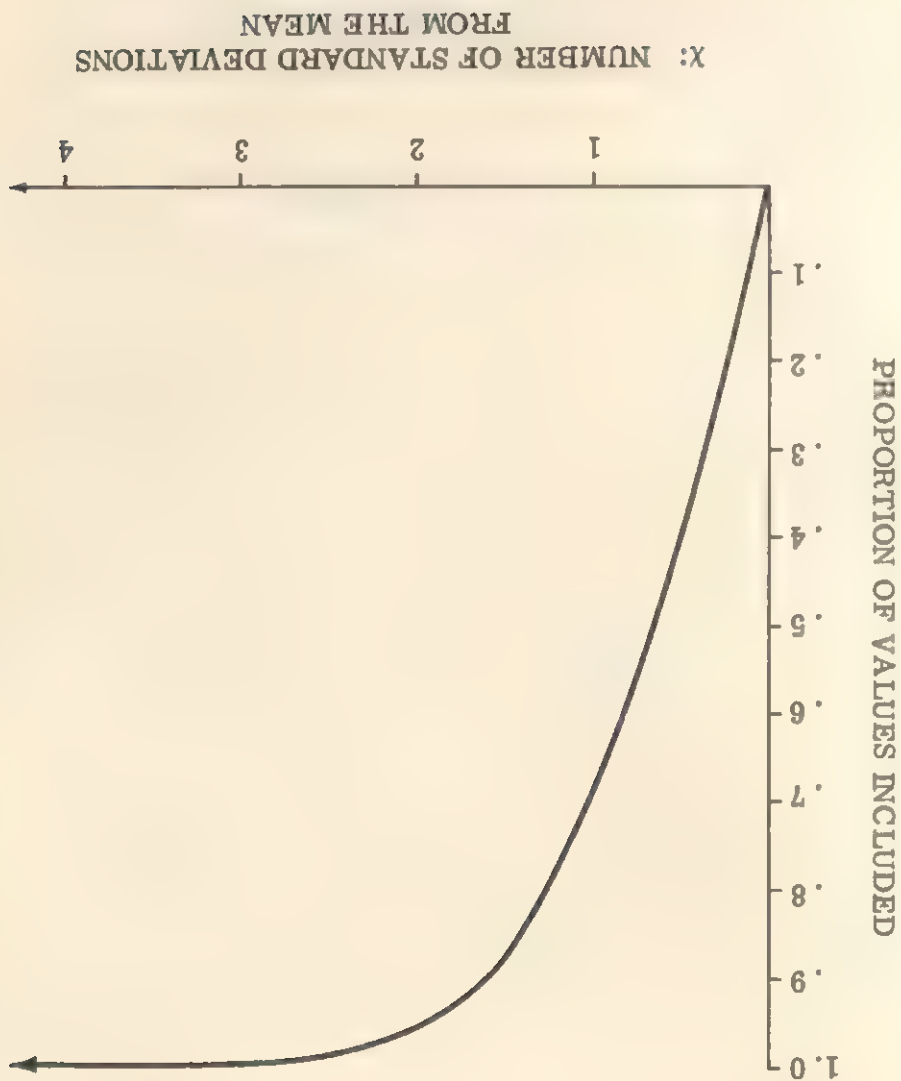
103. Notice that the highest value (the longest time) recorded was _____ seconds, whereas the lowest value recorded was _____ seconds. 149
15

104. Suppose we counted all the "times" of 50 seconds or faster. We would find that exactly five of the observed times fell into this group of times. This group of times would consist of the 32 seconds observed for Subject 7, the 16 seconds observed for Subject 9, the 20 seconds observed for Subject 11, the 15 seconds observed for Subject 18, and the _____ seconds observed for Subject _____. 36
19

105. We could say the frequency of "times" between zero and 50 seconds was five, since there are exactly five observed times that were less than or equal to _____ seconds. 50

We indicated how a normal distribution could be described by the following graph:

PROPORTION OF VALUES WITHIN χ STANDARD DEVIATIONS OF THE MEAN IN A NORMAL DISTRIBUTION



106. Suppose we counted the "times" between 51 seconds and 100 seconds (including times of 51 seconds or 100 seconds). The time recorded for Subject 1 _____ be counted, since this time is between _____ would
would/would not
51 seconds and 100 seconds.

There are exactly _____ observed times that fall 10
between 51 and 100 seconds in the previous table of data. The next group of times that we will consider are those times between 101 and 150 seconds, including any that might be 101 or 150 seconds. Subject 2's time of 128 seconds _____ belong to the group of times does
does/does not
between 101 and 150 seconds. The other times which fall into this group are the time of 149 seconds recorded for Subject 8, the time of 110 seconds recorded for Subject 10, the time of _____ seconds recorded for 120
Subject _____, and the time of 116 seconds recorded for 14
Subject 16.

107. We could summarize these frequencies in the following frequency table of **grouped** data.

TIME	FREQUENCY
0-50 sec.	5
51-100 sec.	10
101-150 sec.	5

Notice the three rows correspond to the three different **groups** of times.

208. In the next section, we will consider the advantage of knowing how often different types of samples can be expected to occur when you draw random samples from a particular type of population. It is only because we use _____ sampling procedures rather than biased sampling procedures that we can use probability theory to calculate theoretical sampling distributions.
209. For example, suppose we had only asked a group of seniors in the French department for their opinions concerning the overseas campus. This would represent a $\frac{\text{biased / random}}{\text{sample from the population}}$ consisting of the 10,000 opinions of the total student body.
210. With a $\frac{\text{random / biased}}{\text{sampling procedure}}$, it is possible to predict the proportion of favorable opinions in the population. If we obtained samples in a $\frac{\text{biased / random}}$ manner, it would not be possible to calculate a theoretical sampling distribution using probability theory, since our sampling procedure would not be a random process.
211. **Binomial distributions** are only one of several common types of distributions that can be used as a theoretical sampling distribution. The **normal distribution** we considered in an earlier section is another type of distribution often used as a theoretical sampling distribution. We described a normal distribution in terms of the proportion of the values that were within each possible number of standard deviations from its mean.

107.

(Continued)

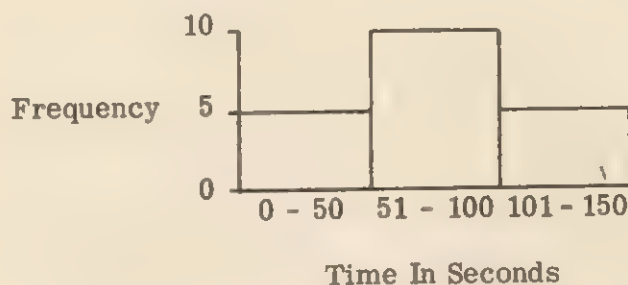
For instance, in the first row "0-50 seconds" indicates that we counted all the times between 0 and 50 seconds. The numeral 5 in the same row but in the frequency column indicates that the _____ of observed times falling within this group was _____.

frequency (number)
five

108.

A frequency table of this sort is called a frequency table of **grouped** data because we have determined the frequency for _____ of values, rather than the frequency for particular values. Below is a graph of the previous frequency table of grouped data:

groups



This frequency distribution gives a reasonable picture of the distribution of times for the different subjects. It indicates clearly that some subjects had times less than 50 seconds, that about the same number had times greater than 100 seconds, and that the most typical times fell between _____ and _____ seconds.

51, 100

205.

Previously, when we considered **experimental** sampling distributions, we were able to determine how frequently to expect different types of samples to occur by actually obtaining a large number of samples. However, it

necessary to actually obtain a large number of is/is not

samples in order to calculate a **theoretical** sampling

distribution using the rules of probability.

206.

It might seem like magic to be able to obtain a sampling distribution without actually having to collect a large number of samples. When we use probability theory to calculate theoretical sampling distributions, we are simply taking advantage of what has already been

determined about random processes. This is perhaps

the major advantage of using **random sampling procedures**

to collect samples. Probability theory can be used to

represent processes. Random samples

are obtained by a process. Therefore,

you can use probability theory to calculate theoretical

sampling distributions.

207.

You have seen that the procedure we considered for

obtaining samples of students' opinions is similar to a

simple random process, such as flipping a coin. It is

the similarity between the random sampling procedure

for obtaining student opinions and the random process

of flipping a coin that allows us to use p

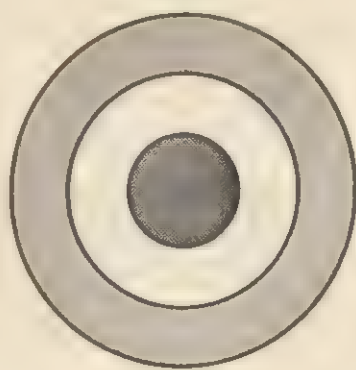
theory to calculate a theoretical

distribution for the sample of student opinions without

having to actually collect a large number of samples.

If you are interested in obtaining a rough picture of the frequency distribution, it is often useful to group the data in this way — that is, not distinguishing between values that fall within the same group.

Just how much you can group your data and still obtain a sufficiently clear picture of the distribution depends on your particular purposes. For example, in the previous illustration in which subjects were throwing a dart at a target, it was sufficient to divide the board into three circles and assign one of three scores depending upon where each dart struck. Consider the two targets shown below. Target A is divided as the target was in the previous illustration. In Target B, however, more circles have been drawn, thereby dividing the target into narrower rings.



TARGET A



TARGET B

If you assigned a score to each dart depending upon which ring it struck, there would be more possible scores on Target $\frac{A}{B}$. The only difference between

B

Target A and Target B, however, is that you could distinguish the position of a dart more precisely on Target $\frac{A}{B}$.

B

200.

You could toss a coin seven times and record the number of heads in those seven tosses. You could then repeat this procedure and record the number of heads in those seven tosses. You could repeat this procedure over and over again until you had a very long list of numbers. According to the previous theoretical sampling distribution, you would expect more of these numbers to be $\frac{3}{2}$ than you would $\frac{3}{2}$.

3, 2

201.

Similarly, according to the previous theoretical sampling distribution, it would be $\frac{\text{more} / \text{less}}{\text{common to obtain}}$

more

a sample of seven tosses in which there were 5 heads than samples of seven tosses in which there were 6 heads.

202.

Finally, the distribution indicates that samples of seven tosses which were all heads (or all tails) would occur very $\frac{\text{seldom} / \text{often}}$ compared to the other types of samples.

seldom

203.

In other words, the types of samples which have a very small probability would be expected to occur very

infrequently

infrequently / infrequently

204.

Specifically, if a particular type of sample had a probability of .1, you would expect that type of sample to occur only about _____ time(s) out of every 10 samples obtained in that manner.

1

110. In a similar way, we might have divided the previous mathematical problem data into smaller groups. For example, we could have divided each of the groups of times into two smaller groups. Instead of considering all the times from 0 to 50 seconds, we would have considered all the times from 0 to 25 and from 26 to 50 seconds. Instead of considering all the times from 51 to 100 seconds as a single group, we would have formed two groups, 51 to 75 and 76 to 100. Finally, we could have divided the group of scores from _____ to _____ seconds into the following two groups: 101 to 125 seconds and 126 to 150 seconds. 101, 150
111. If you become a scientist, one of your chief jobs will be to make observations of variables. Records of these observations are called _____. data
112. We have seen that one way of presenting data is to arrange it in rows and columns to form a _____. table
113. A table listing every observed value of the variable is often referred to as a table of _____ data. raw
114. The number of times a particular value occurs in a table of raw data is referred to as the _____ of that value. frequency
115. You _____ determine the frequency of each value for both a numerical and a non-numerical variable. can
can/cannot

certain information about the random process, it is possible, using simple rules of probability, to calculate a theoretical sampling distribution. This distribution indicates the relative frequency with which each possible type of sample would be expected to occur if we the sampling procedure over and over again a very great number of times.

repeated

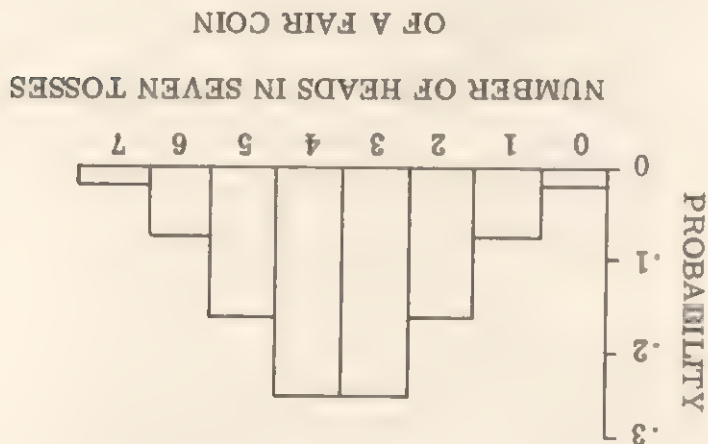
theoretical

binomial

198. Whenever a random process is repeated N times resulting in one of two outcomes each of these N times, you can determine the appropriate binomial distribution to use as a theoretical sampling distribution if you know the probability of each of the two possible outcomes.

binomial

199. The following distribution can be used as a theoretical sampling distribution to describe the probability of obtaining any number between 0 and 7 heads out of seven tosses of a fair coin.



OF A FAIR COIN

Notice that the probability of obtaining 3 heads in seven tosses is larger/smaller than is the probability of obtaining two heads in seven tosses.

larger

116. Instead of presenting a complete list of all of the observed values in your data, it is often useful to **summarize** the data in some way. Any term or number summarizing data in this way is called a s _____. statistic
117. One way we can summarize data is to report the frequency of occurrence for each value in the table of raw data. Therefore, each of these frequencies would be a _____. statistic
118. Since the frequency of a value is determined by counting (enumerating) the number of times it was observed, a frequency is often referred to as an _____. enumerative
119. Any value of the variable not observed (one that does not occur in the table of raw data) would have a frequency of _____. If the data consisted of 20 observations, the greatest possible frequency of any value would be _____. 0
20
120. We can summarize the contents of a table of raw data in a table listing the number of times each value occurred in the data. This summarizing table is often referred to as a _____ table. frequency
121. The enumerative statistics presented in a frequency table indicate how our observations were distributed among the different possible _____ of the variable we were studying. values

the first draw, they were so close to being independent that we could consider them as if they were. Therefore, we could treat the two draws as a series of two completions of a random process, where each completion of the process could have only two possible outcomes: "yes" or "no." The theoretical sampling distribution, therefore, was another example of a _____ distribution.

196.

In situations such as this, all that is required to determine the appropriate binomial distribution to use as a

theoretical sampling distribution is to determine the probability of each of the two outcomes. Thus, knowing that the probability of heads equals $\frac{1}{2}$ (and that the probability of tails equals $\frac{1}{2}$), each time a fair coin is tossed is sufficient to determine the probability of obtaining each possible number of heads in N tosses of a coin. Similarly, the only information necessary to determine the theoretical sampling distribution indicating the probability of each possible number of "one dot" rolls in N rolls of a die is that the probability of getting "one dot" is _____, whereas the probability of getting "more than one dot" is _____ on each individual roll of the die.

197.

We will not attempt to consider exactly how a statistician determines the particular binomial distribution to use as a theoretical sampling distribution for any particular number of tosses of a coin or rolls of a die. The procedure is similar (though more complicated) to the procedure we have already used for calculating the theoretical sampling distribution of two tosses of a coin. The important point we wish to make is that, given

122. The **frequency distribution** of a collection of data is the collection of frequencies for the values in that data. Therefore, the enumerative statistics in a frequency table indicate the _____ of the data. frequency distribution
123. If you were told that a coin had been tossed 100 times and had come up "heads" 75 times and "tails" 25 times, you _____ know the frequency distribution would/ would not would of the data.
124. Suppose you tossed a coin 100 times and observed 50 "heads" and 50 "tails." Instead of reporting the actual frequency of "heads" or "tails," you might describe the distribution of the variable by saying that one-half of the tosses were "heads" and that _____ of the tosses were "tails." one half
125. If you observed 500 "heads" and "500" tails in 1000 tosses of a coin, you could _____ say that one half also/ not also of your observations were heads and one half of your observations were tails.
126. When you say one half of the observations are "heads," you are not indicating the actual frequency of "heads" in the data. Instead, you are indicating what **part** of all the observations is "heads." For example, if I told you I tossed a coin some unknown number of times and that one half of the observations were "heads," you _____ know the actual frequency of heads; would/ would not would not you _____ know, however, what part of the would/ would not would data was made up of observations of "heads."

(for any particular value of N) would be an example of a **binomial distribution**. Thus, the theoretical sampling distribution indicating the probability of obtaining each possible number of heads in 10 tosses of a coin would be an example of a **binomial distribution**.

194. The important feature of repeated tosses of a coin

allowing us to represent them with a binomial distribution is that successive tosses of a coin are independent and can be viewed as repeated completions of a simple

random process with two possible outcomes. Suppose we grouped the outcomes of rolling a die so as to only consider two possible outcomes: "one dot" or "more than one dot." We could view repeated rolls of the die as a series of completions of a random process with only two possible outcomes ("one dot" or "more than one dot"). A theoretical sampling distribution indicating the probability of obtaining each possible number of one dot outcomes in any series of rolls would be an example

of a **binomial distribution**.

195. Statisticians and mathematicians have analyzed processes

of this sort so that it is possible for them to determine the particular binomial distribution which would serve as a theoretical sampling distribution for any series of N completions of a simple random process with only two possible outcomes. Earlier, we considered an example in which we obtained a theoretical sampling distribution indicating the probability of obtaining 0, 1, or 2 favorable opinions in a sample of 2 opinions drawn from a population of 10,000 student opinions. While it is not strictly true that the opinion sampled on the

second draw was **independent** of the opinion sampled on

127. If you knew the frequency of a particular value was 10, you would not know whether observations of that value represented a large part of the data or a small part of the data. If a value had a frequency of 10, observations of that value would represent a **larger part** of your data if the data consisted of $\frac{12}{12/100}$ 12 observations rather than $\frac{12}{12/100}$ observations. 100
128. Instead of reporting the actual frequencies of each value in your data, it often is useful to indicate the **relative** frequency of each value. The relative frequency of a value indicates how often that value occurred in relation to the total number of observations. For example, if you said the value 10 occurred 30 times in the data, you would be indicating the actual or **absolute** frequency of the value 10. If you said one-third of the total number of observations had the value 10, you would be indicating the $\frac{\text{absolute}}{\text{relative}}$ frequency of the value 10. relative
129. Adding together all of the frequencies in a frequency table indicates the total number of observations in the data. To find out what **proportion** or part of the total is represented by a particular frequency, you simply divide that frequency by the total number of observations. If your data consisted of 100 observations and a particular value had a frequency of 50, the proportion of your observations having the value 50 would equal 50 divided by _____, which equals one-half. 100

distribution that indicated the probability of obtaining 0, 1 or 2 heads in two tosses of a coin. In much the same manner, we also could have calculated a theoretical sampling distribution that indicated the probability of obtaining 0, 1, 2, or 3 heads out of three tosses of a coin. If we represented the number of times we tossed the coin by the letter N , we know that the fewest heads that could be obtained in N tosses would be 0, whereas the largest number of heads that could be obtained would be N .

In other words, if we tossed the coin seven times, N would equal _____. If we tossed the coin 20 times, N would equal _____.
 7 20

192. You saw how it was possible to calculate a theoretical sampling distribution indicating the probability of obtaining each of the three possible numbers of heads (0, 1, or 2) when we tossed a coin twice (in other words, when $N = \underline{\hspace{1cm}}$).
 2

193. By using much the same procedure, it is possible to calculate a theoretical sampling distribution indicating the probability of obtaining each possible number of heads in N tosses of a coin, no matter what the value of N . For example, we could calculate a _____ sampling distribution indicating the probability of obtaining 0, 1, 2, 3, or 4 heads out of 4 tosses of a coin (when $N = \underline{\hspace{1cm}}$). The theoretical sampling distribution representing the probability of obtaining each possible number of heads out of N tosses of a coin
 4 theoretical

130. If your data consisted of only 4 observations and a particular value had a frequency of 1, then the proportion of times that particular value occurred is _____ divided by _____, or one-fourth. 1, 4
131. If your data consisted of 1000 observations and you observed a particular value 250 times, the frequency of that value would represent **one-fourth** of the total number of observations, since 250 divided by 1000 equals _____. one-fourth
132. It is possible to represent a **proportion** in several different ways. For example, you could represent the proportion "one-half" either as a **fraction**, which would be written as $\frac{1}{2}$, or as a **decimal**, which would be written as .5. Similarly, you could represent the proportion "one-tenth" as either a **fraction**, which would be written _____, or as a **decimal**, which would be written _____. $\frac{1}{10}$
.1
- If your data consisted of 100 observations of a variable and 10 of those observations had the value 4, you could say the proportion of observations in your data having the value of 4 were _____ (representing the proportion as a **fraction**) or _____ (representing the proportion as a **decimal**). $\frac{10}{100} = \frac{1}{10}$
.1
133. It is possible to convert a particular fraction into another form representing the same value. For example, $\frac{2}{4}$ is the same as $\frac{1}{2}$. Similarly, you can write a decimal in several ways. For example, .2 is the same as $\frac{.20}{.02}$. .20

For example, while you could not exactly predict the character of any particular sample, you could make the following type of confidence statement on the basis of the previous theoretical sampling distribution:

"I would expect to obtain only about $\frac{9}{20}$ samples in

which there were no favorable opinions out of every 100 samples of size 2 obtained in this manner."

189. According to the theoretical sampling distribution, the probability (limiting relative frequency) of samples in which no favorable opinions occurred is .09, which could be interpreted as meaning that only about 9 out of every samples would be expected to contain no

favorable opinions.

190. You have seen how it is possible to represent random processes with probability theory. You have also seen how it is possible to analyze sampling procedures by

using probability theory to obtain theoretical sampling distributions. These sampling distributions are useful because they indicate how often you would

to obtain each of the possible types of samples. We

shall see in the next section how information of this

sort can be used to make decisions about populations

on the basis of random samples from these populations.

191. The theoretical sampling distributions we have just

considered are examples of a very important and useful type of theoretical distribution called a **binomial**

distribution. We calculated a theoretical sampling

Statisticians often find it useful to represent a proportion in terms of **hundredths**. In these terms, one-fourth would equal $\frac{25}{100}$, whereas one-half would equal $\frac{50}{100}$. Similarly, $\frac{3}{4}$ would equal _____ hundredths.

75

134.

If a certain proportion equals $25/100$, we often say that it equals 25 **percent**. Similarly, if a proportion equals $50/100$, we can say it equals fifty **percent**. Representing a proportion as a percentage is simply a convenient way of comparing proportions by converting them all into **hundredths**. For example, you could compare one-half and one-fourth by saying one-half equals _____

50

percent, whereas one-fourth equals $\frac{50}{25}$ percent,

25

(since $1/2 = 50/100$ and $1/4 = 25/100$.) Similarly, the proportion .25 represents $25/100$ or _____ percent.

25

135.

If you said the frequency of a particular value in a collection of data represented 50 percent of the data, you would know the frequency of that value represented _____ hundredths of the data. Thus, if the data consisted of 100 observations, a value would have to have a frequency of _____ in order to represent 50 per cent of the observations, since 50 per cent is equivalent to $50/100$. A value observed on five out of ten observations also represents 50 percent of those 10 observations since $5/10$ equals _____ hundredths.

50

50

50

184. The probability of obtaining two **unfavorable** opinions is simply $\Pr(\quad)$, or .09.

$\Pr(nn)$

185. In order to obtain the probability of obtaining one favorable opinion, however, we must apply the **addition** rule and add $\Pr(\quad)$ and $\Pr(\quad)$.

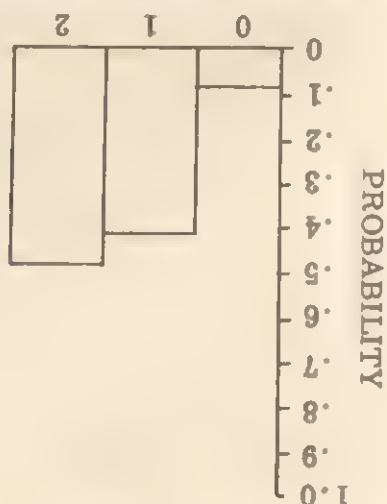
y_n, n_y

186. Thus, $\Pr(\text{one favorable opinion}) = \Pr(y_n) + \Pr(n_y)$
 $= \quad + \quad = \quad$ (Refer to the previous table for the actual probabilities.)

.21, .21, .42

187. Therefore, by means of a probability analysis, we have determined a sampling distribution for samples of size two drawn from a population consisting of 10,000 students' opinions, of which 3,000 were unfavorable and 7,000 were favorable.

188. The importance of this theoretical sampling distribution (shown on the graph below) is that it tells us what proportion of each possible type of sample to **expect** when we obtain random samples from a population of this sort.



NUMBER OF FAVORABLE OPINIONS
IN A SAMPLE OF SIZE 2

136. If your data consisted of one thousand observations, of which 250 had the value 5, you could say that _____ percent of your observations had the value 5. If 250 out of one thousand observations had the value 5, then $250/1000$, or $25/100$, of the observations would have the value 5. This is why you could say that 25 p_____ of the observations had the value 5. percent

137. Instead of the word "percent", the symbol % is often written after a number. Thus, 25% means the same thing as 25 _____ percent

A proportion written as a percent is often referred to as a **percentage**. Therefore, the following is a list of _____ s. percentages

20%, 30%, 5%, 8%, 2%

Remember, $\frac{20\%}{20\%/2\%}$ represents a proportion of $\frac{20}{100}$ 20%

138. While it is often useful to represent a frequency as a proportion or percentage, it is important to consider the actual frequency. For example, suppose an automobile dealer told you $3/4$ of the people to whom he had sold a particular car claimed it was the best car they had ever driven. If he had sold one thousand of these cars, the proportion $3/4$ would represent $\frac{750}{250}$ people. On the other hand, if he had only sold four such cars, only _____ people would have told him it was the best car they had ever driven. 750 3

Remember, 50% of 1000 is _____, whereas, 50% of 100 is _____. 500 50

By using this same reasoning, we could determine the probabilities for each possible pair of opinions. Our results would be those shown in the following table:

Pairs of Opinions	Multiplication Rule	Probability
YY	.7 x .7	.49
YN	.7 x .3	.21
NY	.3 x .7	.21
NN	.3 x .3	.09

In the first column of the table, we have indicated the four possible pairs of opinions. In the second column (in the appropriate row), we have indicated how the multiplication rule would be applied. In the third column, we have indicated the actual probability for each of the possible pairs of opinions. Thus, the probability of obtaining two unfavorable opinions was obtained by multiplying $\frac{\quad}{\quad} \times \frac{\quad}{\quad}$, which indicates indicates $\Pr(m) = \frac{\quad}{\quad}$.

.3, .3
.09

182.

When we considered the samples of two opinions drawn from the population, we were only interested in the number of favorable opinions in the sample. In a sample of size two, it is only possible to obtain 0, $\frac{\quad}{\quad}$, or $\frac{\quad}{\quad}$ favorable opinions.

1
2

183.

We could group the four possible outcomes shown in the previous table, just as we grouped the four possible outcomes for two tosses of a coin where we were only interested in how many heads had occurred. Thus, the probability of obtaining two favorable opinions is simply $\Pr(yy)$, or $\frac{\quad}{\quad}$.

.49

139. It is easy to convert any frequency table into a table listing the proportion of times each value was observed. To do so, you simply divide each frequency in the frequency table by the total number of observations in your data. By doing so, you convert each

$\frac{\text{frequency}}{\text{absolute/relative frequency or proportion}}$ to a(n) $\frac{\text{absolute/relative}}{\text{absolute/relative}}$

absolute, relative

140. Consider the frequency table shown below:

VALUE	FREQUENCY	PROPORTION
red	2	
green	3	
blue	5	

The data represented in this table consist of _____ observations of a _____ variable.
numerical/non-numerical

10

non-numerical

141. In the third column of the above table, we could list the proportion of observations having each of the three values. For example, the value "red" occurred _____ times in the _____ observations. Therefore, the proportion of times Red occurred equals _____ divided by _____.

2

10

2

10

142. The proportion of times that the value "green" was observed equals _____ divided by _____, which equals _____.

3, 10

$\frac{3}{10}$

The two draws making up the sample of size two are not **exactly** independent, since the probability of

obtaining "yes" on the second draw, given that a "no" was obtained on the first draw, is slightly different

from the probability of obtaining a "yes" on the second draw, given a "yes" on the first draw. However, the

population is so very large that drawing out a single

opinion only alters the probability distribution by a very,

very small amount. Thus, we will pretend that the two

draws are independent, which allows us to use the

rule to obtain the probability of addition/multiplication

a "yes" opinion, followed by a second "yes" opinion.

180.

If the probability of obtaining a "yes" opinion is $7/10$,

then the probability of obtaining two yes opinions in a

row (assuming the two draws are independent) is equal

to $.7$ times $.7$, according to the **multiplication** rule.

$$\Pr(yy) = \Pr(y) \times \Pr(y) = .7 \times .7 = .49$$

All this means is that out of every 100 **pairs** of opinions

drawn in this manner, you would expect about 70 of them

to begin with a favorable opinion. Of the 70 pairs of

opinions that began with a favorable opinion, about $7/10$

of them would also have a favorable second opinion.

Since $7/10$ of 70 is 49, you would expect about 49 of the

100 pairs of opinions to consist of two "yes" opinions.

This is indicated by saying $\Pr(yy) = \underline{\hspace{2cm}}$.

.49

143.

The _____ of times the value "blue" occurred in the data equals 5 divided by 10, or $1/2$.

proportion

144.

VALUE	FREQUENCY	PROPORTION
red	2	$2/10$
green	3	$3/10$
blue	5	$5/10$

The numbers in the third column of this table are the _____ we just calculated.

proportions

145.

Each of these proportions tells what **part** of the total number of observations is represented by each value. If a particular value had a frequency of 0, you would know that **none** of the observations had that value; therefore, the proportion of observations having that value would equal 0 divided by the total number of observations, which means the proportion would equal _____.

0

146.

Suppose you asked 100 people to predict which party, Democratic or Republican, would win the next Presidential election. Your data might look like that summarized in the frequency table shown below:

PARTY	FREQUENCY
DEMOCRAT	75
REPUBLICAN	25

represented by NN, NY, YN, and YY, where, for example, NY means a "no" opinion was drawn first and was followed by a _____ opinion.

yes

178. As you recall, we were able to determine the probability distribution appropriate for two successive independent tosses of a coin by application of the **multiplication** rule, whereby we multiplied the probability of obtaining a head on the first toss times the probability of obtaining a head on the second toss, to determine the probability of obtaining two successive heads. There is a very slight difference, however, between this situation and the situation involving successive tosses of a coin. As you recall, we stipulated that the multiplication rule applied to successive, **independent** outcomes of a random process. By independent, we meant that the character of one outcome in no way determined the character of the other outcome. Consider very carefully, however, what occurs when two opinions are drawn from a population in the present illustration to form a single sample of size two. If the first opinion in the sample were a "no," we would have removed one of the unfavorable opinions from the population.

Therefore, when the second opinion in the sample is drawn, there will be 7,000 favorable opinions in the population but only 2,999 unfavorable opinions. Thus, the proportion of unfavorable and favorable opinions on the second draw depends upon what opinion was obtained on the first draw. In this sense, the two draws

are/are not exactly independent.

are not

146. (Continued)

You could describe your data as consisting of _____ 100
observations of a variable you might call "predicted
party," with "Democratic" and "Republican" the two
possible _____ of that variable. values

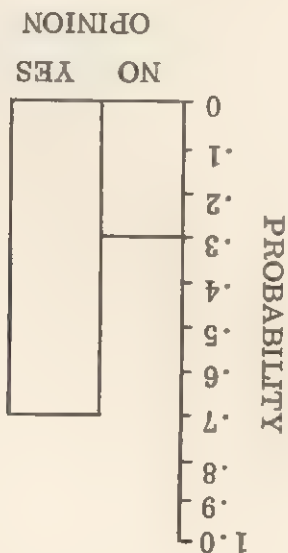
147. The **proportion** of people predicting Democratic _____ 3/4
whereas the proportion of people predicting Republican
was _____. 1/4

148. The number 75 indicates the _____ frequency absolute
absolute/relative
of the value "Democratic" in the previous data, whereas
the number $\frac{3}{4}$ indicates this value's _____ relative
relative/absolute
frequency.

149. Just as the frequencies 75 and 25 describe the absolute
frequency distribution of the previous data, the
proportions $\frac{3}{4}$ and $\frac{1}{4}$ describe the **relative frequency**
distribution of the previous data.

If you tossed a coin 20 times and observed 10 "heads"
and 10 "tails," the numbers _____ and _____ 1/2, 1/2
indicate the relative (proportional) frequency
distribution of your observations.

The graph shown below indicates the probability of obtaining a positive opinion and the probability of obtaining a negative opinion when we randomly draw a single opinion from the population. In other words, the graph indicates a _____ distribution.



probability

Notice the similarity between this probability distribution and the probability distribution for a single toss of a coin. In this case, the sample space consists of the two opinions "no" and "yes," whereas, in the case of a toss of a coin, the sample space consisted of the two outcomes "heads" and "tails." With the coin, both possible outcomes ("heads" and "tails") had the same probability, whereas in this case the two outcomes _____ have _____ the same probability.

Suppose we consider the four possible outcomes that can occur when we draw two opinions from the population. The first opinion we draw can be either a "no" or a "yes." Therefore, the four combinations of opinions could be

150. Suppose you asked 1,000 people, rather than 100 people, to predict who would win the election. You might have obtained the data represented in the following frequency table:

PARTY	FREQUENCY
DEMOCRATIC	750
REPUBLICAN	250

In the example where we asked 100 people, $\frac{3}{4}$ of them predicted "Democratic" and $\frac{1}{4}$ predicted "Republican." In the present example of 1,000 predictions, the proportion of people predicting "Democratic" is _____ whereas the proportion of people predicting "Republican" is _____.

$\frac{3}{4}$

$\frac{1}{4}$

151. The important point to be made here is that even though the _____ frequency distributions were _____ different for the two examples, the _____ frequency distributions were the same.

absolute

relative

152. Let us consider another case in which we change an absolute frequency distribution to a relative frequency distribution by converting each frequency to a proportion. Suppose you have the data represented in the following frequency table:

VALUE	FREQUENCY
0-5	3
6-10	5
11-15	2

173.

What we are really interested in, however, is not which particular student's opinion is sampled, but the nature of his opinion. Therefore, it would be appropriate to

group together the opinions of all students who answered yes and all students who answered no. If 3,000 students answered "no," we would add the probabilities of

sampling each of their opinions to find the probability of obtaining any opinion from the group of negative opinions. Therefore, since each of the 3,000 negative opinions has a probability of 1/10,000, the probability of obtaining 1 or another of these negative opinions is simply 3,000

divided by 10,000, or $\frac{3/10 - 3/100}{}$.

174.

Notice also that the probability of obtaining a negative

opinion is simply the proportion of negative opinions in the population. This is a characteristic of all sample spaces in which every outcome has been assigned the same probability. In other words, if every outcome in

a sample space has the same probability and if we combine some number of these outcomes to form a group of outcomes, the probability of sampling any member of

the group is simply the number of outcomes in the group divided by the total number of outcomes. For example, if a sample space consisted of ten outcomes, each

having the same probability, the probability for obtaining any member of a group of three of these

outcomes is equal to $\frac{\quad}{10}$.

175.

Returning to our illustration, we can say that the

probability of obtaining a negative opinion when we draw a single opinion from the population is equal to 3/10.

Therefore, the probability of obtaining a positive opinion is $\frac{\quad}{}$ since there are only two possible outcomes

and since the sum of their probabilities must equal 1.

173.

What we are really interested in, however, is not which particular student's opinion is sampled, but the nature of his opinion. Therefore, it would be appropriate to

group together the opinions of all students who answered yes and all students who answered no. If 3,000 students answered "no," we would add the probabilities of

sampling each of their opinions to find the probability of obtaining any opinion from the group of negative opinions. Therefore, since each of the 3,000 negative opinions has a probability of 1/10,000, the probability of obtaining 1 or another of these negative opinions is simply 3,000

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174.

Notice also that the probability of obtaining a negative

opinion is simply the proportion of negative opinions in the population. This is a characteristic of all sample spaces in which every outcome has been assigned the same probability. In other words, if every outcome in

a sample space has the same probability and if we combine some number of these outcomes to form a group of outcomes, the probability of sampling any member of

the group is simply the number of outcomes in the group divided by the total number of outcomes. For example, if a sample space consisted of ten outcomes, each

having the same probability, the probability for obtaining any member of a group of three of these

outcomes is equal to $\frac{\quad}{10}$.

175.

Returning to our illustration, we can say that the

probability of obtaining a negative opinion when we draw a single opinion from the population is equal to 3/10.

Therefore, the probability of obtaining a positive opinion is $\frac{\quad}{}$ since there are only two possible outcomes

and since the sum of their probabilities must equal 1.

152. (Continued)

Note that this data consists of $\frac{10}{12}$ observations of 10
a $\frac{\text{numerical}}{\text{numerical/non-numerical}}$ variable. numerical

Furthermore, the frequencies in the table are for
 $\frac{\text{grouped}}{\text{grouped/ungrouped}}$ data. grouped

153. The number 3 represents the number of times a value
between _____ and _____ occurred in the data. 0, 5

154. The number of times a value between 11 and 15 was
observed is _____. 2

155. The group of values between _____ and _____
occurred more often than any other group of values
in the data. 6, 10

156. The **proportion** of times a value was observed between
6 and 10 is equal to _____ divided by _____, which
equals $1/2$. 5, 10

157. The proportion of times a value between 0 and 5 was
observed is _____, whereas the proportion of
times values between 11 and 15 were observed is _____. $3/10$
 $2/10$ ($1/5$)

158. Therefore, the $\frac{\text{absolute}}{\text{absolute/relative}}$ frequency distribution
of the previous data is represented by the 3, 5, and 2,
whereas the $\frac{\text{absolute}}{\text{absolute/relative}}$ frequency distribution is
represented by the numbers $3/10$, $5/10$, and $2/10$. absolute
relative

170. While the theoretical sampling distribution shown in

the same general procedure can be used to determine a theoretical sampling distribution for much more

complicated random sampling procedures. As an

illustration of how we can use the same reasoning

procedure to determine theoretical sampling distributions

in other situations, let's consider how we might

determine the theoretical sampling distribution for the

same situation for which we derived an experimental

sampling distribution. As you recall, we considered a

situation in which random samples of size 2 were

obtained from a population of 10,000 student opinions.

Let's suppose that 3,000 of the student opinions were

negative (the students answered "no"). Therefore,

_____ of the students' opinions were positive (the

students answered "yes").

171.

By thoroughly mixing all of the student opinions together in a large basket, we hoped to give each student's

opinion an equal opportunity of appearing in the sample.

In other words, we hoped that this was a _____

sampling procedure.

172.

Since there were 10,000 student opinions, each opinion could be regarded as having the same probability

because, after many repeated samples of this sort, we would expect each opinion to be sampled about the same proportion of times. If we viewed the 10,000 student

opinions as a sample space consisting of 10,000 possible outcomes, each outcome could be assigned a probability of 1/10,000 with the sum of the probabilities equal to _____

1

TABLE A

VALUE	FREQUENCY
0 - 5	5
6 - 10	3
11 - 15	2

TABLE B

VALUE	FREQUENCY
0 - 5	300
6 - 10	500
11 - 15	200

Of the two tables shown above, the table having the same **relative** (proportional) frequency distribution as the data shown in Frame 152 is Table A/B.

B

160. Notice that Table B consists of data from _____ observations. Thus, each frequency would be converted to a proportion by dividing it by _____.

1000

1000

161. Expressing the frequency of a value as a proportion indicates what part of the total number of observations were of that particular value. The largest possible proportion you could obtain would occur when all the observations in your data had the same value. This proportion would be _____.

one

For example, if the frequency of a particular value were 1,000 in a collection of 1,000 observations, the relative frequency (proportion) of observations having that value equals _____ divided by _____, or 1.

1,000 1,000

162. If a particular value had a frequency of 0, it would be represented in a relative frequency distribution by a proportion of _____.

0

Earlier, we considered an illustration in which we

obtained an **experimental** sampling distribution for

samples of size 2 obtained from a population of 10,000

student opinions (where each opinion was either for or

against a particular proposal). It is possible to view

each pair of tosses of a coin as a sample of size 2 from

an infinite population consisting of an unlimited number

of tosses. In that sense, the probability distribution in

Graph B is a sampling distribution for samples of size 2

from an unlimited population. In this case, however, we

have determined the sampling distribution **not** by actually

obtaining samples, but through a reasoning process

using probability theory. Thus, the sampling

distribution shown in Graph B can be thought of as a(n)

experimental/theoretical sampling distribution.

theoretical

To obtain this **theoretical** sampling distribution, we

begin with a careful consideration of what we mean by a

fair coin. Then, by carefully reasoning out the

implications of our definition of a fair coin, we arrived

at the conclusion that the sampling distribution shown in

Graph B should represent the sampling distribution for

pairs of tosses of a coin. We have also seen how

probability theory can be used to represent a random

process, such as tosses of a coin, and how simple rules,

such as the **addition** and **multiplication** rule, can be used

to represent characteristics of this random process.

The advantage of random processes by probability theory

is that (if our representation is appropriate) it is

possible to use the rules of probability theory in a

mechanical fashion, to arrive at a

experimental/theoretical

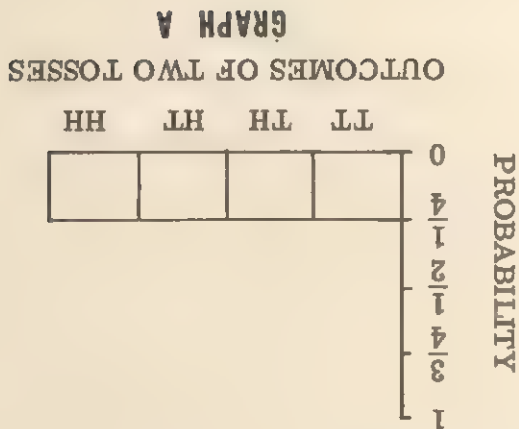
theoretical

sampling distribution.

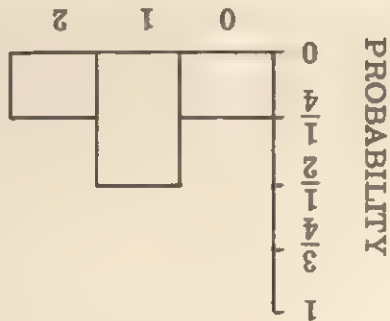
163. Notice that all of the proportions in a relative frequency distribution must lie somewhere between the largest possible proportion of _____ and the smallest possible proportion of _____. 1
0
164. Each of the proportions in a relative frequency distribution represents the part of the total collection of observations having a particular value. All of the parts of something added together must equal the whole. Therefore, the sum of all the proportions in a proportional distribution must always equal _____. 1
165. A little thought will indicate why the total of all of the proportions in a proportional distribution must equal 1. Each proportion is a fraction whose numerator is the frequency of some value and whose denominator is the total number of observations represented in the data. The sum of all these fractions would simply equal the sum of all the numerators over this common denominator. However, the sum of all the numerators is the sum of the frequencies in the frequency table, which _____ equal the total number of observations. Therefore, the sum of all of the fractions or proportions would always equal _____. does
1
- For example, $\frac{2}{10} + \frac{5}{10} + \frac{3}{10}$ equals $\frac{2 + 5 + 3}{10}$, which equals $\frac{10}{10}$, or _____. 1
166. We stated earlier that a statistic is any number or term that summarizes a collection of data. Each of the proportions in a proportional distribution summarizes something about the frequency of a particular value in the collection of data. Therefore, each proportion _____ be called a statistic. would
would/would not

On Graph A (shown below) we have indicated the probability distribution on a sample space consisting of 4 possible outcomes of 2 tosses of a coin. Graph B represents the sampling distribution on a sample space consisting of 3 possible outcomes for 2 tosses of a coin. Notice that in Graph B we only consider the number of heads occurring in each pair of tosses. Getting 0 heads in two tosses is equivalent to the outcome TT on Graph A. (Similarly, 2 heads on Graph B is the same as the outcome HH on Graph A.) So the probability of this event is the same on both graphs. However, obtaining "1 head" on Graph B could be either the outcome TH or the outcome HT, shown in Graph A. According to the simple addition rule of probability theory, therefore, the probability of obtaining 1 head is equal to the probability of TH plus the probability HT (i.e., the probability of obtaining 1 head equals _____).

$$\frac{1}{2}$$



GRAPH A
OUTCOMES OF TWO TOSSES



GRAPH B

167.

Of the three tables shown below, two are

_____ frequency tables and the other is a
 absolute/relative
 _____ frequency table.
 absolute/relative

absolute

relative

TABLE A

TIME	FREQUENCY
0-100 sec.	10
101-200 sec.	90

TABLE B

TIME	FREQUENCY
0-100 sec.	100
101-200 sec.	900

TABLE C

TIME	FREQUENCY
0-100 sec.	1/10
101-200 sec.	9/10

168.

Table C is a relative frequency distribution describing
 the data in Table _____.

A/ B/ A or B

A or B

169.

Even though the enumerative statistics in Table A
 are different from those in Table B, both of these
 absolute frequency distributions can be represented
 by the same _____ frequency distribution.

relative
 (proportional)

163. Similarly, the outcome $\frac{HH}{HT}$ would represent 2 heads in the new list of 3 outcomes.
164. On the other hand, both the outcome _____ and the outcome _____ shown in Graph B correspond to the outcome one head in the new list of 3 outcomes.
165. In other words, in the new sample space of 3 possible outcomes, we **group** the outcomes HT and TH, since they both represent the occurrence of one head. According to the simple **addition** rule, the probability of obtaining either HT or TH is equal to the _____ of their individual probabilities.
166. We could state this formally by stating $\text{Pr}(1 \text{ head})$ equals $\text{Pr}(HT) + \text{Pr}(TH)$. Both $\text{Pr}(HT)$ and $\text{Pr}(TH)$ are equal to $\frac{1}{4}$, as indicated on Graph B. We could substitute the actual values of these probabilities into the previous equation, which would give us
- $$\text{Pr}(1 \text{ head}) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$
- According to this reasoning, in other words, the probability of obtaining no heads is simply the probability of obtaining a tail on the first toss and a tail on the second toss (which is equal to $\frac{1}{4}$). The probability of obtaining 2 heads is simply equal to the probability of a head on the first toss and a head on the second toss (which is equal to $\frac{1}{4}$). To find the probability of obtaining one head, however, we must **group** the two outcomes shown previously on Graph B, to form a new outcome entitled "one head," whose probability equals the **sum** of the probabilities of the two individual outcomes in the group. Therefore, the probability of one head equals _____.

170.

Just as you could present an absolute frequency distribution on a graph, it is also possible to present a proportional distribution in graphic form. In the distribution graph, we represented the frequency of each value by the _____ of a column. In this same way, we may represent each proportion in a proportional distribution.

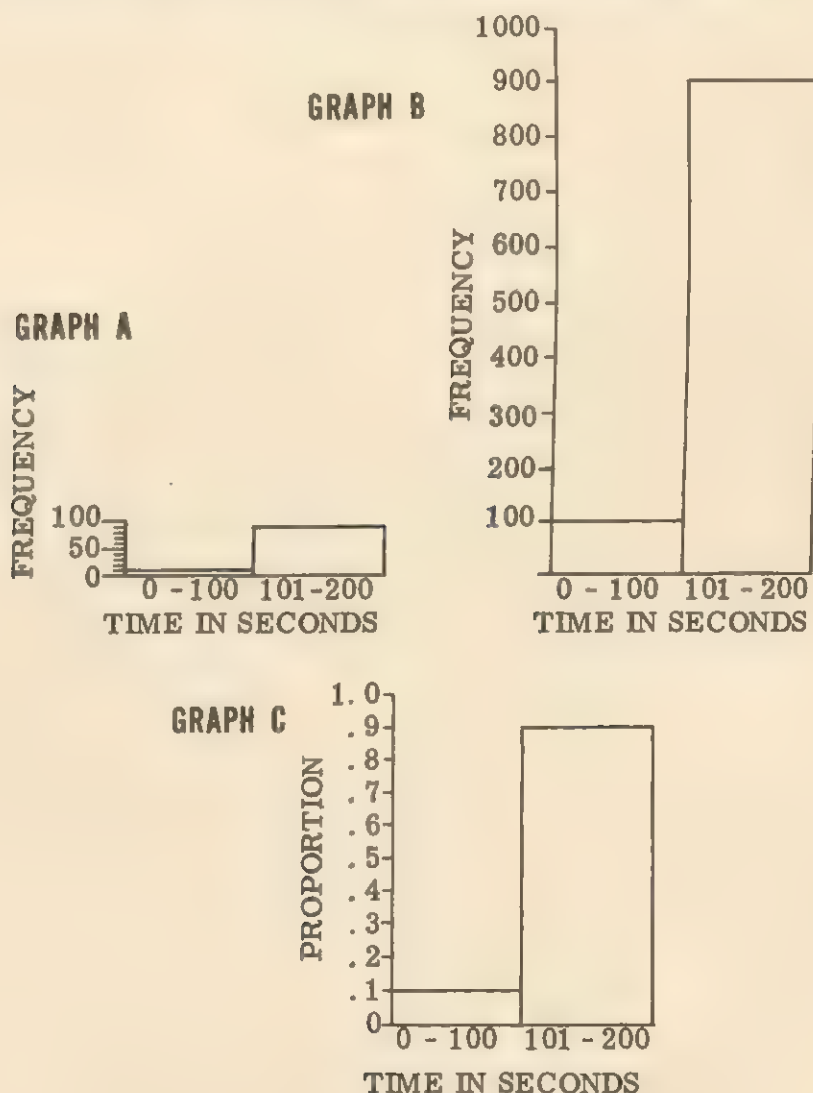
height

171.

The highest possible column that could ever occur on a proportional distribution graph would represent a proportion of _____. The three graphs shown below represent the same data shown just previously in tables A, B, and C. Notice that Graph C is a _____ frequency distribution graph, just as Table C was a proportional frequency table.

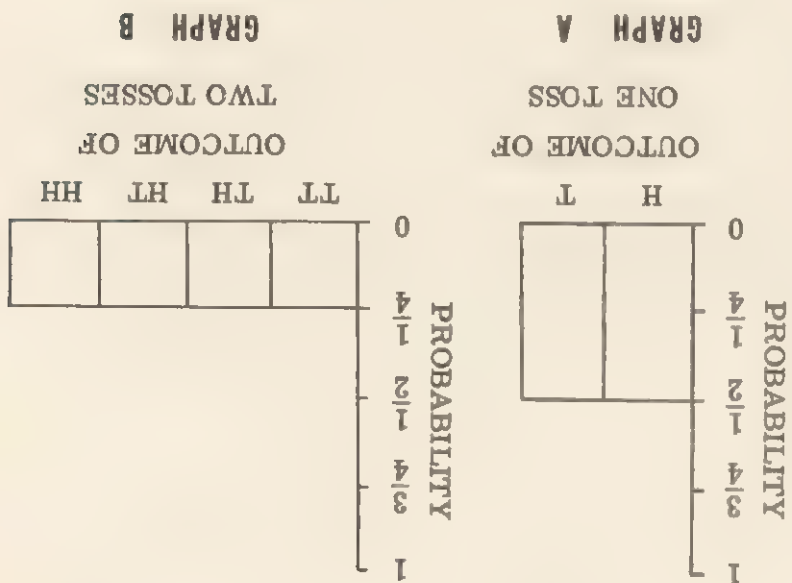
1

proportional



You also considered an illustration in which we determined the probability distribution for a sample space consisting of the 4 possible outcomes for two tosses of a coin, using the simple multiplication rule of probability theory. We determined the probability distribution shown in Graph $\frac{A}{B}$ (below) by noting that the probability of each of the 4 possible outcomes in that sample space was equivalent to the product of 2 probabilities shown in the other distribution.

B



Let us go one step further in our analysis of the random process of tossing a coin. Suppose we **grouped** the 4 possible outcomes of a pair of coin tosses on the basis of how many heads had occurred. Describing the pairs of tosses solely in terms of the number of heads that have occurred would define 3 possible outcomes: no head, or two heads. In other words, the outcome represented by TT in the Graph B (above) would correspond to the outcome _____ heads in the new list of 3 possible outcomes.

172. Notice that it is difficult to compare the two absolute frequency distributions in Graph A and B because the number of observations in each collection of data was the same/different. different
173. The difference in the size of the collection of data could easily obscure the similarity of the two distributions. This similarity is the fact that the absolute/relative relative frequency distributions of the two groups of time are the same.
174. Therefore, if you wish to compare two collections of data when there are different numbers of observations in each collection, it is often useful to use a(n) absolute/relative or relative, proportional frequency.

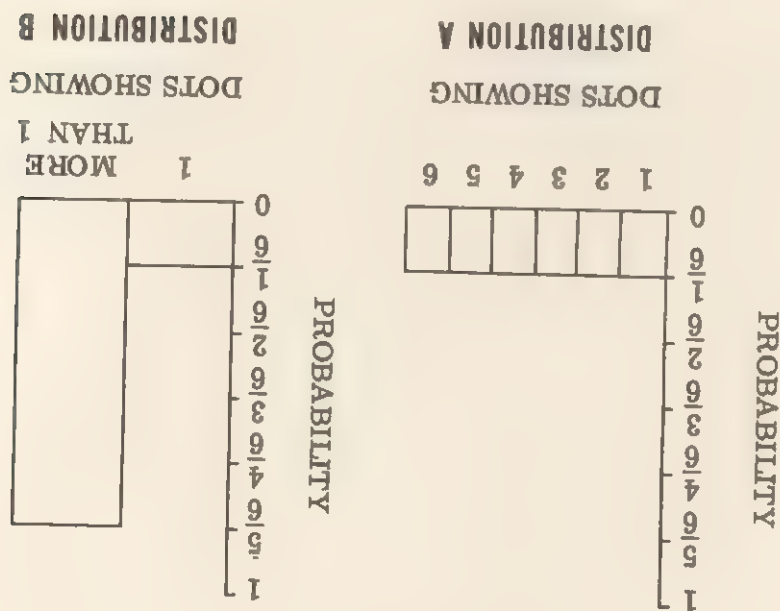
Let's review the two rules we have just considered:

- 1) the simple addition rule
- 2) the simple multiplication rule

By using the addition/multiplication rule, we were able

addition

to determine the probability distribution for the sample space shown in the Distribution B (below), since it is based on grouping some of the outcomes in the sample space shown in Distribution A.



In other words, you determine the probability of the group of outcomes represented by the title (more than one) by adding the probabilities of the individual outcomes in the group. This, therefore, was an application of the addition rule.

addition

REVIEW I

FILL IN THE BLANKS:

1. If a value is not recorded in the data, the value has a frequency of _____. zero
2. A collection of frequencies in a frequency table is called a frequency _____. distribution
3. Let us assume that we have two red marbles, five green marbles, and four blue marbles in a box. The proportion of red marbles is _____. $\frac{2}{11}$
4. Let us assume that we have two groups in an experiment. Group A has ten brunettes and five blonds. Group B has twenty brunettes and ten blonds. The relative frequency of the two groups is _____. the same
the same/different
5. The largest possible proportion is _____. 1
6. The sum of the proportions in a proportional distribution must equal _____. 1

7.

Totals				
Student	1	2	3	4
Bob	4	0	9	14
Mary	2	1	8	18
Lon	3	7	12	19

What is the score in the second row, fourth column?

18

H, T

$$\Pr(\text{HT}) = \Pr(\text{H}) \times \Pr(\text{T})$$

find the probability of a head followed by a tail, as follows:

158.

Similarly, you would apply the multiplication rule to

Thus, $\frac{1}{2}$ of $\frac{1}{2}$ is equal to $\frac{1}{2}$ times $\frac{1}{2}$, or $\frac{1}{4}$.

 $\frac{1}{4}$

simply states that of the pairs beginning with a head

the second toss. The multiplication rule
 a head, you would expect $\frac{1}{2}$ of them also to be a head on
 begin with a head. Of the $\frac{1}{2}$ of the pairs that began with
 expect about $\frac{1}{2}$ of the pairs of tosses of a fair coin to

already considered. In other words, you would

representing a characteristic of the process we have

The multiplication rule is simply a formal way of

157.

of obtaining a head on the second toss.

head on the first toss the probability

on the next toss is equal to the probability of obtaining a

probability of obtaining a head on one toss and a head

This formula can be interpreted verbally as follows: the

156.

$$\Pr(\text{HH}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

 $\frac{1}{4}$

values to obtain,

Since we know $\Pr(\text{H}) = \frac{1}{2}$, we can substitute actual

$$\Pr(\text{HH}) = \Pr(\text{H}) \times \Pr(\text{H}).$$

state this in terms of the previous illustration as

product of the probabilities of each outcome. We can

sequence of two independent outcomes is equal to the

states that the probability of obtaining a particular

The simple multiplication rule of the probability theory

toss followed by a head on a second toss should equal $\frac{1}{4}$.

We just illustrated why the probability of a head on one

155.

MULTIPLE CHOICE:

8. A list of things, one underneath the other, is called a:
- a. column.
 - b. row.
 - c. radial.
 - d. none of the above _____ a
9. A list of things arranged side by side is called a:
- a. column.
 - b. radial.
 - c. row.
 - d. none of the above _____ c
10. If we list all the observations and do not summarize them, we have:
- a. finished data.
 - b. theoretical distribution.
 - c. raw data.
 - d. none of the above _____ c
11. A number, or term, that summarizes or describes a collection of data is called:
- a. raw data.
 - b. a statistic.
 - c. a theoretical distribution.
 - d. none of the above _____ b

independent

independent

150. It would seem reasonable to assign each of these four outcomes the probability _____, since each outcome would then have the same probability, and the sum of the outcomes would equal one.
151. Notice that $\Pr(HT) = 1/4$ can be interpreted to mean that about one out of every four, or about 25 out of every 100, or about _____ out of every 1,000, pairs of tosses will begin with a head and end with a tail.
152. $\Pr(HH) = \frac{1}{4}$ does not mean that exactly 1 out of every 4 (or 25 out of every 100) pairs of tosses will consist of a head on the first toss and a head on the second toss.
153. The multiplication rule of probability theory is a formal way of representing this relationship between independent outcomes of a random process. We stated earlier that if two outcomes were independent, the character of one outcome has no bearing on the character of the second outcome. For example, the outcome of one toss of a coin has no bearing on (does not determine) the outcome of another toss of the coin. Therefore, the two tosses are said to be independent.
154. Similarly, when you roll a die, the outcome of one roll in no way determines the outcome of another roll. Therefore, rolls of a die are said to be independent.

1/4

25

1

250

12. Frequencies are often called:

- a. raw data.
- b. inoperative.
- c. enumerative statistics.
- d. none of the above

c

TRUE OR FALSE:

13. We refer to something that does not change during an experiment as a constant.

true

14. Something that does change in an experiment is called a variable.

true

15. The numbers one to ten are written on pieces of paper and placed in a hat. The number 4 is drawn from the hat. The number 4 is an observed value.

true

16. Let us assume that we have an ordinary die. On such a die, the number 7 is a possible value.

false

17. The number of nickels in a cookie jar would be an example of a continuous variable.

false

18. Numerals can only be used to represent numbers.

false

19. Only a list of observed values can be referred to as data.

true

144. You could use precisely the same reasoning to determine approximately how many pairs would begin with a head and end with a tail. Thus, of the 100 pairs, you would expect about _____ of them to begin with a head. Of these 50 which began with a head, you would expect about _____ of them to end with a tail.
- 25
145. Thus, of 100 pairs of tosses, you would expect about _____ pairs to begin with a head and end with a tail.
- 25
146. It is clear that you would expect about 25 of each of the four possible pairs: HH, HT, TH, TT, where, for example, HT represents a head on the first toss and a tail on the second toss, and TH represents a _____ on the first toss and a _____ on the second toss.
- head
147. Imagine a **sample space** consisting of the four possible outcomes for a pair of tosses: HH, _____, TH, TT.
- HT
148. Note that this is a perfectly acceptable sample space, since **one and only one** of these outcomes can occur with each pair of tosses. In other words, the four outcomes are mutually exclusive and _____.
- exhaustive
149. You would expect each of these four possible outcomes to occur about the same number of times. In other words, you would expect each of these possible outcomes to occur about _____ times out of every 100 pairs of tosses.
- 25

The review that you have just completed can assist you in evaluating your progress at this point in the program. If you had no difficulty with the review, proceed to the next section. If you did have trouble with any of the review questions, return to the place in the program where this material is presented and make sure you understand the material before going on to the next section.

Follow this procedure with each of the reviews in the program.

139.

Let's consider only those pairs of tosses beginning with the toss of a head. In other words, we know the first

head

140.

Since a coin cannot **remember** what happened to it on the previous toss, the second toss in each pair of tosses is said to be **independent** of what occurred on the first toss. In other words, the behavior of a coin on the second of two tosses does/does not depend upon how the coin fell on the first toss.

does not

141.

Successive tosses of a coin are said to be independent since the outcome of any one toss **does not depend** upon what has occurred on any of the other tosses.

142.

Therefore, considering only the 50 pairs of tosses which began with the toss of a head, you would expect about of the second tosses in each of these 50 pairs to be a head since about 1/2 of a series of tosses should be heads.

25

143.

To summarize, out of 100 pairs of tosses of a fair coin, you would expect about 50 (1/2 of the 100) to begin with a head. Of these 50 pairs that **began with a head**, you would expect about 25 (1/2 of the 50 pairs) to end with the toss of a tail. In other words, out of the 100 pairs of tosses, you would expect about of them to consist of a head followed by a head.

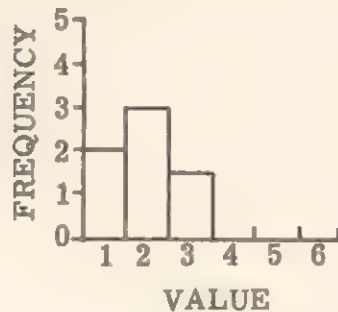
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Section III: Central Tendency

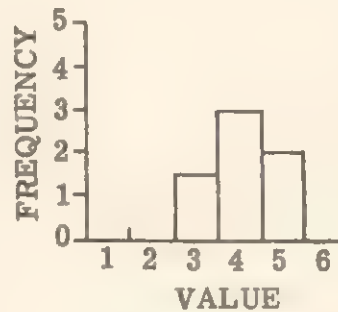
1. You have seen how any collection of data can be regarded as a distribution of values. By "distribution" we mean the number of times each of the possible values has been _____.

recorded (observed)

2. Next, we shall consider some of the ways in which distributions differ and how these differences can be described. Consider, for example, the two distributions shown below.



DISTRIBUTION A



DISTRIBUTION B

You would probably say that the typical observed value in Distribution B is _____ than the typical value of Distribution A.

larger

3. While the two distributions have approximately the same shape on the graph, the **center** of Distribution B is at a _____ value than the center of Distribution A.

larger

137.

Suppose you were to toss a coin twice. You could list four mutually exclusive and exhaustive outcomes for this process as follows.

Outcome 1:

a head on the first toss and a head on the second toss.

Outcome 2:

a head on the first toss and a tail on the second toss.

Outcome 3:

a tail on the first toss and a head on the second toss.

Outcome 4:

a tail on the first toss and a tail on the second toss.

138.

If you were using a fair coin, the probability of heads would equal the probability of tails on each toss of the coin. In other words, $\Pr(\text{heads}) = \Pr(\text{tails}) = \frac{1}{2}$.

Bearing this in mind, let's consider how you would assign probability values to a sample space consisting of the four possible outcomes of the two tosses of a coin that were listed above. First of all, let's consider about how many heads you would expect to obtain in 100 tosses of the coin. If the coin were fair, you would expect about _____ heads to occur in 100 tosses.

Therefore, if you tossed a coin twice and repeated this procedure 100 times (if you made 100 pairs of tosses), you would expect about _____ of the 100 pairs to begin with the toss of a head.

4. You could say the **central tendency** of Distribution A is different from the central tendency of Distribution B. In other words, if the values in one collection of data are generally larger than the values in another collection of data, you could say the distributions of the two

collections of data have the same/a different **central**

a different

tendency.

5. Consider the three collections of data listed below:

Data A: 2, 4, 3, 12, 6, 4

Data B: 10, 12, 14, 16, 50

Data C: 1, 3, 2, 3, 2, 6

Notice that the values in Data A are **more** similar to the

values in Data $\frac{C}{B}$ than they are to the values in

C

Data $\frac{\quad}{C/B}$.

B

6. The distributions of Data A and Data B seem to have

similar/different central tendencies, whereas the

different

distributions of Data A and Data C have similar/different

similar

central tendencies.

7. If the values in two distributions were quite similar, we would describe the two distributions as having similar

central tendencies

8. Consider the two sets of data shown below.

Data A: 4, 3, 6, 6, 4, 6

Data B: 21, 23, 20, 21, 23, 20

You could describe Data $\frac{A}{B}$ as having a larger typical

B

value than the other collection of data.

133. The second outcome in Distribution B (more than one dot showing) consists of five of the possible outcomes shown in Distribution A. Applying the simple rule of probability theory, we have assigned the second outcome in Distribution B, a probability equal to the total of the probabilities assigned to the five individual outcomes included in that **group** of outcomes.

134. The **addition rule** as it would apply to outcomes in the type of sample space we have considered can be stated formally in the following manner: If A and B are two different outcomes in a sample space, the probability of either outcome A or outcome B is simply the probability of A plus the probability of B. Therefore, we could represent the **addition rule** symbolically as:

$$\Pr(A \text{ or } B) = (\Pr(\quad)) + \Pr(\quad)$$

A, B

135. Extending this rule, we would apply it to the previous illustration as follows.

$\Pr(\text{more than one dot}) = \Pr(\text{two dots or three dots or four dots or five dots or six dots}) =$
 $\Pr(\text{two dots}) + \Pr(\text{three dots}) + \Pr(\text{four dots}) +$
 $\Pr(\text{five dots}) + \Pr(\text{six dots})$

Substituting the actual probability values, we could write

$$\Pr(\text{more than one dot}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6}$$

136. To summarize, the simple rule of

addition

Probability Theory says that when we group outcomes in a sample space, the probability of an outcome in the group occurring is simply the of the

probabilities of each of the outcomes in the group.

sum (total)

addition

Suppose the two collections of data represented the ages of people at two different parties. In other words,

Data $\frac{\quad}{A/B}$ would have been collected at a children's

A

party and Data $\frac{\quad}{A/B}$ would have been collected at a

B

party for young adults.

9. A person who was 5 years old would be more representative of people at a party corresponding to Data $\frac{\quad}{A/B}$ than

A

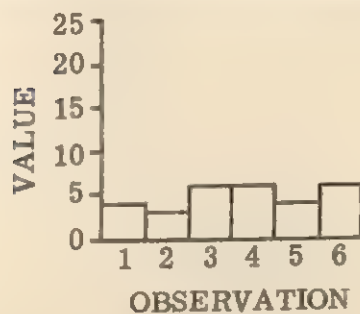
he would be to the data collected at the other party.

10. Even though it does not occur in Data A, the number 5 is more representative or typical of that collection of data than it is of the other collection. In a similar sense, the number 22 is more typical of Data $\frac{\quad}{A/B}$ than it is of

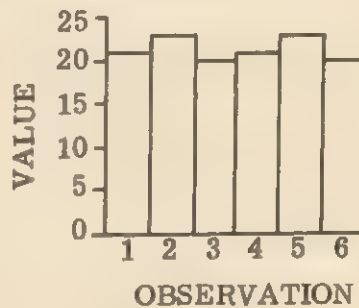
B

the other collection of data.

11. The two previous collections of data differ mainly in the magnitude or size of the recorded values. This can be illustrated by the two graphs of raw data shown below.



DATA A



DATA B

Notice how the values in Data $\frac{\quad}{A/B}$ all seem to be close

B

to (cluster around) the value 22, whereas the values in the other data seem to cluster around the value $\frac{5}{10}$.

5

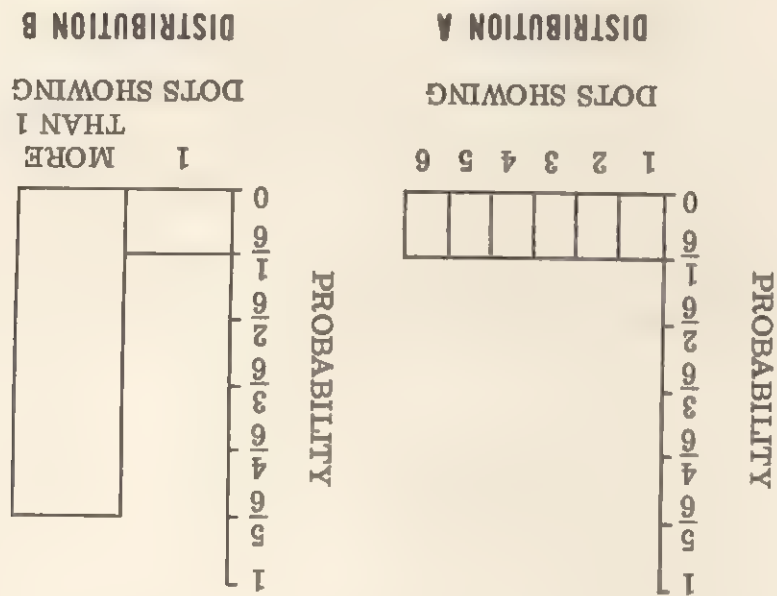
131.

The probability of obtaining more than one dot is simply the sum of the probabilities for each of the five outcomes grouped in this category. This could be stated symbolically as follows:

$$\Pr(\text{more than one dot}) = \Pr(\text{two dots}) + \Pr(\text{three dots}) + \Pr(\text{four dots}) + \Pr(\text{five dots}) + \Pr(\text{six dots})$$

132.

This application of the simple addition rule can also be illustrated by the following two graphs:



Note that Distribution A is the distribution on the sample space consisting of six possible outcomes, whereas Distribution B is the distribution on the sample space consisting of two possible outcomes.

12. Suppose a student named John received grades of 75, 70, 76, 75, 76 in a course. Another student, Dick, received grades of 95, 95, 96, 75, 98. A grade of 75 would be more typical or representative of _____ grades, even though both students had John's/Dick's received a grade of 75 during the course.

John' s

13. The reason a grade of 75 is more typical of John's grades is that a score of 75 was unusual for Dick. Most of Dick's grades seem to be clustered around 95/72.

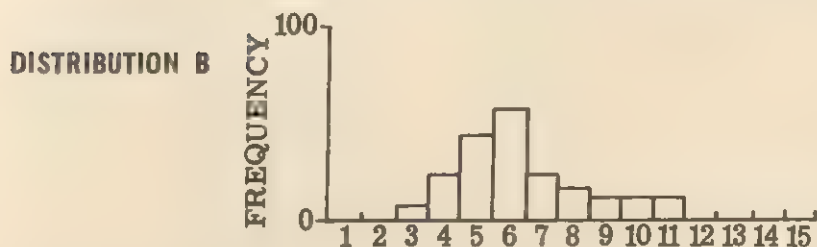
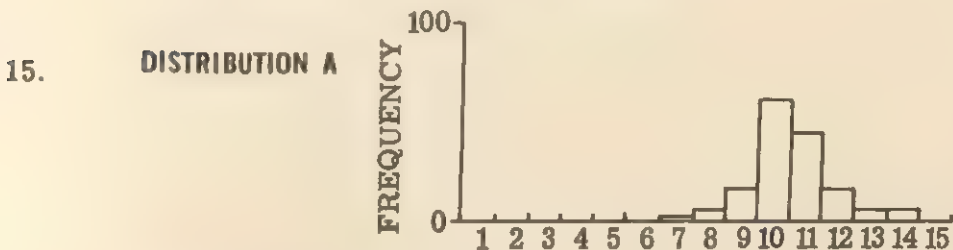
95

We would say, therefore, that the central tendency of John's grades seems to be higher than the central tendency of Dick's grades.

lower

14. If someone asked you to characterize John's work by reporting his typical grade, you would be more likely to answer $\frac{75}{85}$ than you would $\frac{75}{85}$.

75, 85



Notice how the values in Distribution A seem to be clustered around the value $\frac{5}{10}$, whereas the values

10

in Distribution B are clustered around the value $\frac{5}{10}$:

5

126.	The phrase expected frequency in the heading of column two implies each of the six possible outcomes will not necessarily occur exactly 100 times out of the 600 rolls. " Expected frequency " implies that each of the six possible outcomes would occur about equally often "in the long run" (in a very long series of successive rolls). Thus, _____ is our best bet concerning the number of times we would expect each of these six outcomes to occur in a series of 600 successive rolls.	100
127.	In Table B, we have indicated the two possible outcomes in the new sample space. Naturally, we would still expect about _____ rolls of a single dot in a series of 600 successive rolls of the die.	100
128.	The second outcome shown in Table B represents all rolls in which " more than one dot " was shown on the face of the die. Expected frequency of all outcomes falling in this group is simply the _____ of the frequencies for the five outcomes that fall into this group.	sum (total)
129.	According to Table B, the expected frequency of a single dot in 600 rolls of a die is _____. Therefore, the expected frequency of "more than one dot," which is the only other possible outcome, is _____.	100 500
130.	If a particular outcome is expected about 500 times out of 600 rolls, it would have a probability of $\frac{500}{600}$, or _____-sixths.	5

16. In other words, you could describe the difference between the two distributions shown in the previous frame by pointing out that the **central tendency** of one distribution was different from that of the other. For example, if you said you were referring to the distribution whose typical or central value was 10, it would be clear that you were referring to Distribution rather than to the other distribution. A
A/ B
17. The characteristic of distributions we have been referring to as the "typical value" is generally a value near the "center" of the distribution. In other words, we can usually think of the values in a distribution as being clustered around (close to) a typical or central value. That is why we refer to this characteristic of a distribution as its central . tendency
18. Up to this point we have not been very specific about what we mean by "central tendency." We have purposely not done so because there is more than one way to define the typical value or central tendency of a distribution. In other words, there more than one acceptable is
is/ is not way of defining the central tendency of a distribution.
19. If one value occurred more frequently in a distribution than any other value, you might consider that value the most common or typical value of the distribution. Therefore, one way of characterizing the central tendency of a distribution be to report the most would
would/ would not frequently occurring value in that distribution.

124. six possible outcomes. These five outcomes have been grouped under the heading "more than one dot." These five outcomes are: two dots, three dots, four dots, five, six four dots, and _____ dots. _____

125. In the new sample space we **group** all outcomes in which more than one dot are showing. (We do not distinguish between outcomes in which more than one dot is showing.) The tables shown below indicate the expected frequency of each of the possible outcomes in 600 rolls of the die in the two sample spaces.

Dots showing	Expected Frequency in 600 rolls	Dots showing	Expected Frequency in 600 rolls
one	100	one	100
two	100	more than one	500
three	100		
four	100		
five	100		
six	100		

Table A
Table B

In Table A we have listed six possible outcomes and indicated that each of these outcomes would be expected to occur about _____ times in 600 rolls of the die.

20. Statisticians use the most frequently occurring value in a distribution to characterize the central tendency of that distribution. Statisticians call this most frequent value the **mode** of the distribution. Thus, the value having the highest frequency in a frequency table _____ would be called the mode of the distribution represented in that frequency table. would

21.

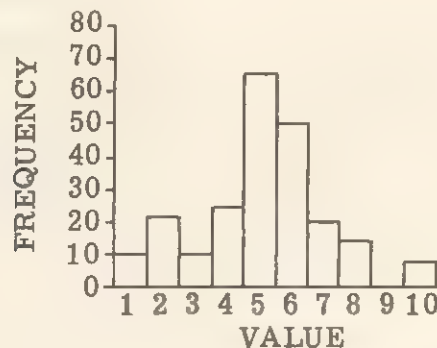
Subject	Score
1	40
2	20
3	10
4	40
5	40

The value occurring most frequently in this collection of data is _____, since this value has a frequency of _____. 40, 3

22. The **mode** is the most frequently occurring value in a distribution; therefore, 40 is the _____ of the distribution shown in the previous table. mode

23. The number _____ could be used to characterize the central tendency of the distribution shown in the previous table if you used the mode to characterize the central tendency. 40

24. The _____ column on a frequency distribution graph indicates the most frequently occurring value in that distribution. highest (tallest)



of sample space we have been considering and is nothing more than a convenient mathematical way of summarizing a characteristic of the random sampling process. For example, let's consider how this **addition rule** could be applied to rolls of a die. We have seen how certain properties of successive rolls of a fair die can be characterized by a probability distribution. Assigning each of the six outcomes the probability $\frac{1}{6}$ indicates that we would expect each of these six outcomes to occur about _____ of the times we rolled the die in a very large number of rolls.

$$\frac{1}{6}$$

121.

The preceding probability distribution indicates that each of the six possible outcomes in the sample space has been assigned the probability _____ to indicate the limiting relative frequency (or proportion) of times each of these outcomes would be expected to occur in an unlimited number of successive rolls of the die.

122.

You could represent the probability of obtaining a single dot (a one) on any roll of the die by the symbol **Pr(one dot)**. Similarly, you would represent the probability of obtaining two dots by the symbol _____.

Pr(two dots)

123.

Let's consider how the simple **addition rule** would be used to define the probability for a new sample space consisting of the following two outcomes:

Outcome 1:

a single dot

Outcome 2:

more than a single dot

In the new sample, in other words, we have grouped five of the outcomes in the previous sample space containing

24. (Continued)

The value having the highest column in the graph shown above is _____. We could characterize the central tendency of this distribution by reporting that its _____ was 5.

5

mode

25. You can indicate that 5 is the most frequently occurring value in the previous data by saying that 5 is the **modal** value. Thus, if the mode of a distribution is 12, you would say 12 was the m_____ (most frequently occurring) value in that distribution.

modal

26. Suppose you were considering a distribution in which the largest frequency was 10. Suppose, however, that more than one value had a frequency of 10. In this case, **both** of these values could be referred to as the mode of that distribution. For example, suppose you had a collection of data consisting of 10 observations of a variable with 3 possible values: a, b, and c. If the frequency of both a and b was 4, and the frequency of c was 2, you _____ say that **both** a and b were modes in that _____ distribution.

could

27. Consider the table of data shown below.

Observations	Value
1	11
2	11
3	21
4	11
5	21
6	33
7	11
8	21
9	21
10	33

The value 11 has a frequency of _____.

4

The value 21 has a frequency of _____.

4

The value 33 has a frequency of _____.

2

group the opinions of all those students who were in favor of the overseas campus. We decided not to distinguish between opinions that represented the same point of view concerning the overseas campus. In the sample space consisting of 10,000 outcomes, we assigned the probability $1/10,000$ to each outcome. In the sample space consisting of the **grouped** outcomes, we assigned the probability $1/10$ to the outcome

"unfavorable opinion" and $9/10$ to the outcome

"favorable opinion." The probability $1/10$ was

appropriate since 1,000 of the 10,000 students had an unfavorable opinion and, therefore, if each particular student's opinion tended to occur equally often, the unfavorable opinion would tend to occur about

times out of every 10,000 opinions sampled.

Whenever we ask what is the probability of obtaining one of a particular **group** of outcomes in a sample space, we are really asking the question what is the probability of obtaining the opinion of one member of the group, or the opinion of another member of the group, or of another member of the group, and so on.

120.

The rule of probability theory that can be applied when we assign a probability to a group of outcomes is called the **simple addition rule**. According to this rule, the probability of obtaining any one outcome in the group of outcomes is simply the total of the probabilities assigned to the individual outcomes in the group. This simple **addition rule** of Probability Theory applies to the type

The two modes in this collection of data are the values _____ and _____.

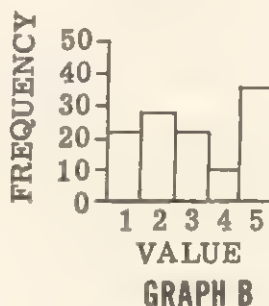
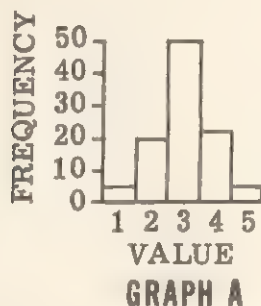
11, 21

- 28 It would be possible to find the value that occurred more frequently than any other value in either numerical or non-numerical data.

Therefore, a collection of non-numerical data can be described in terms of the modal, or most frequently occurring value. For example, if you tossed a coin 5 times and observed 3 "heads" and 2 "tails," you could say _____ was the **modal** (most frequently occurring) value in your data.

heads

29. Although the mode is often a useful way of characterizing the **central tendency** of a distribution, it is sometimes misleading to describe a distribution by its mode. Consider the two frequency distribution graphs shown below:



Notice how the mode of Distribution _____ is near the

A

center of the distribution and is typical of the values in that distribution, whereas the mode of the other distribution is not near the center of the distribution and is not particularly typical of the values observed in that collection of data.

116.

This suggests the probability distribution appropriate for our new sample space (the space consisting of the two outcomes "favorable opinion" and "unfavorable opinion") would be one in which the probability $9/10$ was assigned to the outcome "favorable opinion" and the probability _____ assigned to the outcome "unfavorable opinion."

117.

Assigning the probability $1/10$ to the outcome "unfavorable opinion" simply represents a characteristic of the random sampling procedure for obtaining a single opinion from this particular population. It is consistent with the limiting relative frequency procedure we discussed earlier, since if we were to repeat the sampling procedure over and over again, the proportion of unfavorable opinions obtained in this manner would tend to approach the value _____.

118.

It is as if the list of outcomes for repeated draws of a single opinion from the jar containing 10,000 opinions were viewed as a sample from an unlimited population consisting of an unlimited number of draws of a single opinion. As this sample (list of outcomes for successive draws) becomes larger and larger, the proportion of unfavorable opinions in this sample tends to approach a steady value of $1/10$. This is all that is meant by assigning the probability _____ to the outcome "unfavorable opinion" and the probability _____ to the outcome "favorable opinion".

119.

Whenever you group together the outcomes in one sample space to form a new sample space (as we did in the previous illustration), you can apply a simple rule of probability to assign probabilities to the new sample space. In the previous illustration, we decided to

30. Partly because the mode of a distribution is not always representative of the central or typical value in a distribution, statisticians have defined other ways of characterizing the central tendency of a distribution.

Another way of representing central tendency is to report a value which is **smaller** than the same number of observations as it is larger than. This value is called the **median** of the distribution. For example, suppose your data consisted of the observations 4, 5, 7, 8, and 10. The value 7 would be greater than $\frac{3}{2}$ of the remaining observations and smaller than $\frac{3}{2}$ of the remaining observations. Therefore, we could call _____ the **median** of these five observations.

2

2

7

31. The easiest way of finding the median of a collection of data is to list all the observed values in the order of their size. This procedure is called **ranking** the data. If your data consisted of the values 4, 3, 8, and 7, then the list of values 8, 7, $\frac{3}{4}$, and $\frac{3}{4}$ would be a ranking of the data.

4, 3

32. If you were to rank the values 10, 6, 11, and 4, you would start with the largest value and end with the smallest to form the list _____, _____, _____, and _____.

11, 10, 6

4

simply decided **not** to distinguish between particular students' opinions if their opinions are the same. In other words, we have **grouped** the opinions of all those students in **favor** of the overseas campus as one outcome, and the opinions of all those students who are/are not in favor of the overseas campus as the other outcome.

114.

For purposes of illustration, let's suppose only 1,000 of the 10,000 students were **opposed** to the overseas campus. We have already indicated that if the sampling procedure were truly random each **particular** student's opinion would be chosen about the same number of times as the sampling procedure was repeated over and over again. That is why we assigned the probability $1/10,000$ to each **particular** student's opinion. However, while each particular student's opinion would be sampled equally often, you would expect to obtain more opinions in favor of/opposed to the overseas campus by repeated sampling, since only 1,000 of the 10,000 students were opposed to the overseas campus in the population.

115.

While each particular student's opinion would tend to be sampled about the same number of times, only about 1,000 out of every 10,000 opinions sampled (one out of every ten) would be opposed to the overseas campus, since only 1,000 of the 10,000 students' opinions were unfavorable. In other words, if you repeated the sampling procedure over and over again, you would find about 1,000/10,000 or $1/10$ of the opinions sampled were unfavorable and about /10 of the opinions were favorable.

33. You could rearrange the observations in a table of raw data, arranging the values in the order of their size rather than in the order in which they were observed. Thus, we would be forming this new table by ranking the observations.

Notice that the data shown in Table B were/were not formed by ranking the data in Table A.

were not

Observation	Value
1	10
2	12
3	11
4	8

TABLE A

Rank	Value
1	12
2	11
3	8
4	10

TABLE B

34. Table D (shown below) was/was not formed by ranking the data in Table C.

was

Observation	Value
1	10
2	12
3	11
4	8

TABLE C

Rank	Value
1	12
2	11
3	10
4	8

TABLE D

35. We stated earlier that the **median** value in a collection of data was a value which was _____ than half of the other observed values and _____ than the other remaining values.

smaller
larger

36. One way of finding the median of a collection of data is to **rank** the data and locate the value which divides this list of ranked values in half. If 10, 7, 6, 2, and 1 were a list of ranked values, the value _____ would divide the list in half. **Two** of the other values would be larger than 6, whereas _____ of the remaining of the other values would be smaller than 6.

6

2

113. We could define another sample space for the process of randomly drawing a single opinion from the population of 10,000 opinions. Suppose you defined a sample space consisting of the two outcomes: a favorable opinion and an unfavorable opinion. This would be a sample space, since each opinion sampled would correspond to one and only one of these two outcomes. Actually, we have

outcomes in the sample space.

should be assigned to each of the 10,000 possible

sample space, the number (probability) _____

frequency procedure for assigning probabilities to a

This suggests that according to the limiting relative

opinion would be sampled the same proportion of times.

proportions would equal 1 and since each student's

samples of a single opinion, since the total of these

to be sampled about 1/10,000th of the time in repeated

$\frac{1}{10,000\text{th}} - \frac{2}{10,000\text{ths}}$

Each opinion would tend

repeat samples of one opinion each would be about

could be expected to be sampled in a large number of

opinions, the proportion of times each student's opinion

procedure to be about the same. Since there are 10,000

in repeated applications of this random sampling

proportion of times each particular opinion was obtained

student opinions were truly random, you would expect the

single student opinion from the population of 10,000

To summarize, if the procedure by which you obtained a

111. To express this in terms of relative frequency, you

would expect the proportion of times each particular

student opinion was sampled (as you repeated this

procedure over and over again) to be about the

same

1/10,000th

1/10,000th

36. (Continued)

Since 6 is larger than half of the remaining values and smaller than the other half, the **median** would be _____.

6

37. It is a simple matter to find the median of a distribution when you have an odd number of observations. You simply rank the observations and find the middle value in this list of ranked observations. This middle value would be the _____ of your data.

median

38. If you had an even number of observations, there would not be a value in the list of ranked data such that the same number of observations fell above and below that value.

To illustrate this problem consider the table of ranked data shown below:

Rank	Value
1	80
2	60
3	40
4	30
5	20
6	10

Notice how we have indicated the rank of each value in the first column of the table. The largest observed value has a rank of $\frac{1}{6}$ and the smallest observed

1

value has a rank of $\frac{6}{6}$.

6

Notice also that there are _____ observations larger than the value 40 and _____ observations smaller than the value 40. Thus, 40 _____ the median. There are _____ is/ is not

2

3

is not, 3

observations larger than the value 30 and _____ observations less than the value 30, indicating that 30 _____ the median. is/ is not

2

is not

107.

We have now considered how probability distributions can characterize observations from an infinite population. Let's consider how a probability distribution could be defined characterizing the process of drawing samples randomly from a finite population. Earlier, we considered an example in which the opinions of 10,000 college students, each of which was either for or against an overseas campus, were all written on slips of paper. These slips of paper were placed in a basket, thoroughly mixed, and then withdrawn to form a sample of student opinion. This process was represented as a random

random

procedure for obtaining a sample of student opinion from the finite/infinite population of 10,000 students' opinions.

finite

108.

The population of student opinions would be **finite** since there was a limited/unlimited number of opinions in the population.

limited

109.

Suppose you defined a **sample space** of all possible outcomes for the random process of drawing a single student opinion from the jar of 10,000 opinions. There would be 10,000 possible outcomes, one outcome for each **particular** student opinion. Therefore, a sample space could consist of outcomes, one for each particular student opinion.

10,000

110.

According to our definition of a random sampling process, each of the 10,000 student opinions would have the same opportunity of being included in the sample. Therefore, if the random process by which we obtained a single student opinion was repeated over and over again for a very great number of times, you would expect each particular student's opinion to be sampled about the number of times as would any other student's opinion.

same

38. (Continued)

Neither 30 nor 40 is the median, since too many values are smaller than $\frac{30}{40}$, whereas too many values are larger than $\frac{30}{40}$.

40

30

39. Strictly speaking, any value between 30 and 40 could be called the median of this data. However, statisticians have agreed upon a rule for finding the median of an **even** number of observations. They would say that the median of the previous collection of data was a value halfway between 30 and 40. In other words, 35 $\frac{30}{40}$ be called the median, because would/ would not would

35 is halfway between 30 and 40.

40. Suppose 8, 6, 4, and 3 were ranked data. The value halfway between 6 and 4 would be the value 5. Therefore, 5 would be the median of this data.

41. Three lists of ranked data are shown below.

Data A: 5, 4, 1

Data B: 6, 3, 2

Data C: 5, 3, 1, 1

The middle value in Data A is the value 4; therefore, 4 would be the median of Data A.

The middle value in Data B is the value 3. This value, therefore, would be the median of that data.

The proportion of times a baseball player has gotten a hit out of all the times he has officially been to bat is often referred to as his batting average. In other words, if he has been to bat 10 times and gotten 3 hits, his batting average would be .3 or, as it is often written, .300. If a batter had a batting average of .400, this would mean that he had gotten a _____ on exactly _____ of his times at bat. $\frac{4\text{-tenths}}{400}$

hit
4 tenths

Suppose you viewed the process that determined whether or not a particular batter obtained a hit as a **random process**. You might view each time at bat as an additional observation from an infinite population consisting of an unlimited number of times at bat. Suppose the batter had been at bat a great number of times and had obtained a hit about 3-tenths of the time. You might represent his performance as a probability distribution. The **sample space** to which you could assign probabilities would consist of the two mutually exclusive and exhaustive outcomes of a time at bat: a hit or _____ hit. (We will consider anything other than a hit as "no hit".) Since the proportion of hits by this particular batter approached the value .3 after a great number of times at bat, you could assign the probability .3 to the outcome "hit" and the probability _____ to the outcome "no hit".

The probability of a hit (.3) is a way of characterizing the batter's performance if you view his performance as a _____ process. This probability implies that the proportion of hits in a very large number of observations (official times at bat) appears to approach the value _____.

random

41. (Continued)

Notice that Data A and B consist of an _____ number of observations, whereas Data C consists of an _____ number of observations. Using the rule for _____ data consisting of an even number of observations, we would find the value halfway between _____ and _____; this is the value _____. Therefore, we would say the median of Data C is _____.

42. Consider the table of data shown below:

Observation	Value
1	100
2	200
3	160
4	30
5	180

The largest value in this table is _____ and the 200
smallest value is . 30

43. If we **rank** the data in the preceding table, our list of values would/would not be 200, 160, 180, 100, 30. would not

44.	The previous list was incorrectly ranked. The value	
	"180" should have come before _____ rather than after	160
	it, since 180 is larger than _____.	160

45. We refer to the largest value in a table of ranked data as having rank 1. The value having rank 1 in the previous collection of data would be _____. The value having rank 2 would be _____.

101.

You have just seen that the relative frequency or

of observations of a particular value

proportion

in a random sample tends to vary less and less as the

size of the sample is increased.

102.

This limiting relative frequency can be thought of as the

population proportion of an infinite population. Since an

infinite population consists of an unlimited number of

observations, we could never calculate the actual

population statistic. It is often useful, however, to

view the sample statistic in a very large sample as if it

were the population statistic. This is reasonable,

because we know the difference between a sample statistic

and the population statistic in a very large sample will

tend to be quite $\frac{\text{large/small}}{\text{large/small}}$ according to the law of

small

large numbers

103.

A probability distribution can represent an infinite

population consisting of an unlimited number of outcomes

produced by a random process. You have seen, for

example, how a p distribution can

represent an infinite population consisting of an unlimited

number of tosses of a coin. When you say that the

probability of heads equals one-half, you simply mean

that the limiting r

of heads is one-half as the sample size is made very

large.

relative frequency

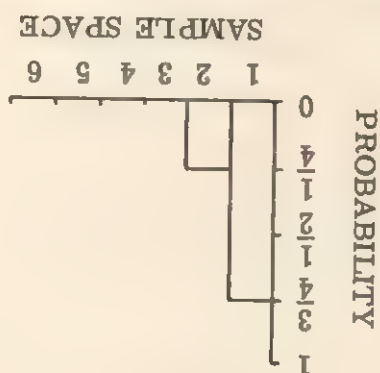
probability

46. The **median** of the previous collection of data is the value _____, since the values _____ and _____ are larger and the values _____ and _____ are smaller than this median value. 160, 180, 200
100, 30
47. However, if the values in the previous collection of data were 200, 180, 160, and 100, the median would be a value halfway between _____ and _____. 160, 180
48. To find the difference between 180 and 160, you subtract 160 from 180. Half of this difference is _____, which is what you would add to the value 160. Therefore, the median is _____. 10
170
49. We have now considered **two** ways of representing the central tendency of a distribution: by the _____ and by the _____. median
mode
50. The _____ of a distribution is the most frequently occurring value in the distribution, whereas the _____ is a value that is less than half of the other values and greater than half of the remaining values. mode/median
mode
median
51. Suppose your data were the values:
10, 5, 1, 10 and 4.
The median of the distribution is the value _____, whereas the mode is the value _____. 5
10

It is important to keep in mind that probabilities assigned in this fashion characterize the **process** rather than any particular outcome of the process. In other words, the probability $\frac{1}{2}$ assigned to the outcome "heads" for repeated tosses of a **fair** coin does not indicate what will happen on any particular toss. What it represents is the tendency for the proportion of heads in a large number of tosses to approach the value _____. This is a characteristic **not** of one toss but of the whole series of tosses — that is, the **process** of tossing a fair coin.

100.

This interpretation of probabilities is often referred to as the **limiting relative frequency** concept of probability. As a simple illustration of another procedure for assigning probabilities, consider the following example. Suppose you were about to roll a die and you asked someone to assign a number to each of the six possible outcomes, such that the sum of numbers totaled 1 and the size of each number was related to his "**subjective degree of certainty**" that a particular outcome would occur. In other words, if he had a hunch that 6 dots was more likely to occur than 2 dots, he should assign a larger number to the outcome 6 than to the outcome 2. The probability distribution shown below could be used to characterize a possible assignment of numbers by this procedure.



According to this assignment, the subject expected the number of dots showing on the die to be _____ or _____.

52.

Observation	Value
1	10
2	9
3	10
4	11
5	7
6	10
7	5
8	2
9	1
10	5

The mode of the collection of data shown above is _____
 since this value occurs _____ times.

10

3

53.

The value 9 _____ the median of the previous
 is/ is not

is not

collection of data since there are _____ observations
 with values larger than 9 and _____ observations with
 values smaller than 9.

4

5

While 9 is too large to be the median, 7 is too _____
 to be the median.

small

54.

To find the median of the previous data, you would find
 the value halfway between _____ and _____. Therefore
 the median of the previous collection of data is _____.

9, 7

8

55.

The median (like the mode) can sometimes give a
 misleading picture of a distribution. For example,
 consider the two collections of data shown below:

Data A: 100, 99, 98, 97, 96.

Data B: 100, 99, 98, 4, 2.

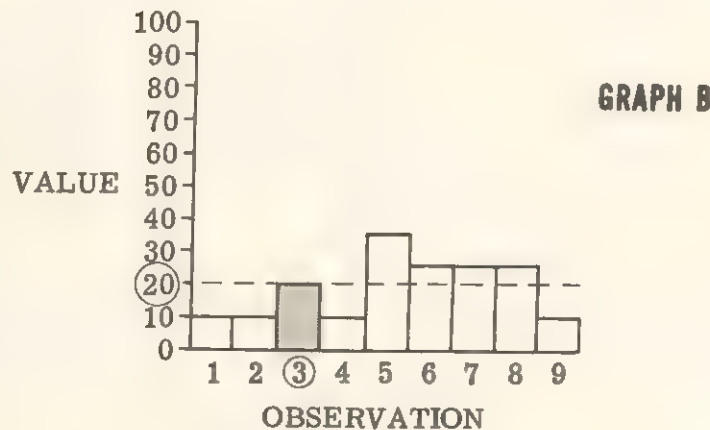
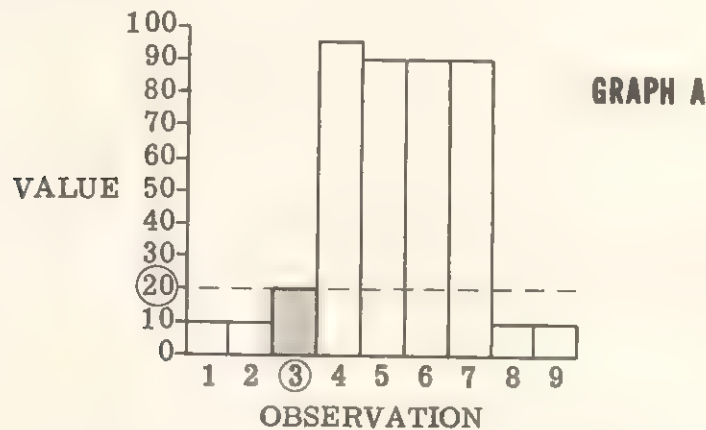
The median of Data A is _____ and the median of Data B
 is _____.

98

98

56. While both distributions have the same median, the values below the median in Data A/B differ much more from the median than do the values above it. B

57. The median only indicates the value dividing the list of ranked data into two equal parts. The median does not indicate **how much** smaller or **how much** larger are the values falling above it or below it in the list. This can be illustrated by the following raw data graphs.



Notice that in **both** collections of data _____ 4
 observations had a value **larger than** the value of
 observation 3 and _____ observations had a value 4
smaller than the value of observation 3. Thus, _____ 20
 is the median of both distributions since it is the value
 of observation _____. 3

90.

The Graph A indicates that the first roll of the die resulted in one dot showing. Thus, on this graph "one dot" has a proportion of 1 and all the other outcomes have a proportion of _____ after the first roll of the die.

0

91.

After 6 rolls (Graph B) the only outcomes that had not occurred at least once were 2, 4, and _____ dots showing.

6

92.

After 60 rolls (as indicated on Graph C) the proportions of times each of the 6 possible outcomes had occurred were more similar. After 600 (Graph D) rolls each of these proportions was very close to $\frac{1}{6} / \frac{2}{6}$.

 $\frac{6}{1}$

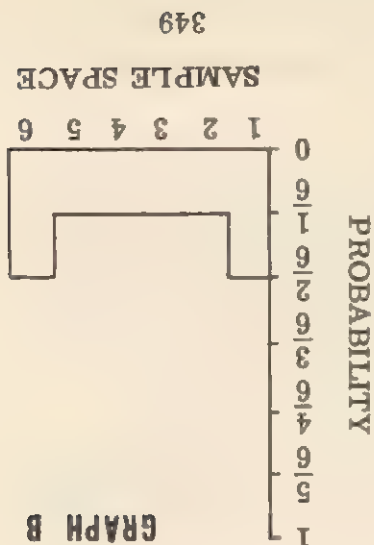
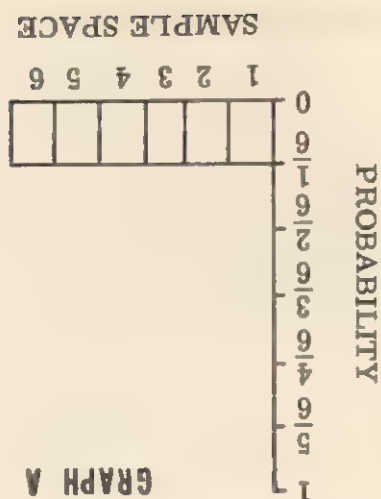
93.

Finally, after 6,000 rolls (Graph E) all 6 sample proportions were extremely close to _____.

 $\frac{6}{1}$

94.

You could represent this characteristic of repeated rolls of a die by the probability distribution shown in Graph $\frac{A}{B}$. It indicates that the proportion of each of the possible outcomes tended to approach a value of $\frac{1}{6}$ as the number of observations was increased.



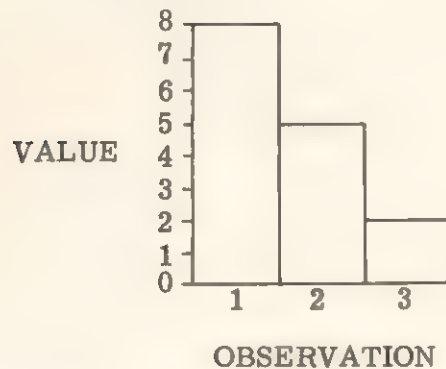
58. The table of data shown below contains _____ observations of a _____ variable.
numerical/non-numerical

3
numerical

Observation	Value
1	5
2	8
3	2

59. The graph of raw data shown below _____ does/does not represent the data in the previous table.

does not

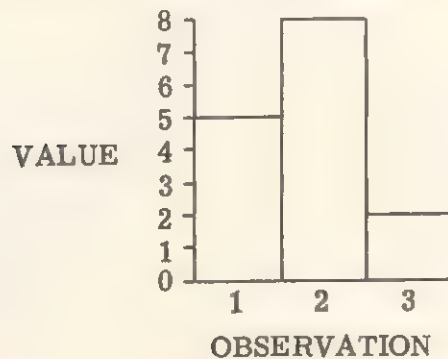


60. The previous graph does not represent the previous table of data, since observation 1 had a value of 5 and observation 2 had a value of 8 in the table, whereas observation 1 had a value of _____ and observation 2 had a value of _____ in the graph.

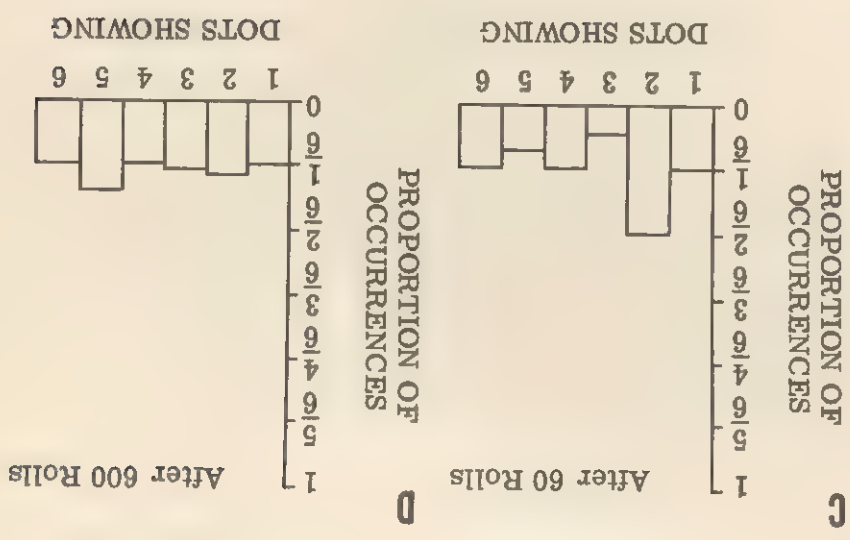
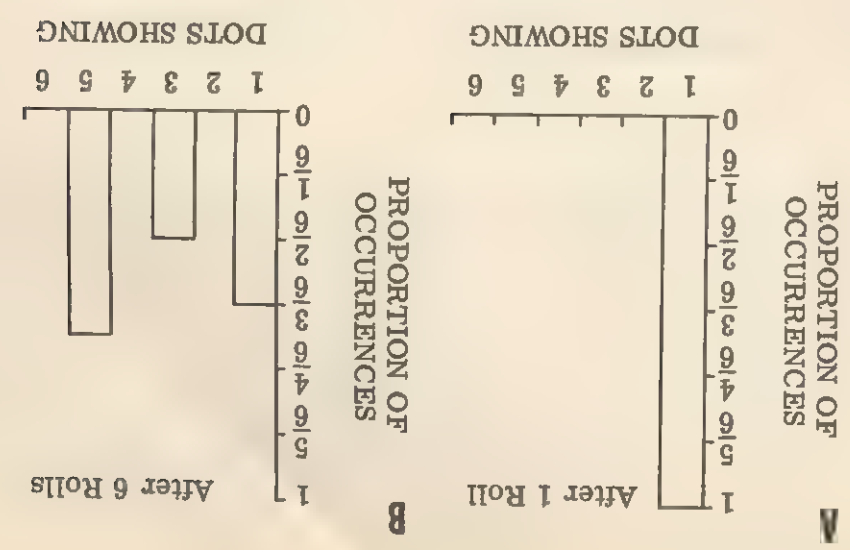
8
5

61. The graph of raw data shown below _____ does/does not represent the data shown in the previous table.

does



(Continued)



61. (Continued)

All the observed values in the previous graph _____ are not
are/are not
the same.

Differences in the observed values are indicated on the
graph by differences in the _____ of the 3 columns. heights

Thus, the value of observation 1 is clearly less than
that of observation 2, since the height of column 1 is
less than that of column 2. Similarly, the value of
observation 2 is _____ than the value of greater than
less/greater
observation 3, since column _____ is higher than 2
column _____. 3

62. While you can always compare the value of one
observation to the value of another observation, it
wouldn't make much sense to simply say, "Observation
3 is less than." The immediate question would be:
"Less than what?" It is often convenient to pick a
reference value for comparison with the observed values.
For example, suppose you chose the value 4 as a
reference value. You could then describe the data in
the previous graph by saying, "Observation 1 is greater
than 4, observation 2 is greater than 4, and
observation 3 is _____ than 4." less

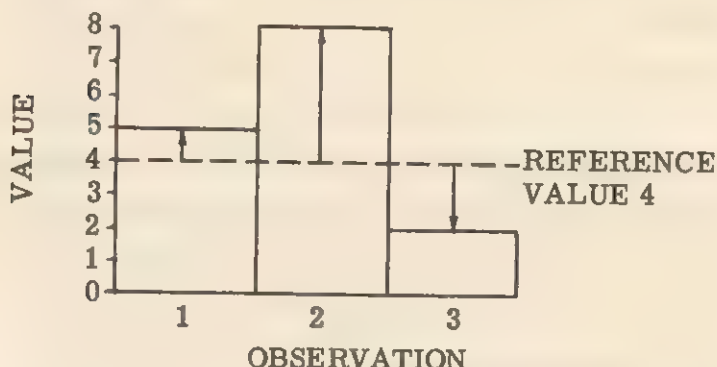
a value of .5. In other words, it appears as if the successive tosses of the coin were all randomly drawn from an infinitely large population in which the population proportion of heads equalled ____.

88. Notice that if the proportion of heads tends to approach a value of .5 as the number of observations is increased, the proportion of tails must approach a proportion of ____ (since the total of the two proportions must equal ____).

89. A probability distribution can be defined which represents this characteristic of repeated tosses of a coin. You simply assign the number .5 to the outcome "heads" and the number ____ to the outcome "tails."

90. This probability distribution **characterizes** something interesting about repeated tosses of a coin — namely, the tendency for the proportion of heads and tails to approach the values indicated in the probability distribution as the number of tosses (observations) of the coin is increased. Let's consider a similar illustration, only this time, instead of successively tossing a coin, imagine successively rolling a die. Suppose the die were rolled 6,000 times. You could calculate a proportional distribution of the 6 possible outcomes (one dot, two dots, etc.) after one roll, after six rolls, 60 rolls, 600 rolls, and after 6,000 rolls. The following proportional distribution graphs indicate the manner in which these sample proportions tend to approach a steady value as the number of observations is increased.

63. The relationship between the reference value 4 and the value of each of the three observations is made clear by the following graph.



While this graph represents the same data as does the previous graph, we have indicated the reference value 4 with a dotted line drawn across the graph at a height equal to the value _____. We have also indicated the difference between the reference value and each observed value with an arrow. The arrow in column 1 is pointing upwards, since the observed value 5 is greater than the reference value 4. Similarly, the arrow points _____ in column 2, since the value of observation 2 is _____ than the reference value 4.

4
up
greater

64. The value of observation 3 is _____ than the reference value 4. Therefore, the arrow on our graph points _____. Notice that the **difference** between the value of observation 1 and the reference value is less than the difference between the value of observation 2 and the reference value. This is indicated on the graph by the fact that the arrow in column 1 is _____ than the arrow in column 2. Thus, the **size of the difference** between the observed value and the reference value is indicated by the _____ of the arrow representing that difference.

less
down
shorter
length

85.

Looking down the column of sample proportions

(column 3), it is clear that as more and more

observations were included in the sample, the sample

proportion tended to change less and less and seemed to

settle near a value of

$$\frac{.4}{.5}$$

86.

The fact that a sample statistic (such as a proportion)

tends to approach a steady value as the size of the sample

is increased is an example of the law of large

numbers. It is as if the sample statistic

were approaching closer and closer to the value of the

population statistic as you increased the size of the

sample.

It is this tendency of the sample statistics of random

samples from an infinite population to approach a steady

value as the sample size is increased that allows us to

define the population proportion. The population

proportion of an infinite population is defined as that

value which the sample statistic tends to approach as

the size of the sample is increased. According to the

previous table, therefore, it would appear that the

population proportion in the population consisting of an

unlimited number of tosses of a coin was approximately

$$\frac{.5}{.4}$$

87.

This characteristic of repeated tosses of a coin allows us

to say something about the uncertainty concerning the

outcome of any particular toss. While you cannot predict

exactly what will occur on any particular toss, you can

say that as the number of tosses is increased, the

proportion of heads seems to grow closer and closer to

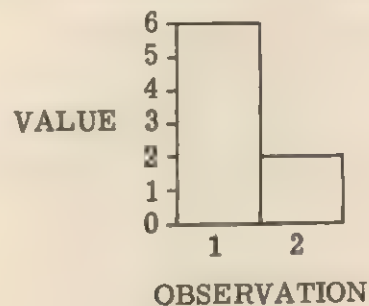
65. The length of the arrow represents the difference between the reference value and the observed value, regardless in which direction the arrow is pointing. Thus, the arrow points up in column 2 since the second observed value is greater than the reference value, whereas the arrow points down in column 3 because the third observed value is _____ than the reference value. less
However, the value of observation 3 is closer to the reference value than is the value of observation 2. This is indicated by the fact that the arrow in column _____ 3
is _____ than the arrow in column 2. shorter
shorter/longer

66. Differences between observed values and a reference value (represented with arrows in the previous graph) are referred to as **deviations** by statisticians. If a value is greater than the reference value, that value has a **positive deviation** from the reference value. If a value is less than the reference value, that value has a **negative deviation** from the reference value. Since the value of observation 1 (on the previous graph) is greater than the reference value 4, you would say that the value of observation 1 had a positive deviation from the reference value 4. Similarly, since the value of observation 2 is _____ than the reference value 4 greater
greater/less
you would say the value of observation 2 had a _____ deviation from the reference value 4. positive
positive/negative

Since the value of observation 3 is less than the reference value 4, we would say the value of observation 3 had a _____ deviation from the negative
positive/negative
reference value 4.

67. The amount by which a particular value deviates from the reference value is indicated by the **length** of the arrow in the previous graph. Whether the deviation is positive or negative is indicated by the **direction** in which the arrow points. In other words, whenever an observed value is greater than the reference value, the arrow will point , indicating a deviation. Whenever the observed value is less than the reference value, the arrow will point , indicating a deviation.
- up, positive
up/down positive/negative
down
negative

68. The graph of raw data shown below represents 2 observations having a value of and .



6, 2

69. Suppose you chose 0 as the reference value for describing the two observations in the previous graph. Both of the observed values would be than the reference value and would represent deviations from the reference value.
- greater
greater/less
positive

outcomes for 10 tosses of a coin as a sample from this infinite population. You could also view the list of

outcomes for 20 tosses as a larger _____ from

this infinite _____.

sample population

80. According to the law of large numbers, you would expect

the sample proportion, p , to be closer to the population

proportion, p , for a $\frac{\text{large}}{\text{small}}$ sample rather than for

large

a $\frac{\text{large}}{\text{small}}$ sample.

small

81. As you increase the size of a random sample from an

infinite population, a statistic (such as a sample

proportion) will appear to grow closer and closer to some

particular value. This can be illustrated by the following

table.

Figure 6

Sample proportion tends to approach a steady value as sample size is increased.

Number of tosses completed	Number of heads observed	Proportion of heads in sample
1	0	0
2	0	0
5	3	.60
10	3	.30
50	20	.40
100	48	.48
500	260	.52
1,000	510	.51
5,000	2,542	.50
10,000	5,028	.50

70. Suppose you choose 8 as a reference value. Since the value of both observations is _____ than 8, both deviations would be _____.
 greater/less
 negative/positive

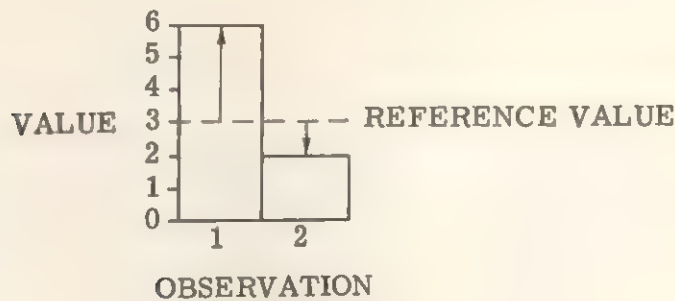
less
 negative

71. Suppose you chose a reference value somewhere between 6 and 2. The value of observation 1 would represent a _____ deviation from the reference value, _____
 positive/negative
 whereas the value of observation 2 would represent a _____ deviation.
 positive/negative

positive
 negative

72. We have redrawn the preceding graph and indicated below the deviation of each observation from a **reference value** of _____.

3



The reference value is closer to the value of observation 2 than it is to the value of observation 1. This is indicated by the fact that the arrow pointing up is _____ than the arrow pointing down. We can calculate the actual size of the deviation simply by **subtracting the reference value from the observed value**. For example, you would calculate the deviation of observation 1 from the reference value by subtracting 3 from _____.

longer

6

actually lands on one side or the other.) These two outcomes represent a sample space, since the outcomes are mutually exclusive (only one outcome can occur each time the coin is tossed). The two outcomes are also exhaustive because they represent possible outcomes.

all

76. Suppose we tossed the coin repeatedly, each time listing the outcome of the toss. This list would constitute a collection of data because it is a list of observed values of a variable called "outcome," whose two possible values are _____ and _____.

heads, tails

77. Imagine repeating the tossing procedure over and over an unlimited number of times. This would produce an unlimited collection of data. If this collection of data is regarded as a population, it would be described as a(n) finite/infinite population, since it consists of an unlimited number of observations.

infinite

78. To determine the proportion of heads for a finite (in other words, limited list of outcomes), you could simply divide the number of _____ observed by the total number of observations.

heads

79. Since you could not possibly collect all of the data in an infinite population, a problem arises concerning how you would define the population proportion, p . The law of large numbers provides us with a way of defining this population proportion. You can view a collection of

73. The deviation of observation 1 from the reference value 3 equals 3 subtracted from 6, which equals _____. 3

74. Similarly, you would calculate the deviation of observation 2 from the reference value 3 by subtracting 3 from _____. 2

75. Notice that 2 minus 3 equals -1. This means the deviation of observation 2 from the reference value 3 is $\frac{-1}{+1}$. -1

When we subtract a reference value from a smaller value, our answer will be negative. Therefore, all **negative** deviations will be represented by $\frac{\text{negative}}{\text{positive}}$ negative numbers. On the other hand, all $\frac{\text{positive}}{\text{positive}}$ positive deviations will be represented by positive numbers, since the observed value is, in this case, greater than the reference value.

76. In the table shown below, we have summarized the information concerning deviations of the observed values from the reference value 3.

OBSERVATION	VALUE	DEVIATION FROM 3
1	6	3
2	2	-1

77. We have simply added another column to the previous table of raw data and listed the deviations of each of the observed values from the value _____. Thus, the numeral -1 in the last row of the third column represents the deviation of observation 2 from the reference value 3. We obtained the deviation of observation 2 from the reference value 3 by subtracting _____ from _____, to give us an answer of _____. 3, 2, -1

Let's consider how this characteristic of random samples serves as a basis for assigning probabilities to the members of a sample space. The two outcomes, heads and tails, represent a sample space for a single toss of a coin. (We shall only consider tosses where the coin

75.

You also saw that a sample mean is more likely to be close to the population mean if the sample is large than if the sample is small. This is another example of the _____ of _____.

74.

This law does not imply that a sample proportion from a large sample is always more similar to the population proportion than is one from a small sample. It says that it _____ to be closer. In other words, it is **more often** closer than is one from a small sample.

tends

large numbers

The idea that the sample proportion will tend to be closer to the population proportion the larger the size of the sample is, therefore, the main point of the law of

73.

size $\frac{10}{100}$.

10

This characteristic of random samples is part of what is meant by the **law of large numbers**. The main point of this law is that a sample statistic, such as a proportion, is more likely to be similar to the population proportion if the size of the sample is large. In other words, if the population proportion were .8, the sample proportion from a sample of size $\frac{10}{100}$ would tend to be closer to .8 than would the sample proportion in a sample of

100

78. The following table contains the same two observations. However, we have left room to indicate deviations from the reference value _____ rather than 3 (as in the previous table).

4

OBSERVATION	VALUE	DEVIATION FROM 4
1	6	
2	2	

79. To find the deviation of observation 1 from the reference value 4, you subtract 4 from $\frac{6}{2}$.

6

80. Therefore, the deviation of observation 1 from the reference value 4 is _____.

2

81. The following table _____ correct.
is/ is not

is not

OBSERVATION	VALUE	DEVIATION FROM 4
1	6	2
2	2	2

82. The previous table is incorrect because observation 2 was **smaller** than the reference value. Therefore, the deviation of observation 2 from the reference value would have to be represented by _____ rather than 2.

-2

83. The following table _____ correct.
is/ is not

is

OBSERVATION	VALUE	DEVIATION FROM 4
1	6	2
2	2	-2

67. Each of these probabilities can be no larger than _____, nor smaller than _____, and the total of all of these probabilities must equal _____.
68. Any particular assignment of probabilities to members of a sample space is called a _____.
69. Up to this point, we have only described some of the basic ideas involved in **Probability Theory**. Earlier we indicated that Probability Theory was a mathematical system which could be used to represent random processes, such as games of chance.
70. Although Probability Theory was originally developed to represent games of chance, it has been found to be useful in representing a wide variety of processes. Therefore, the general definition of a random process is any process that can be represented by _____.
71. Next, we shall consider procedures for assigning probabilities that make the probability distribution a useful representation of an actual sampling process. Earlier, we indicated that a **large** random sample tends to be more representative of the population than does a small random sample. In general, the larger the size of the random sample, the more/less similar will be the sample statistic to the population parameter.

more

Probability Theory

probability
distribution

1
0
1

84. There is something special about using 4 as the reference value. The positive deviation represents the same difference between the observed value and the reference value as does the negative deviation. In other words, if we represented deviations on a graph as we did previously, the length of the two arrows would be the same/different (although one arrow would be the same pointing up and other down).

85. Because the reference value 4 has the unique property of being as close to the value 6 as it is to the value 2, we say that 4 is the **mean** of the values 6 and 2. Since 6 and 2 are the two observed values in the previous collection of data, we could say that the **mean** of that collection of data is 4.

86. If we added the positive deviation from 4 and the negative deviation from 4, our answer would be 0 because 0 +2 added to -2 equals 0.

87. Another way, therefore, of describing that unique characteristic of the reference value 4, which makes it the **mean** of the previous collection of data, is to say the **sum of the deviations** from 4 equals 0.

88. Suppose your data consisted of 3 observations instead of only 2 — for example, 8, 3, and 1. If you chose 10 as your reference value, all the deviations would be negative. If you chose 0 as your reference value, however, all the deviations would be positive.

OBSERVATION	VALUE	DEVIATIONS FROM 6
1	8	2
2	3	-3
3	1	-5

Suppose the die were rolled and 2 dots were showing. In that case, both outcome _____ and outcome _____ in the previous list would have occurred.

62.

Two dots would represent an "even number of dots" and at the same time it would represent "more than one dot showing." Therefore, two of the outcomes in the previous list of four outcomes would have occurred. This list of four outcomes would not be suitable as a sample space since more than one outcome could occur at the same time. The list is **exhaustive** because any outcome would be included in the list. Since more than one outcome can occur at the same time, however, the outcomes are not _____.

mutually exclusive

63.

A list of outcomes can describe a sample space if the outcomes are both _____ and _____.

exhaustive, mutually exclusive

64.

A simple way of stating that a list of outcomes is both _____ and _____ is to say that one and only one of the outcomes must occur.

exhaustive, mutually exclusive

65.

The fact that **one** of the outcomes must occur implies that the list is _____ and the fact that **only one** can occur implies that the list is _____.

mutually exclusive

66.

The numbers assigned to the outcomes in a sample space are called _____.

probabilities

89. To check the deviation of observation 3 in the previous table, you would subtract the value _____ from the value _____. This would indicate that the deviation in the table was _____.
correct/incorrect

6
1
correct

90. We mentioned that a reference value is called a **mean** when the sum of the deviations from that reference value equals 0. For the set of data we considered earlier, for example, the deviations from the mean value 4 were +2 and -2, giving a sum of deviations equal to $(+2) + (-2)$, which equals _____.

0

91. Even when the data consist of more than two observations, we can define the **mean** in the same way. In other words, if we **add all the deviations from a particular reference value** and our answer is 0, that reference value is the **mean** of those observations. Consider the previous table of deviations from the reference value 6. To find the sum of the deviations, we would add +2, -3, and -5, which yields an answer of _____.

-6

92. Since the sum of the deviations of our three observed values from the reference value 6 does not equal 0, the value 6 _____ the **mean** of these three observations.
is/is not

is not

93. We could record the deviations of each observation from the reference value _____ in the following table:

4

OBSERVATION	VALUE	DEVIATIONS FROM 4
1	8	
2	3	
3	1	

We would find the deviation of observation 1 by subtracting _____ from _____, indicating that the deviation of observation 1 from the reference value was _____.

4, 8
4

58. Notice the probability of outcome _____ on Graph B is the smallest possible value of a probability, since it equals _____.
59. Let's review the two characteristics of a list of outcomes which are necessary for the list to be a sample space. First, the list must include **all** possible outcomes. A list of this sort is said to be **exhaustive**, since it exhausts **all** possible outcomes (it is a **complete** list of **all** possible outcomes). The second necessary characteristic is that **only one** of the outcomes may occur at one time. Such a list of outcomes is said to be **mutually exclusive** (the occurrence of one outcome **excludes** the occurrence of any of the other outcomes since only one outcome may occur at a time). Therefore, a **sample space** consists of a list of outcomes that are both exhaustive and mutually exclusive. The list is exhaustive/mutually exclusive if it includes all possible outcomes, and it is exhaustive/mutually exclusive if only one of the outcomes can occur at a time.
60. A list of outcomes for rolls of a die consisting of the two outcomes "one dot showing" or "two dots showing" could be described as **mutually exclusive**, but it could not be described as exhaustive, since all possible outcomes are not included.
61. On the other hand, consider the following list of outcomes for a role of a die.
- Outcome A: an even number of dots showing
 Outcome B: an odd number of dots showing
 Outcome C: one dot showing
 Outcome D: more than one dot showing

94. In the same way, we would find that the deviation of observation 2 was ____ and that the deviation of observation 3 was ____.

-1

-3

95. We have recorded the deviations from the reference value 4 in the following table:

OBSERVATION	VALUE	DEVIATIONS FROM 4
1	8	+4
2	3	-1
3	1	-3

We said that the **mean** or **average** of a group of numerical observations is that particular reference value yielding deviations whose total is 0. To find the total of the deviations from the reference value 4 for the three observations in this table, we would add ____, ____, and ____.

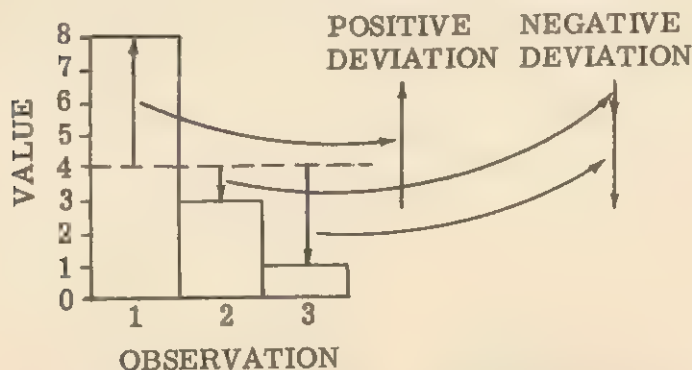
4, -1

-3

Thus, 4 is/ is not the mean of this group of observations.

is

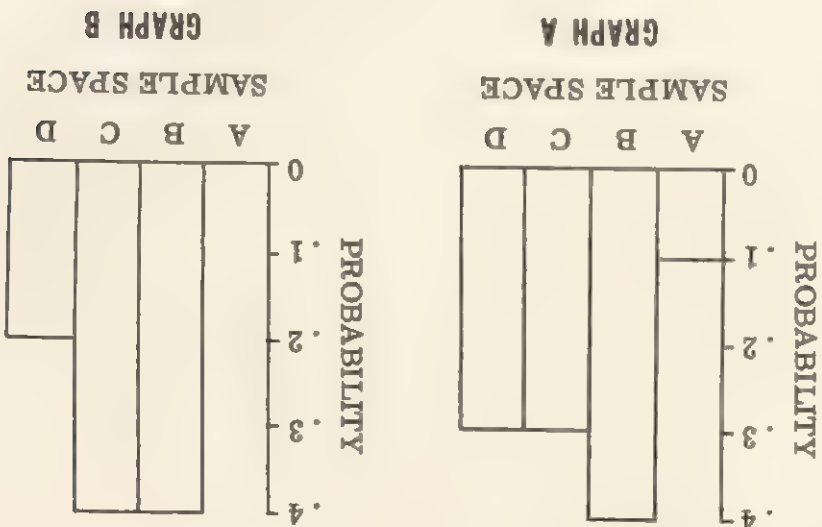
96. We can illustrate this graphically as follows:



The arrow labeled "positive deviation" represents the one positive deviation in the graph. The two downward pointing arrows, which are connected together, represent the two negative deviations in the graph. In order for the sum of the deviations to equal 0, the positive deviations

55.

One of the two graphs shown below could not represent a probability distribution. Try to find that graph.



56.

You would be correct in saying Graph $\frac{A}{B}$ could not represent a probability distribution.

A

57.

On the other hand, Graph B could represent a probability distribution, since

$$\Pr(A) + \Pr(B) + \Pr(C) + \Pr(D) = 0 + .4 + \underline{\quad} + \underline{\quad} = \underline{\quad}.$$

.4, .2, 1

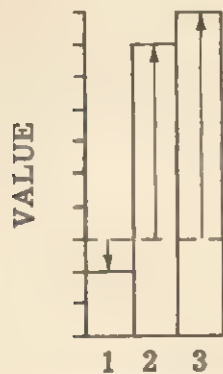
96. (Continued)

must exactly balance the negative deviations. In other words, the total length of the arrows representing the positive deviations must be exactly the _____ as the total length of the arrows representing negative deviations; this is apparently true when the reference value is 4. Thus, _____ is the **mean** of these three observations.

same

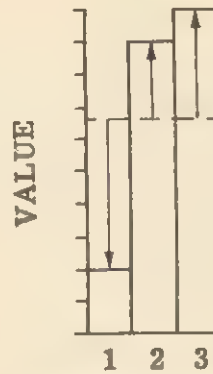
4

97. Consider the following two graphs.



OBSERVATION

GRAPH A



OBSERVATION

GRAPH B

Remember, the mean is that reference value from which the sum of the deviations equals 0. Keeping this in mind, it would appear that the reference value shown in Graph A/B

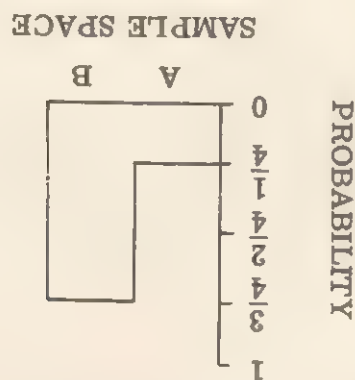
B

was more likely the **mean** than is the reference value in the other graph.

98. The reference value in Graph B is more likely the mean because the two positive deviations added together almost equal the negative deviation in size, whereas the two positive deviations in Graph A added together appear to be _____ than the size of the negative deviation in that graph.

larger

51. Another way of representing a probability distribution is with a graph, such as the one shown below.



Notice that the **sample space** consists of two possible outcomes: _____ and _____.

A, B

52. The number assigned to each member of the sample space by the probability distribution is indicated by the height of the column over each outcome. Thus, the _____ (number) assigned to outcome A equals $\frac{1}{4}$.

53. The probability assigned to outcome B equal _____.

$\frac{3}{4}$

54. $\Pr(A) + \Pr(B)$ equals _____ + _____, which equals _____, just as the total of all probabilities must in any probability distribution.

$\frac{1}{4}, \frac{3}{4}, 1$

99. The mean is a useful way of representing the central tendency of a distribution. However, we do not yet have a simple way of calculating the mean from an actual collection of data. Of course, you could try one reference value after another, calculating the deviations from each of these reference values. You could eventually locate a reference value from which the sum of the deviations equaled _____, which would indicate that reference value was the **mean** of the collection of data.

zero

If the collection of data consisted of only a few observations, you could probably find the mean in this manner. However, if the data consisted of many observations, the procedure outlined in the previous frame would not be practical. Therefore, we will consider a procedure whereby we can calculate the mean of any collection of data according to a simple rule. A **rule** for calculating the value of some statistic is called the **formula** for that statistic. Thus, a rule for calculating the mean would be called a _____ for the mean.

formula

100. In order to discuss ways of calculating specific statistics, it is often useful to talk about data in general rather than about particular observed values. For example, consider the two tables of raw data shown below.

TABLE A

Observation	Value
1	20
2	10
3	42
4	52

TABLE B

Observation	Value
1	10
2	20
3	0
4	52

Each table lists the values for $\frac{4}{6}$ observations of a numerical variable. (You might wish to insert a book-mark here since we will refer to these tables in later frames.)

4

46.

If the four outcomes listed in the first column of the following table represented a sample space, the missing probability in column 2 would equal _____.

1/4

Outcome	Probability
A	1/4
B	1/4
C	1/4
D	_____

47.

Pr(D) must equal $1/4$, since the sum of the probabilities for A, B, and C equals _____ and the total of all four probabilities must equal _____.

3/4
1

48.

Notice that of the two tables shown below, Table _____ could not possibly represent a probability distribution.

B

A		B	
Outcome	Probability	Outcome	Probability
A	.2	A	.2
B	.2	B	.3
C	.6	C	.6

49.

Table B (above) could not be a probability distribution since the total of the probabilities equals _____, whereas it should equal _____.

1.1 (1 $\frac{1}{10}$)
1

50.

Just as a proportion can be represented by either a decimal or a fraction, a probability can be represented by either a decimal or a fraction. If a sample space consisted of two possible outcomes — A and B — and the probability of A equalled $\frac{1}{4}$, the probability of B could be represented by either the fraction _____ or _____ by the decimal _____.

$\frac{3}{4}$
.75

101. The term "**observation 1**" specifies a particular value in each table. It specifies the value 20 in Table A and the value ____ in Table B. 10
102. Similarly, the term **observation 3** refers to the value ____ in Table A and the value ____ in Table B. 42, 0
103. Instead of writing out **observation 1** or **observation 2** statisticians have found it simpler to use the symbol X_1 to represent the same value as does the term **observation 1**. Thus, X_1 represents the value 20 in the previous Table A and the value ____ in Table B. 10
104. The number which appears after and just below the capital letter X in X_1 is called a **subscript**. The subscript indicates which particular observation you are representing. Thus, the subscript ____ in X_1 indicates you are representing observation 1. 1
105. Notice that the symbol X_2 has a s _____ 2 instead of a subscript 1. subscript
106. Similarly $\frac{X_2}{X_3}$ has a subscript 3 instead of a subscript 2. X_3
107. Since the symbol X_1 indicates the "**observation 1**," you would use the symbol X_2 to represent "**observation ____**." 2
108. Notice that the symbol X_2 can represent the first/second observed value in any table of raw data, regardless of the particular value of that observation. second

41. Since $\Pr(A)$ represents the probability of outcome A the largest possible value of $\Pr(A)$ would be _____ and its smallest possible value would be _____.
42. Therefore, no matter what the actual probability distribution, $\Pr(A)$ must be a number between _____ and _____.
43. Imagine a sample space consisting of two outcomes — outcome A and outcome B. If $\Pr(A)$ was equal to 1, then $\Pr(B)$ must equal _____, since the sum of the probabilities must equal _____.
44. Just as a frequency distribution or a proportional distribution can be represented by either a table or a graph, it is possible to represent a probability distribution by either a table or a graph. For example, the table shown below represents a probability distribution for a sample space consisting of three possible outcomes. Notice that the three possible outcomes are listed in the _____ column of the table and that the numbers _____ first/second assigned to each of these possible outcomes are listed in the _____ first/second column of the table.
- | | | | |
|-------------|-----|-----|-----|
| Outcome | A | B | C |
| Probability | 1/4 | 2/4 | 1/4 |
45. Each of the numbers in the second column is a _____, since it is the number assigned to a member of a sample space by a probability distribution.

109. In the same way, you could represent the third observation in any table of raw data by $\frac{X_1}{X_3}$. The subscript X_3 , 3 indicates you are referring to observation 3.
110. Considering the two previous tables of data, the term X_1 would refer to the value 20 in Table A and the value 10 in Table B.
 X_3 would refer to the value 42 in Table A and the value 0 in Table B.
111. Notice how you could represent any collection of **three** observations with X_1 , X_2 , and X_3 .
 If your data consisted of X_1 , X_2 , X_3 , and X_4 , then there would be four observations in your collection of data.
112. Statisticians often use a capital letter N to represent the **number of observations** in a collection of data. If the collection of data consisted of four observations, N would equal 4. If the data consisted of ten observations, N would equal 10.
113. Since N represents the number of observations in a table of raw data, you could represent any collection of N observations with a column of X's starting with X_1 , then X_2 , and so on, until X_N .
 This way of representing tables of data is useful because you can describe a general rule or formula for calculating some statistic without speaking of particular values. For example, you could represent the procedure for getting the **sum** or **total** of a collection of five observations with the formula:

$$\text{Total} = X_1 + X_2 + \text{---} + \text{---} + X_5$$
 X_3, X_4

34. The largest probability you could assign to any member of a sample space would be _____ and the smallest would be _____.
1 0
35. Notice the similarity between numbers (probabilities) assigned to members of a sample space by a probability distribution and the proportions in a relative frequency distribution. The largest value of a probability is _____, and the largest value of a proportion is _____.
1 1
36. Similarly, the smallest value of a proportion would be _____, and the smallest value of a probability would be _____.
0 0
37. We will use the symbol $\Pr(A)$ to represent the probability of outcome A. In other words, $\Pr(A)$ represents the number assigned to outcome A, where A is a member of a _____.
sample space
38. Similarly, if B is another member of a sample space, you can represent the number assigned to it by a probability distribution with the symbol $\Pr(_)$.
B
39. Suppose, A, B, and C represent members of a sample space consisting of 3 possible outcomes. According to our definition of a probability distribution,
 $\Pr(A) + \Pr(B) + \Pr(C) = _$.
1
40. The total of the three probabilities in the previous frame would have to equal 1, since the total of the numbers (probabilities) assigned by a probability distribution to members of a sample space must equal _____.
1

114. You could write a general formula for finding the total of any collection of data consisting of three observations as follows:

$$\text{Total} = X_1 + X_2 + X_3$$

It would become awkward to write a formula of this sort if the collection of data consisted of very many observations.

For example, if the data consisted of one hundred observations, we would have to write out a string of X values with plus signs between them, beginning with X_1 and ending with an X having the subscript ____.

100

115. One way you could simplify the formula would be to write out the phrase: **sum all the X 's**, which would mean add/multiply together the values of all of the observations in the table of data.

add

116. Statisticians have a special shorthand way of writing the phrase: **sum all the X 's**. Instead of writing out the whole phrase, they simply write $\sum X$. Thus, if your data consisted of two observations, $\sum X$ would equal X_1 plus X_2 . If your data consisted of three observations, $\sum X$ would equal ____ + ____ + ____.

X_1, X_2, X_3

117. You are now in a position to write a simple formula for the total of all the observations in any collection of data. Using the shorthand way of writing "**sum all the X 's**," you could write the formula for the total as

$$\text{Total} = \text{____} X$$

\sum

29.

Since each of the numbers assigned to a member of a sample space is called a probability, we could describe a probability distribution as an assignment of a

p _____ to every member of a sample

space, such that the sum of these probabilities equals 1.

30.

Three different assignments to numbers to the sample space for one toss of a coin are listed below. Assignment $A/B/C$, however, could **not** be a probability distribution.

Assignment A:

heads - $\frac{1}{2}$, tails - $\frac{1}{2}$

Assignment B:

heads - .9, tails - .1

Assignment C:

heads - .8, tails - .4

31.

The assignment of the number .8 to **heads** and number .4 to **tails** could **not** be a probability distribution since the total of the numbers does/does not equal 1.

does not

32.

Since the total of all the numbers assigned to the members of the sample space must equal 1, the largest number you could assign to any member of a sample space would be _____.

1

33.

If one outcome in a sample space were assigned the number 1, then all the other outcomes would have to be _____ assigned the value _____.

0

118. The symbol Σ is a capital Greek letter called **sigma**.
Thus, the Greek letter _____ in the expression ΣX indicates to you that the expression represents the _____ of all the X 's (observations). sigma
sum
119. The Greek sigma (Σ) is often referred to as a **summation** symbol, since ΣX represents the _____ of the observed values. sum
120. Let's return now to the original task of finding a formula for the mean. Basically, this involves stating the formal relationship between the mean of the data and the observed values by a mathematical equation, and then (by simple algebra) rearranging this equation until it is in the form of a formula for the mean.

We shall begin by considering a collection of data consisting of three observations. In other words, $N =$ _____ for this collection of data. three
121. Just as we can represent observation 1 by X_1 without indicating any particular value, we will represent the **mean** by \bar{x} . In this way, we can represent the mean without indicating any particular _____ for the mean. value
122. You could represent the **deviation** of observation 1 from the mean as $X_1 - \bar{x}$. In a similar way, you would represent the deviation of observation 2 from the mean as (_____ - \bar{x}). X_2
123. If the collection of data consisted of three observations, you could represent the sum (total) of the three deviations from the mean by $(X_1 - \bar{x}) + (X_2 - \bar{x}) + (\text{___} + \text{___})$. X_3, \bar{x}
124. If your three observations had the values 10, 2, and 4, you could put these actual values in the previous formula and write it as $(10 - \bar{x}) + (\text{___} - \bar{x}) + (\text{___} - \bar{x})$. 2, 4

five possible outcomes occurred. The six possible outcomes are said to be **mutually exclusive** since the occurrence of any one outcome excludes the occurrence of any other possible outcomes.

25. To review, we have begun our description of Probability Theory by defining a **sample space** as a list of all

_____ of a process.

26. Furthermore, we will suppose that **one and only one** of

the outcomes can occur. That is what is meant when we

said that the outcomes are mutually exclusive.

27. The next basic concept of Probability Theory we will

consider is that of **probability distribution**. A probability distribution is simply the assignment of a non-negative

number to each member of a sample space such that the

sum of all these numbers equals 1. Using the sample

space for one toss of a coin as an example, therefore, we could assign the number $\frac{1}{2}$ to the outcome **heads** and the

number $\frac{1}{2}$ to the outcome **tails**. Each of the outcomes in

the sample space has been assigned a number such that

the total of these numbers equals 1. This assignment

of numbers to members of a sample space is an example

of a **probability distribution**.

28. Each of the numbers assigned to a member of a sample

space is called a **probability**. Therefore, the number $\frac{1}{2}$

assigned to the outcome heads in the previous illustration

is a _____.

125. According to our previous discussion of the mean, you know the sum of the deviations from the mean must equal _____. Therefore, the following formula _____ zero, could not be true if \bar{x} is the mean of these three observations:

$$(10 - \bar{x}) + (2 - \bar{x}) + (4 - \bar{x}) = 5$$

126. If your data consisted of three observations and the following equation were true, 10 would be the _____ mean of these three observations.

$$(X_1 - 10) + (X_2 - 10) + (X_3 - 10) = 0$$

127. In other words, without actually stating the values of the observations nor the actual value of the mean, we know (from the definition of the mean) that the following equation is _____ for any collection of three observations. true

$$(X_1 - \bar{x}) + (X_2 - \bar{x}) + (X_3 - \bar{x}) = 0$$

128. According to simple algebra, we _____ remove the parentheses in the previous equation and write it as follows: could

$$X_1 - \bar{x} + X_2 - \bar{x} + X_3 - \bar{x} = 0$$

129. Furthermore, we could rearrange the symbols on the left-hand side of the previous equation so that the equation reads as follows:

$$X_1 + X_2 + \underline{\hspace{1cm}} - \bar{x} - \bar{x} - \underline{\hspace{1cm}} = 0 \quad X_3, \quad \bar{x}$$

21. While these games of chance are **examples** of random processes, a random process can be defined in general as **any** process that can be represented by _____

Probability Theory

22. A complete introduction to the mathematical theory of probability is far beyond the scope of this program. It will be possible, however, to illustrate the character of Probability Theory and to introduce some of its more fundamental concepts. One of the basic concepts involved in Probability Theory is the idea of a **sample space**. Whenever a process can have several possible outcomes, a list of all possible outcomes can be thought of as a **sample space**. For example, when you toss a coin the two possible outcomes are heads and tails. (We will ignore the possibility of its landing on its edge for this illustration.) Thus, a **sample space** for the simple process of tossing a coin is represented by the following list of possible outcomes: _____ and _____.

heads, tails

23. A **sample space** for a single roll of the die consists of a list of all possible outcomes. In other words, the list 1, 2, 3, 4, 5, and 6 represents the _____ for a single roll of a die, where each of the six numbers is a possible number of dots showing after each roll.

sample space

24. Notice that **only one** of the possible outcomes in the sample space can occur each time the die is rolled. In this sense, the occurrence of a particular outcome **excludes** the occurrence of any of the other outcomes. Thus, to say that a die was rolled and that one dot was showing is equivalent to saying that none of the other

130. Remember, the left-hand side of the previous equation is the **sum of the deviations from the mean**. Notice how this sum is made up of the total of all the scores from which we subtract the mean $\frac{\text{three}}{\text{three/four}}$ times. three

131. We can represent the total of the three observations by $\sum X$. Also, subtracting the mean three times is the same as subtracting $3\bar{x}$. Therefore, we could write the previous equation as $\sum X - \underline{\hspace{1cm}} \bar{x} = 0$. 3

132. The previous equation says that when we subtract $3\bar{x}$ from the sum of the three observations, our answer is zero. Therefore, the total of the three observations ($\sum X$) must be exactly equal to $3 \underline{\hspace{1cm}}$. \bar{x}

133. We started with an equation that said the sum of the deviations from the mean equals zero, and we have proceeded to the following equation:

$$\sum X = 3\bar{x}$$

If we divide both sides of the previous equation by 3, the result would be

$$\frac{\sum X}{3} = \frac{3\bar{x}}{3}$$

The 3's on the right-hand side of the equation cancel each other out, leaving us with the equation:

$$\frac{\sum X}{3} = \underline{\hspace{1cm}} \quad \bar{x}$$

18. Probability theory can be defined simply as a mathematical way of presenting _____ processes. In fact, mathematicians originally developed probability as a way of representing the random process in a game of chance. Later, they found Probability Theory was a useful way of representing processes other than in games of chance. It became useful to think of a random process as **any** process that could be represented by Probability Theory. Therefore, it is possible to say that any process that can be represented by Probability Theory is a _____ process.
19. As a psychologist, you will be interested in Probability Theory primarily because it is a way of representing the uncertainty involved in random sampling procedures. It is even possible to use Probability Theory to predict the form of sampling distributions without actually collecting the samples necessary to form an **experimental** sampling distribution. We noted earlier that an experimental sampling distribution is based on an actual collection of samples, while a **theoretical** sampling distribution is **not** based on actual samples but rather on logical or theoretical considerations. Thus, the sampling distribution predicted by means of Probability Theory would be a(n) _____ experimental/theoretical sampling distribution.
20. Remember, Probability Theory was originally devised as a mathematical way of representing _____ of _____, such as dice and roulette.

games
chance

theoretical

random

random

134. This equation is a **formula** for the mean, since it says that if we added together our three values and divided this sum by the number of observations (3), the result would equal the mean.

Let's see if this formula works for a particular example.

Suppose your data consisted of the values 2, 8, and 2.

The formula says to first add the observations. This

would give you a sum of _____. Dividing this sum by 3

12

would give you a result of _____.

4

135. In other words,

$$\frac{\sum X}{3} = \frac{6 + 2 + 2}{3} = \frac{12}{3} = 4$$

Therefore, according to the formula, the **mean** of the

three observations is _____. We can check to see whether

4

4 really is the mean of the previous observations by

considering the following table:

OBSERVATION	VALUE	DEVIATION FROM 4
1	8	4
2	2	-2
3	2	-2

The sum of the deviations from the reference value 4

equals _____. Therefore, we know that 4 is the _____

zero, mean

of the 3 observations.

13. Each time you roll a die, there are _____ possible outcomes since the die can fall with either one, two, three, four, five, or six dots showing on its face.
14. While you are uncertain exactly how many dots will show on any particular roll of the die, you can describe this uncertainty. For example, you would expect each of the possible outcomes to occur about as often as the other possible outcomes. Thus, you might make the following kind of confidence statement:
"I would expect the die to fall with one dot showing about _____ times out of every 100 600 tosses."
15. The answer to the previous frame was 100 since each of the six possible outcomes would be expected to occur about as often as any other. Since there are _____ possible outcomes, each outcome would occur about 100 times in every 600 tosses.
16. Another example of a **random process** is the kind of **random sampling procedure** we described earlier. According to this sampling procedure, each member of a population has the _____ opportunity to be included in a sample as has any other member.
17. In this section, we will consider a type of mathematics called **Probability Theory**, which is useful in describing random processes. In other words, this type of mathematics _____ be useful in describing _____ would/ would not _____ games of chance.

would

same

6

100

six

136. The formula we derived was for a collection of **three** observations. Suppose the data consisted of 4 observations. We could write the following equation, which specifies the relationship between the mean and those 4 observations:

$$(X_1 - \bar{x}) + (X_2 - \bar{x}) + (X_3 - \bar{x}) + (X_4 - \bar{x}) = \underline{\hspace{2cm}}. \quad 0$$

137. We could remove the parentheses in the previous equation and rearrange the terms so that the equation reads as follows:

$$X_1 + X_2 + X_3 + X_4 - \bar{x} - \underline{\hspace{1cm}} - \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = 0 \quad \bar{x}, \bar{x}, \bar{x}$$

138. Using our shorthand way of writing this, we could say that:

$$\underline{\hspace{2cm}} X - 4\bar{x} = 0 \quad \sum$$

139. Remember, the sum of the deviations from the mean equals $\sum X - 3\bar{x}$ when the data consists of 3 observations, and $\sum X - 4\bar{x}$ when the data consists of 4 observations. No matter how many observations there are in the collection of data, it is not hard to see that the following equation $\underline{\hspace{2cm}}$ be true. would
would/would not

$$\sum X - N\bar{x} = 0$$

where N is the number of observations.

140. Just as we did earlier, we could rearrange this equation to read: $\sum X = \underline{\hspace{1cm}} \bar{x}$. Then, dividing both sides of the equation by N, we would finally arrive at the following formula for the mean: N

$$\frac{\sum \bar{x}}{N} = \frac{N\bar{x}}{N} = \underline{\hspace{2cm}} \quad \bar{x}$$

9. You are often uncertain about the outcome of many processes. For example, when you roll a die, you are certain/uncertain regarding the outcome. When you draw a random sample from a population, you are certain/uncertain about the particular sample you will obtain.

10. A process in which there is uncertainty concerning outcomes can be described as a **random process**. The most common examples of a random process are games of chance, such as dice or roulette. It is the uncertainty of how many dots will show when the die is thrown and the uncertainty over where the ball will come to rest in roulette that makes these games **random processes**.

11. A simple game of chance is one in which a person bets which face will show when a coin is tossed. A simple game of this sort would also be a random process because it is uncertain which face will show on a given toss.

12. While you would be uncertain about the outcome of tossing a coin, it is possible to say something about your uncertainty in this kind of random process. For example, if the coin were a normal one, tossed in a normal manner, you would expect heads to occur about as often as you would tails. In other words, you might make the following kind of confidence statement concerning the outcome of any particular toss.

"I would expect to obtain about _____ heads out of every 100 tosses of the coin."

This formula says the mean of a collection of N observations is equal to the total of all of the observations divided by the _____ of those observations.

number

141. Thus, if you had a collection of 10 observations and the total of all the observations were 50, the mean of the observations would equal _____ divided by _____. In other words, the mean would equal _____.

50, 10
5

142. Let's test this formula on the following collection of 5 observations.

OBSERVATION	VALUE	DEVIATIONS FROM 6
1	10	4
2	6	0
3	6	0
4	6	0
5	2	-4

Notice that $\sum X =$ _____ for this data (because $\sum X$ is a shorthand way of writing "sum of all the values").

30

143. $\sum X$ equals 30, and N equals _____; thus, our formula for the mean says to divide _____ by _____, which would give a result of _____.

5
30, 5
6

144. The formula says 6 is the mean of these five observations. In the third column of the table, we have listed the deviations of each observation from the reference value _____. It is clear that the sum of these deviations equals _____. This indicates that 6 _____ the mean of _____ is/is not these 5 observations.

6
0, is

In everyday conversation, it is often sufficient to say that one thing is more likely than another in order to express different degrees of uncertainty. As a scientist, however, it is often important to describe your **degree of uncertainty** more precisely. The **confidence statements** we considered earlier are examples of a more precise way of describing uncertainty. For example, both of the following confidence statements imply uncertainty that the mean of a sample drawn from a population in a particular manner will be within one standard deviation of the population mean:

Statement A

Nine-tenths of all samples obtained in this manner had means within one standard deviation of the population mean.

Statement B

Six-tenths of all the samples obtained in this manner had sample means within one standard deviation of the population mean.

Although both statements imply uncertainty, you would be **more** certain of obtaining a sample whose mean is within one standard deviation of the population mean if Statement $\frac{A}{B}$ were true.

A

You would be **more** certain of obtaining a sample mean within one standard deviation of the true mean if Statement A were true, since, according to Statement A, $\frac{.9}{.6}$ of the samples obtained in that manner had

.9

sample means within one standard deviation of the true mean, whereas according to Statement B, only $\frac{.6}{.9}$

.6

had means within one σ of μ .

145. We have considered three ways of representing the central tendency of a distribution: the m _____, the m _____ and the m _____.
mode
median, mean
146. Often a distribution will have a different mean, median, and mode. The data shown in the previous table, however, has a mode of _____, a median of _____, and a mean of _____.
6, 6
6
147. Six is the _____ of the previous collection of data since this is the most frequently occurring value. Six is also the _____ of the previous collection of data since there is, in the collection of data, one value greater than six and one value less than six. Finally, 6 is the _____ of the previous collection of data because the sum of the deviations from 6 equals zero.
mode
median
mean
148. The mean is a useful way of representing the central tendency of a distribution because its value depends on every value in the distribution. The value of the _____ is determined only on the basis of mean/median/mode
mode
the most frequently occurring value in the collection of data. Finally, while you know that there are the same number of values above the _____ as mean/median/mode
median
there are below it, you _____ know how far above do/do not
do not
or how far below the median these values are. Thus, each of the various ways of representing the central tendency of a distribution has its own peculiar features.

4. Almost any event you speak of **as if** it were a certainty will be revealed to have some uncertainty when examined closely in this way. Naturally, there are many events which are not spoken of as if they were certainties. For example, you might say, "It is **very likely** the Yankees will win the World Series this year." This statement implies that you are/are not certain the Yankees will win the World Series.

5. You might tell someone that you will probably be home this evening and the word **probably** implies you are/are not absolutely certain whether you will be home or not.

6. It is possible for a statement to imply more or less uncertainty. For example, both the statement "I will **probably** be home tonight" and the statement "I **might** be home tonight" imply that you are certain/uncertain about being home.

7. While both statements imply some uncertainty, the statement "I will probably be home tonight" implies that you are more/less certain of being home than does the statement "I might be home tonight."

more

uncertain

are not

are not

Section IV: Variability

1. We have just seen that the mean, median, and mode are three _____ that characterize the _____ of a distribution.

statistics
central tendency

2. Besides their central tendency, there is another important characteristic of distributions, one that is **not** represented by either the mean, the median or the mode. Some collections of data are composed of many similar values, while in other distributions the values might vary considerably. For example, the values in Data A/B (below) are all very similar to each other, while the values in Data A/B are much more widely separated.

A

B

Data A: 20, 21, 20, 19, 20

Data B: 2, 38, 20, 5, 35

3. We have listed the two previous collections of data in the following tables:

Observation	Value	Deviation From 20
1	20	0
2	21	1
3	20	0
4	19	-1
5	20	0

TABLE A

1.

You sometimes speak **as if** an event in the future could be predicted with **certainty**. For example, you might say, "When I press this button on the wall, the light on the ceiling will turn on." This statement implies you are certain/uncertain the light will turn on when you press the switch.

certain

2.

Naturally, there is always the possibility that the switch may have been broken or that the light bulb burned out since the last time you pressed the switch. If the switch has almost always worked in the past, however, it is often simpler to speak a _____ _____ you were certain that the light would go on, even though there is some possibility that it will not.

as if

3.

If you were explaining what would happen when you tossed a coin, you might say it will land either **heads** or **tails**, **as if** it were certain that the coin would not land balanced on one edge and, therefore, be neither heads nor tails. Once again, it is simpler to speak **as if** you were certain the coin would land either heads or tails, since the proportion of times it would land balanced over a very large number of tosses is so small/large that the possibility can be ignored for most purposes.

small

3. (Continued)

Observation	Value	Deviation From 20
1	2	-18
2	38	+18
3	20	0
4	5	-15
5	35	+15

TABLE B

Notice that the tables indicate the deviation of each value from the reference value _____. 20

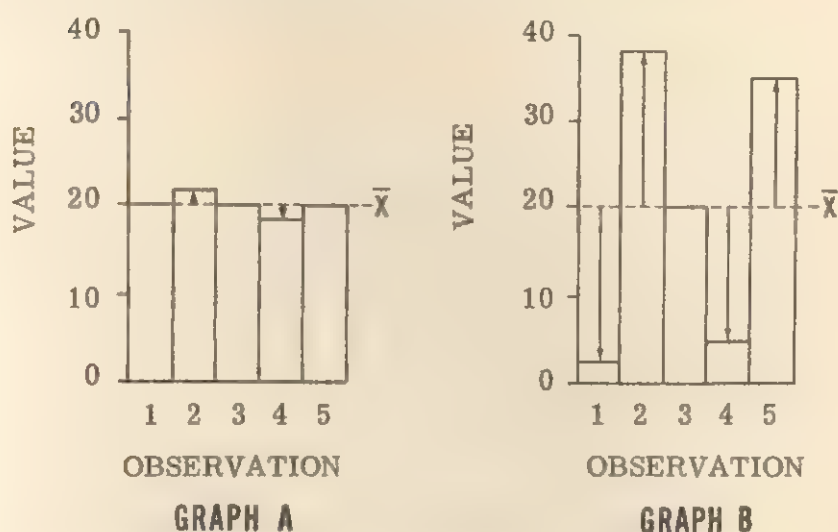
4. The sum of the deviations in Table A is _____. 0
Therefore, the mean of Data A is _____. 20

5. The sum of the deviations in Table B is ____; therefore, 0
the mean of Data B and the mean of Data A are
_____.
the same/different the same

6. While the means of both distributions are identical,
there is an interesting difference in the two distributions.
If we ignore whether or not a deviation is positive or
negative and only consider its absolute size, it is clear
that the deviations in Table _____ tend to be larger B
A / B
than the deviations in the other table. In other words,
the values in Table _____ are more spread out B
A / B
(dispersed) around the mean value of 20.

6. If a population consists of a specific number of observations, it is called:
 a. an infinite population.
 b. an unlimited population.
 c. a disconnected population.
 d. none of the above
-
- TRUE OR FALSE:
7. If the true mean of a population were 27 and your estimate were 25, your absolute error of estimate would be 2.
 true
8. Sampling procedure in which all members of the population have the same opportunity of being selected is called a random sampling procedure.
 true
9. The larger the size of the samples, the more frequently you would expect the sample mean to be far from the population mean.
 false
10. The risk of obtaining a highly unrepresentative sample increases with the size of the sample.
 false

7. The manner in which the deviations from the mean are different in the two previous collections of data can be illustrated by the following graphs of raw data.



We have represented deviations from the mean with arrows, as we have done previously. If we ignore the direction in which an arrow is pointing and only consider its length, it is clear that the deviations from the mean in Data A / B are larger than those of the other collection.

B

8. Notice how the heights of the columns in Graph A / B are very similar, whereas the heights of the columns in the other graph tend to change or **vary** much more from one observation to another.

A

9. Since the height of a column represents the value of that observation, we could say that the values on Graph A / B change or **vary** more than do those on the other graph.

B

FILL IN THE BLANKS:

1. Distributions in which most of the scores are piled up near one end are called _____.

skewed

2. If a distribution has most of its observations piled up near the high values, it is said to be _____.

negatively skewed

3. If a distribution has a standard deviation of 3 and a mean of 10, then a score of 16 would represent a standard score of _____.

2

MULTIPLE CHOICE:

4. A value identical to the mean has a standard score of:
 - a. 0.
 - b. 1.
 - c. 2.
 - d. none of the above

a

5. The sample, as used by statisticians, refers to:
 - a. a very small, but complete, collection of data.
 - b. a collection of data which is considered as part of a larger, complete collection of data.
 - c. bell-shaped.
 - d. none of the above

b

10. The more the values in a collection of data vary, the more **variability** that collection of data is said to have. Thus, we would say that the **variability** of Data $\frac{A}{B}$ was greater than the variability of the other collection of data.

B

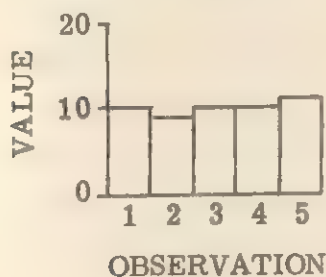
11. Data composed of many widely different values is said to have a great deal of **variability**. On the other hand, a collection of data in which the values are very similar or close together could be described as having little **variability**. Therefore, of the following two collections of data, Data $\frac{A}{B}$ would be described as having more **variability** than the other collection.

B

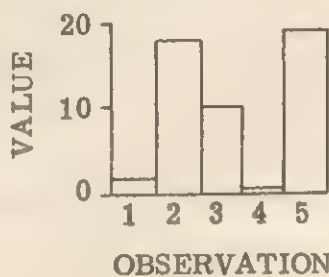
Data A: 10, 9, 10, 10, 11

Data B: 2, 18, 10, 1, 19

12. We could illustrate the difference in **variability** of the two previous distributions with the following graphs:



GRAPH A



GRAPH B

Earlier we defined a variable as something that changed or varied. The more something changes or varies, the more **variable** it is said to be. Thus, the observed values are more variable in Graph $\frac{A}{B}$ than in the other graph.

B

this risk. It is important to bear in mind that the

sampling distribution is determined both by the

characteristics of the population you are sampling and

the procedure by which you have obtained the samples.

The experimental sampling distributions we have just

considered are based on a

sampling procedure.

264.

In the earlier illustration, in which samples of size 10

were drawn from the population of 10,000 college students,

we were interested in student opinion on a particular

question. Suppose the same procedure were followed

to obtain 100 samples from the same students but on an

entirely different question. If the population proportion

of favorable opinions on this question were not the same

as the population proportion of favorable opinions on the

previous question, you would expect the experimental

sampling distribution to be

similar/different.

similar

265.

Suppose you obtained 100 samples of opinions using a

biased sampling procedure. In this case, you

know whether to expect the new

would/would not

experimental sampling distribution to be similar to the

old experimental sampling distribution. In the next

section, we will consider the manner in which you can

predict the distribution of sample statistics obtained by

a **random** sampling procedure without actually collecting

the samples. In other words, we will consider how to

determine a

theoretical/experimental

sampling

theoretical

distribution.

13. The more variable the values in a collection of data, the more **variability** that collection is said to have. In other words, the more the values in a collection of data change or vary from observation to observation the more _____ that collection is said to have. variability
14. If all the values in a collection of data were very similar, the data would **not** have much variability. If all the values are very similar, you would expect the difference between the largest value and the smallest value to be _____. small
large/small
 In other words, if the difference between the largest and smallest value in a collection of data were very small, the data _____ have much variability. would not
would/would not
15. We often use the **difference** between the largest and smallest value in a collection of data as a statistic representing the variability of that data. We call this statistic the **range**. In other words, you could represent the variability of a collection of data by finding the difference between the largest and the smallest value in that collection of data. This difference is a statistic called the _____. range
16. If your data consisted of the values 100, 50, 10, 75, and 80, then the _____ of these values would be 90, range
 since this value is the difference between the highest value (100) and the lowest observed value (10).

On the other hand, every banana used in your sample would be unsuitable for sale. This is an example of what is called **destructive** sampling, since the bananas included in each sample are **destroyed**. To determine the size of your sample, you would have to compare the $\frac{\text{gain/loss}}{\text{gain/loss}}$ associated with having a large and more likely representative sample to the $\frac{\text{gain/loss}}{\text{gain/loss}}$ produced by destroying those members of the population that would be included in the sample.

loss

gain

261.

A tire manufacturer submitted **some** of the tires produced at his factory to pressure until they exploded. In this way he determined the maximum pressure he could expect tires produced at his factory to withstand. This would be another example of destructive sampling.

262.

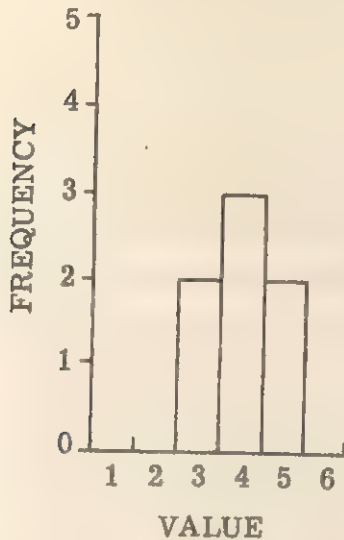
Sometimes it is impossible to obtain a complete collection of data. In such a case you would have to settle for a sample. For example, suppose your population consisted of the heights of all the people on the North American continent 100 years before the arrival of Columbus. If you only knew the heights of three skeletons found by archeologists and identified as belonging to that population, you would only have a sample, population from the _____.

263.

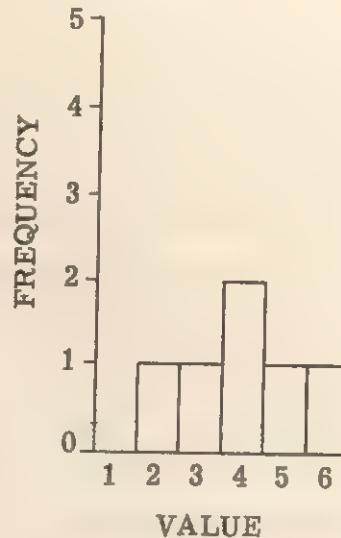
While there are many advantages to obtaining samples, you always run the **risk** of obtaining an unrepresentative sample. You have seen how knowledge of the distribution of sample statistics can help you in specifying

17.

The relation between the range and what we have referred to as the variability of a distribution can be illustrated on the following frequency distribution graphs:



GRAPH A



GRAPH B

The data shown on both of these graphs have the same mean, but the data on Graph B is more **variable** than the data on the other graph.

Notice that the **difference** between the highest and lowest observed values on Graph A equals ____ minus ____.

5, 3

The difference between the highest and lowest value on Graph B equals ____ minus ____.

6, 2

18.

Since the range of a collection of data is the difference between the highest and lowest values, the **range** of the data on Graph A is ____ and on Graph B, ____.

2, 4

258.

It is quite apparent that the frequencies of sample

means whose absolute difference from the population

mean was 1 or less would

increase/decrease

as the size

increase

259.

In this section of the program, we have been considering

the problem of characterizing a population on the basis

of a sample. There are many advantages to simply

obtaining a sample rather than a complete collection of

data on the population. For example, it might be very

difficult to obtain a complete collection of data. If you

wanted to know something about every person in the

United States, you could do one of two things. You could

conduct a national census (an extremely difficult task,

which the government only attempts to undertake every

few years), or you could obtain only a

_____ from this population, rather than obtaining a complete

collection of data.

260.

Suppose you were a merchant who had just received a

shipment of 50,000 bananas from South America. You

could determine the condition of these bananas by

peeling and tasting each of them. This is

obviously not a very sensible proposition since a

merchant could not market 50,000 peeled and nibbled

bananas. On the other hand, you might answer your

question satisfactorily by merely taking a sample from

this population. You could peel and taste only some of

the 50,000 bananas. You would have some incentive for

making your sample large since a large sample is

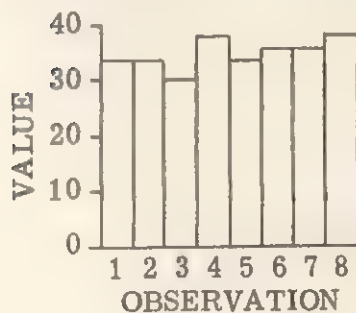
more likely to be representative of the population.

19. Notice that the distribution in which the values are **spread** farthest from the mean has the largest range. In other words, the distribution which is most v _____ variable would tend to have the largest range.

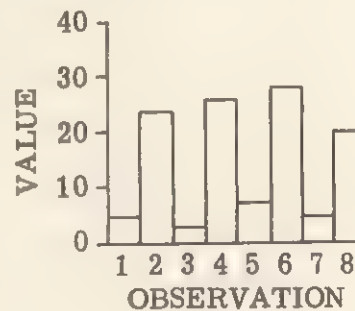
20. Suppose the largest value in a collection of data were 100 and the smallest value were 10. The range of that collection of data would be _____, since this equals _____ minus 10. 90, 100

21. Instead of considering frequency distribution graphs, let's consider how a graph of raw data indicates the range. The **range** of a collection of data is immediately obvious in a graph of **raw data**, since the range is the _____ difference in height of the column representing the smallest observed value and the height of the column representing the largest observed value.

22. The variability of a collection of data does **not** depend on the size of the values in the collection. It only depends on the **differences** in the sizes of the values. For example, consider the two graphs of **raw data** shown below. (Do not confuse these graphs with frequency graphs.)



GRAPH A

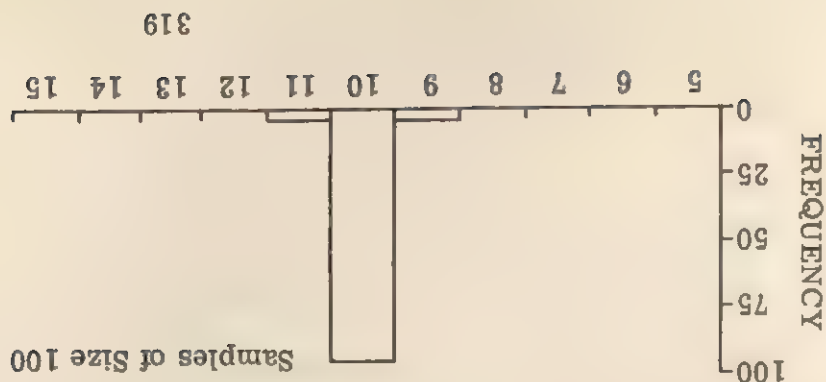
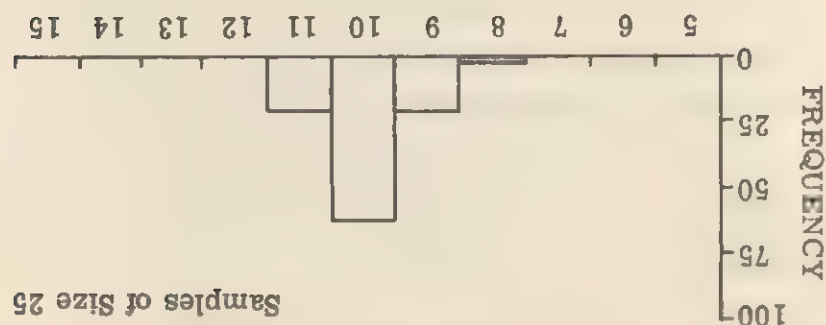
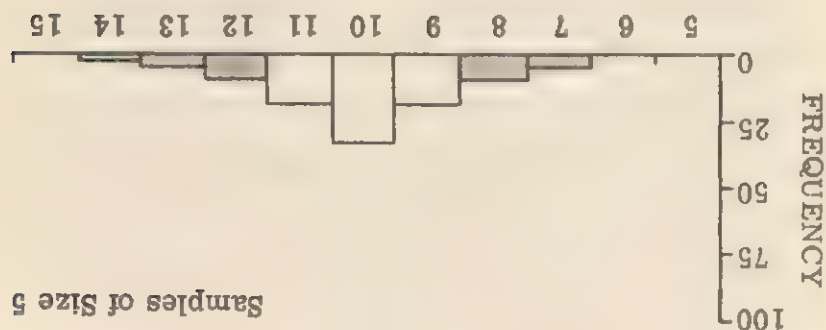
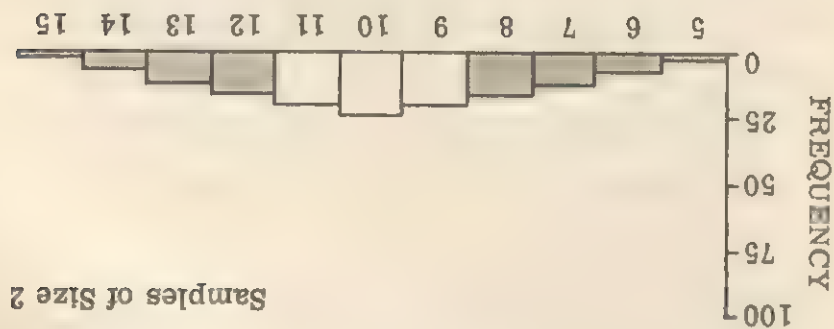


GRAPH B

Notice that the values on Graph _____ are all larger than _____ A / B

those on the other graph. However, the variability of the data on Graph _____ is greater than the variability _____ B of the other collection of data.

Frequency distributions of 100 sample means based on samples drawn at random from a large population. The population has an approximately normal distribution with a mean of 10 and a variance of 8.



23. The range of the data in Graph A / B would be found

A

by subtracting the value of observation 3 from the value of observation 4.

24. The **range** of a collection of data can also be determined from a **frequency** table by finding the difference between the smallest value having a frequency greater than 0 and the largest value having a frequency greater than 0. In the following frequency table, for example, the largest value having a frequency greater than 0 is _____ and the smallest value having a frequency greater than 0 is _____. Therefore, the range of this distribution is _____. The value 13 was a possible value which _____ was/was not

12

10

2

was not

observed. This is why we only consider values with frequencies greater than zero.

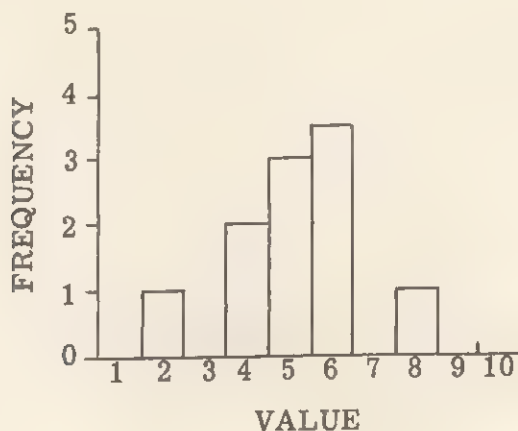
Value	Frequency
10	2
11	20
12	3
13	0

25. The **range** of the distribution shown in the following frequency distribution graph is _____, since the smallest **observed** value is _____ and the largest observed value is _____.

6

2

8



You might think about the change in the variability of sample means as the size of the sample is increased in the following manner. If the sample is very large, you would expect to obtain many values in the center of the distribution as well as some values from each tail. The sample mean would tend, therefore, to be quite close to the true population mean. On the other hand, if the sample were very small (for example, a sample of 2), you could more easily obtain both values in your sample from the same tail of the population distribution. In this case, the sample means would be quite

similar to/different from
the population mean.

different from

In the case of small samples, you run a greater risk of obtaining values from one extreme of the distribution. This would give you a highly unrepresentative picture of the population. On the other hand, the larger the size of your sample, the more likely it is that the sample will contain values from all parts of the distribution and would therefore be representative/unrepresentative of the population.

representative

The following figure contains the four theoretical/experimental sampling distributions we

experimental

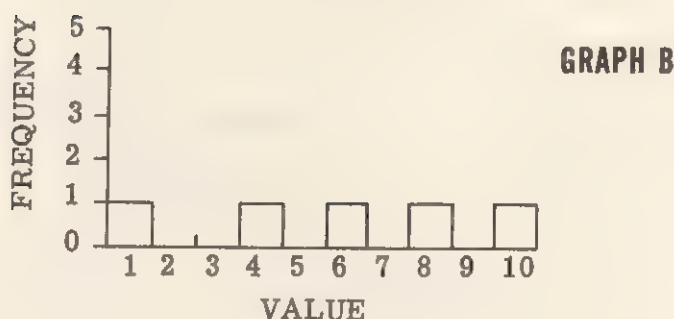
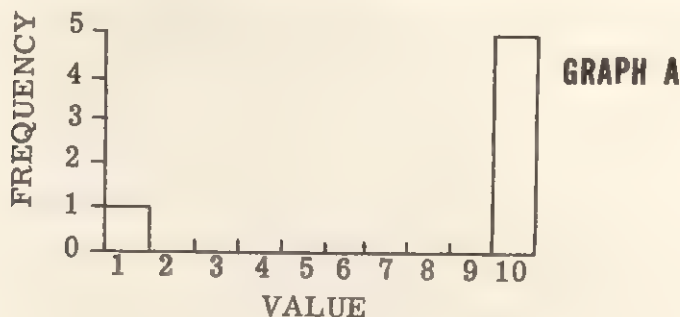
considered a moment ago. However, we have shaded all the columns representing frequencies of sample means which would have led to an absolute error of estimation greater than _____. The unshaded columns represent the frequencies of samples in each distribution which would have led to an error of estimation of _____ or less.

1

1

26.

The range can sometimes give a misleading picture of the variability of a distribution. For example, consider the following two frequency distribution graphs.



Notice that the range of distribution in Graph A is _____ 9
and the range of distribution in Graph B is _____. 9

Notice that all the values except one are identical in
Graph A / B, whereas the values in the other graph were A
all different. Therefore, even though both distributions
have the **same** range, the distribution in Graph A / B A
appears to be more variable than does the other
distribution.

The problem in representing the variability of a
distribution with the range is that it is determined solely
by the **largest** and **smallest** observed values.

252. The value of μ (in other words, the population _____) is _____. You could say the **central tendency** of all four experimental sampling distributions was/was not approximately 10.

mean 10 was

253. While the experimental sampling distributions had similar central tendencies, the **variability** of the sampling distributions increased/decreased as the size of the sample was increased.

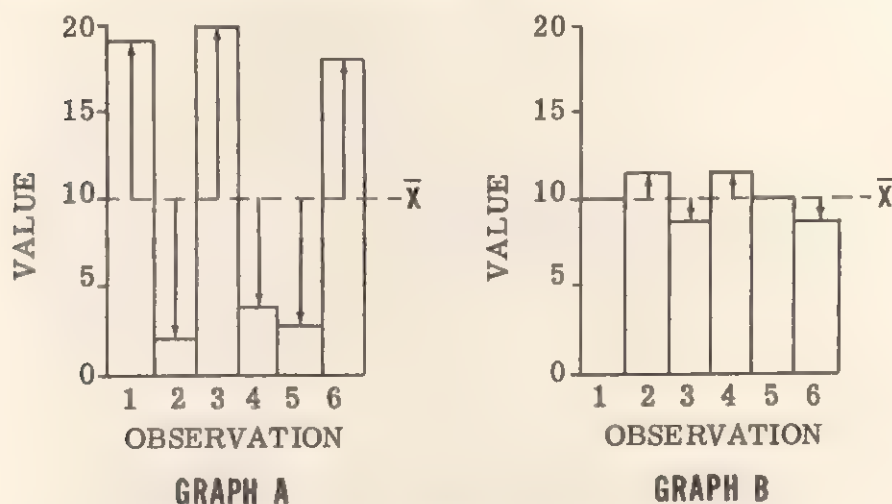
decreased

254. Notice that all four experimental sampling distributions are approximately symmetrical and somewhat bell-shaped. Since the population has been described as having a distribution that was approximately normal, you know that the population distribution was _____ and had that special form symmetrical/asymmetrical of a bell-shaped distribution in which the proportion of values within any number of standard deviations was described by the graph we considered earlier.

Most of the values in a **normal distribution** are grouped about the mean with progressively fewer values at greater distances from the mean. In other words, the values in the tails of a normal distribution have the lowest/highest frequency.

lowest

27. Earlier, we pointed out how the variability of a collection of data is related to the deviations of the values from their mean. For example, consider the two graphs of raw data shown below:



We have shown deviations from the **mean** value on both of these graphs. Thus, the mean of both distributions is the same/different since it equals 10 on Graph A and 10 on Graph B.

the same

While we know that the sum of the deviations from 10 equals 0 in both distributions, there is, nevertheless, an important difference between the two distributions.

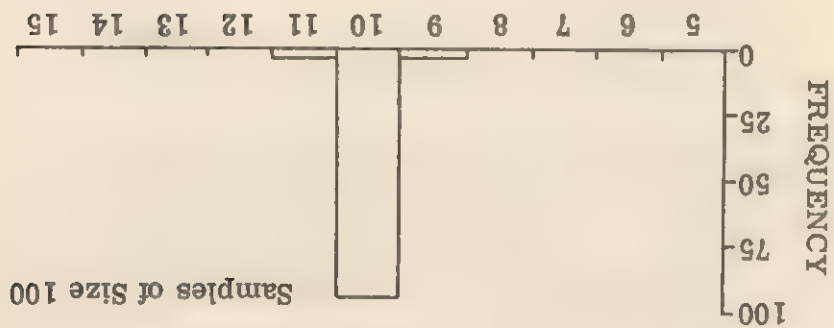
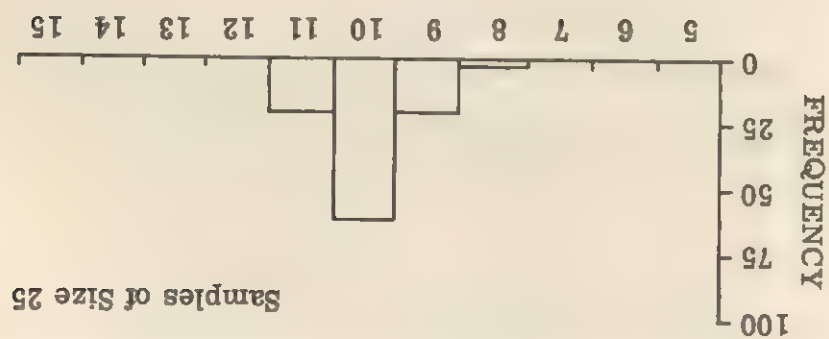
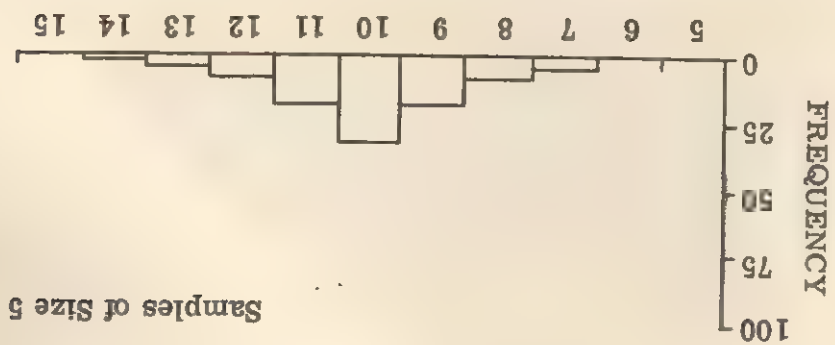
0

28. It is apparent that the deviations from 10 tend to be larger for the data shown on Graph A / B than for the data in the other graph.

A

29. You could describe the differences between the two distributions by saying that the value of each observation tended to vary or change more on Graph A / B than on the other graph.

A



Notice that these four frequency distribution graphs include one graph for samples of size 2, another for sample means from samples of size 5, another for sample means from samples of size 25, and the last (bottom graph) for sample means from samples of size _____.

30. Data containing many widely separated values of a variable is said to have more **variability** than data containing very similar values. Thus, of the two previous distributions, the Distribution in Graph A/B could be described as having the most **variability**.

A

31. In other words, we could say that the two distributions shown in Frame 27 are similar in terms of their central tendencies/variability and different in terms of their central tendencies/variability.

central tendencies

variability

32. The relationship between the variability of a collection of data and the size of the deviations from the mean is illustrated by the following two tables.

Observation	Value	Deviation from 10
1	15	5
2	8	-2
3	12	2
4	5	-5

TABLE A

Observation	Value	Deviation from 10
1	9	-1
2	10	0
3	11	1
4	10	0

TABLE B

Notice that the deviation of observation 4 from the mean in Table A was obtained by subtracting 15 from 10, to yield an answer of -5.

10, 5

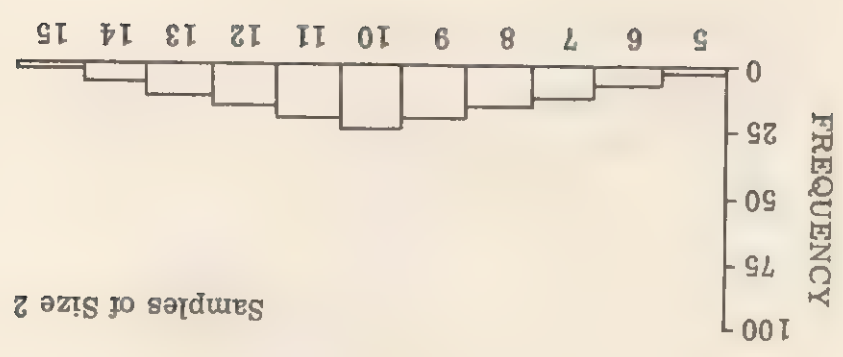
-5

249. In other words, you would be $\frac{\text{confident that}}{\text{more/less}}$ your absolute error of estimate would be 1 or less if the sample size were 5 than if the sample size were 25.

250. In general, then, as you increase the size of your sample, it appears that you can be $\frac{\text{confident}}{\text{more/less}}$ that the sample mean will be close to the population mean in value.

251. The previous illustration is even more apparent when you consider the following graphs representation of the four experimental sampling distributions we have just considered.

Frequency distributions of 100 sample means based on samples drawn at random from a large population. The population has an approximately normal distribution with a mean of 10 and variance of 8.



33. Notice that the sum of the deviations in Table A equals _____ and the sum of the deviations in Table B equals _____. This indicates that the _____ of both distributions is 10. 0
0, mean

34. It is clear that the values tend to be farther away from the mean in Table $\frac{A}{B}$ than they do in the other table. A

Without considering the deviations from the mean, it is apparent that the value of the observations changed or varied more in Table $\frac{A}{B}$ than they did in the other table. Therefore, the _____ of the data in Table A appears to be greater than the variability of the data in the other table. A
variability

35. We could represent the difference in variability by calculating the **range** of each collection of data. The range of the data in Table A would be _____, since this equals _____ minus _____. 10
15, 5

36. Since the range of the data in Table B is _____, the range of the data in Table A is _____ than the range of the data in Table B. 2
greater

37. The range is not the only statistic you can use to represent the variability of a distribution. There is another way of characterizing the difference in variability of the two previous collections of data. Notice that if we ignore whether a deviation is positive or negative, the deviations in Table $\frac{A}{B}$ tend to be larger than those in the other table. A

245.

We might put this another way. We might say: Of the 100 samples of size 100, there were _____ samples which would have led to an absolute error of estimation greater than 1.

246.

We have compared all four experimental sampling distributions in terms of the number of sample means which differed from the true population value (10) by 1 or less (in other words, sample means of 9, 10, or 11). In summary, there were 45 samples of size 2 which would have led to an absolute error of estimation of one or less. There were 76 such sample means for samples of size 5. There were 99 such samples of size 25, and 100 such samples of size 100.

247.

According to the preceding statement, you could make the following **confidence statement**: "If I obtained samples of size _____ from this population by this random sampling procedure, I would expect the absolute error between my sample mean and the population mean to be 1 or less about 66 times in every 100 samples." (Once again, we shall assume that the 100 samples of each size are reasonably representative of the distribution of samples you would obtain through additional sampling.)

248.

If the previous **confidence statement** had specified a sample of size 25 instead of size 5, you could have said: I would expect the absolute difference between the sample mean and the population mean to be one or less in about _____ out of every 100 samples.

99

You can think of the **variability** of a collection of data as the degree to which the values are spread out from the mean. In other words, if all of the values in a collection of data are very similar, they will all be clustered very close to the mean and the data will **not** have much variability. If the values in a collection of data are widely dispersed (spread out), the deviations from the mean will tend to be very _____ and the distribution could be described as having a great deal of _____.

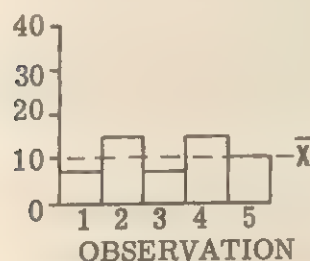
large

variability

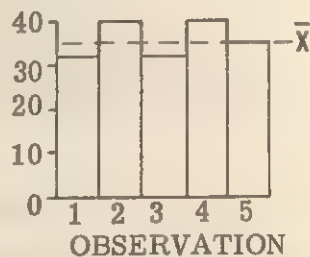
38.

We have seen several illustrations in which two collections of data have the **same mean** but **different variability**. It is also possible for two collections of data to have **different means** and the **same variability**. For example, consider the two graphs shown below. Notice that the mean of the data in Graph _____ is smaller than the mean of the other collection of data.

A



GRAPH A



GRAPH B

39.

However, the variability of the two collections of data (shown above) about their respective means _____ is almost identical. Thus, the two distributions could be described as having similar _____ but different _____.

is

variability

means

239. You could look at this same characteristic of the sampling distribution from a different point of view. Consider the question: How many samples of size 2 would have led to an absolute error of estimation of 1 or less? Only samples with means of 9, 10, or 11 would lead to an absolute error of estimation of 1 or less. In the 100 samples of size 2, there were 19 + 20 + 16 samples with means that would have led to an absolute error of estimation of _____ or less.
240. Similarly, considering the experimental sampling distribution of sample means from samples of size 5, there were _____ samples that would have led to an absolute error of estimation of 1 or less.
241. The answer to the previous frame was obtained by adding the frequencies of samples with means of 9, 10, or 11. These 3 frequencies were _____, _____, and _____. Therefore, the sum of these frequencies was 76.
242. According to the experimental sampling distribution for samples of size 25, there were _____ samples that would have led to an absolute error of estimation of 1 or less.
243. The answer to the previous frame was obtained by adding the frequency for sample means of 9, 10, and 11 — in other words, by adding _____, _____, and _____. 15, 67, 17
244. Finally, when the sample size was increased to 100, there were _____ samples which would have led to an absolute estimation error of 1 or less. 100

40. Therefore, simply knowing that two collections of data are similar in terms of their central tendencies _____ indicate whether they are similar in terms of their variability. does not

41. Another illustration of the lack of any relationship between the mean of a distribution and its variability is given by the following two tables of raw data:

Observation	Value	Deviation From 4
1	6	2
2	4	0
3	2	-2

TABLE A

Observation	Value	Deviation From 8
1	10	2
2	8	0
3	6	-2

TABLE B

Notice that the data in each table consists of _____ observations of a _____ variable. 3
numerical

42. The particular reference value from which the deviations are calculated in each of the previous tables is the _____ of that collection of data, since the sum of the deviations in each table equals _____. Thus, the mean of the data in Table A is _____, whereas the mean of the data in the Table B is _____. mean
0
4
8

Frequency distributions of 100 sample means based on samples drawn randomly from a large population. (The population has an approximately normal distribution, with a mean of 10 and a variance of 8.)

Sample Size	Sample mean										
		5	6	7	8	9	10	11	12	13	14
2	5	0	0	0	1	8	11	19	20	32	21
5	25	0	0	0	1	15	67	17	0	0	0
100	100	0	0	0	0	0	0	0	0	0	0

* The mean of each sample was rounded off to the nearest value shown in column one.

237.

Let us compare the four distributions in terms of the number of absolute errors of estimation greater than 1. For example, in the 100 samples of size 2, there were samples that would have led to an absolute error of estimation greater than 1.

238.

The answer to the previous frame was obtained by adding the frequencies 1, 2, 8, 13, 6, 3, and 1. These are the frequencies of all sample means of size 2 which would have led to an absolute error of estimation greater than 1, (i.e., which had sample means that differed from μ [10] by more than 1).

43. We could represent the **variability** of each collection of data by its range. The range of the data in Table A is _____ and the range of the data in Table B is _____.

4, 4

44. Earlier, we pointed out how the variability of a collection of data could be thought of as the degree to which the observed values were spread out or dispersed about the mean of that collection of data. The _____ is a useful measure of variability because it is the difference between the value having greatest **positive** deviation from the mean and the value having greatest **negative** deviation from the mean.

range

45. We also noted earlier that the range is **not** a completely satisfactory way of representing variability. Two distributions may have identical ranges and yet one distribution may appear to be much more variable than the other. For example, consider the two collections of data shown in the following tables:

Observation	Value
1	15
2	5
3	5
4	5
5	5

DATA # A

Observation	Value
1	15
2	5
3	12
4	7
5	8

DATA # B

The smallest value in each table is _____ and the largest value is _____. Notice that the range of both collections of data equals _____.

5
15
10

46. While the range is the same in both collections of data, all the values, except one, are identical in Data _____.

A

A / B

232. Samples with means less than 7 or greater than 13 would have an absolute difference between \bar{x} and μ greater than 3. Thus, there are only _____ samples of size 2 that would have lead to an absolute error of estimation greater than 3.

7

233. Only one sample of size 2 had a mean of 5. Two samples had a mean of 6, _____ samples had a mean of 14, and one sample had a mean of _____. Thus, there were, in all, _____ samples of size 2 that would have led you to make an absolute error of estimation greater than 3.

3
15
7

234. Whereas there were 7 samples of size 2 which would have led to an absolute error of estimation greater than 3, only _____ sample(s) of size 5 would have led to an absolute error of estimation that large.

1

235. The one sample of size 5 you obtained that would have led to an absolute error of estimation greater than 3 was the sample with a mean of _____, since the difference between this mean and the true mean was _____.

14
4

236. There were _____ samples of size 25 and of size 100 which would have led you to make an absolute error of estimation greater than 3. (We have repeated the previous table for your convenience.)

0 (no)

47. This illustration points up one of the disadvantages of the range as a way of representing variability. The range is determined by only two of the observed values in the collection of data:

- 1) the _____ observed value, and largest
- 2) the _____ observed value. smallest

48. Since the largest observed value in each table is _____ 10
 and the smallest observed value is _____, the collections 1
 of data shown in the following two tables have
 _____ ranges. the same
 the same/different

Observation	Value	Deviation From 5
1	1	-4
2	4	-1
3	5	0
4	5	0
5	5	0
6	10	4

TABLE A

Observation	Value	Deviation From 5
1	1	-4
2	1	-4
3	1	-4
4	8	3
5	9	4
6	10	5

TABLE B

49. We have listed the deviations of the values in each table from their common mean of _____. The largest and 5
 smallest values in both collections of data are the same.
 However, the other observed values in Table _____ are all A
 A/B
 very close (or identical) to the mean. All the values in the other distribution are almost as far away from the mean as are the two extreme values.

Frequency distributions of 100 sample means based on samples drawn randomly from a large population. (The population has an approximately normal distribution, with a mean of 10 and a variance of 8.)

Sample Size	Sample			
	mean	2	5	25
5	1	1	0	0
6	2	0	0	0
7	8	1	0	0
8	11	10	1	0
9	19	23	15	3
10	20	32	67	93
11	16	21	17	4
12	13	9	0	0
13	6	3	0	0
14	3	1	0	0
15	1	0	0	0

* The mean of each sample was rounded off to the nearest value shown in column one.

231.

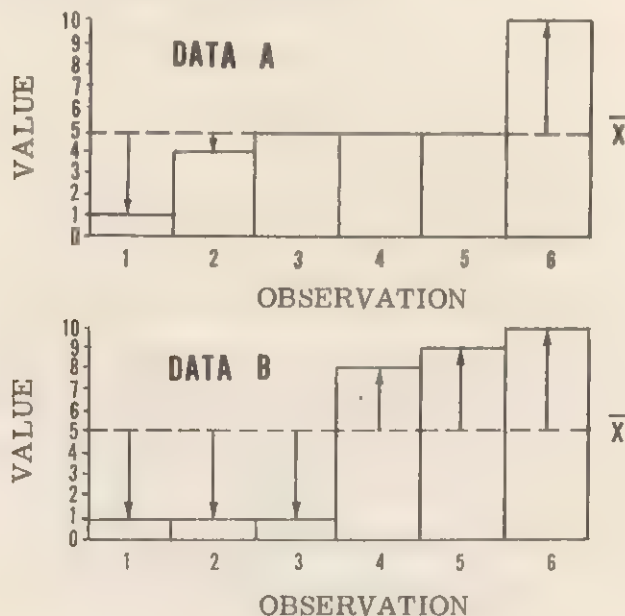
If you used each sample mean as an estimate of the population mean, you would have made an absolute error of estimation greater than 4 only two times in the 100 samples of size two. Only sample means of 5 or 15 would lead you to make an absolute error of estimation greater than 4. Only one sample of size 2 had a mean of 5 and only one sample of size 2 had a mean of 15. Thus, _____ samples of size 2 would lead to an error of estimation greater than 4.

50.

Even though the two extreme deviations in both collections of data are the same, the typical size of a deviation in Data $\frac{A}{B}$ is greater than in the other

B

collection of data. This feature can be illustrated by the following graphs of raw data:



We have indicated the deviations from the mean in both collections of data with arrows, just as we have done previously. The lengths of these arrows indicate how most of the observed values in Data $\frac{A}{B}$ are very

A

close to the mean, whereas the values in the other graph tend to be farther away from the mean.

51.

We have already seen that the mean of a collection of values can be thought of as the typical value. Therefore, one way we might represent the typical size of a deviation would be to find the mean, or the average of the deviations.

However, our formula for finding the mean of a collection of values ($\sum X/N$) says that the first thing to do is to add together all the values. No matter what the variability of a distribution, the sum of the deviations from the mean would always equal ____.

227. After rounding off (grouping) the sample means in this manner, we calculated the frequencies in each collection of 100 sample means. The preceding table lists these frequencies for samples of sizes _____, _____ and _____.

2, 5, 25
100

228. According to the table, there are only two samples of size 2 with means (rounded off) of 5. There were also only two samples of size 2 that had means equal to 6. There were 8 samples of size 2 having means equal to _____, and 11 samples of size 2 with means equal to _____. 7, 8

229. The column on the far right of the table contains the experimental sampling distribution of sample means in samples of size _____. There were 93 samples of this size that had means of _____. 100 10

230. Notice that the most frequently occurring sample mean in each of the four experimental sampling distributions was a sample mean of _____. 10

You _____ say, therefore, that the modal _____ could

sample mean in each of the four experimental sampling distributions was equal to μ . (The table we presented earlier is repeated here for your convenience. Refer back to this table whenever necessary as you answer the following frames.)

52. Statisticians have found it useful to represent the variability of a distribution by the typical or average size of the **squared** deviations from the mean. Whenever you square a number, your answer will be positive, whether the number you are squaring is positive or negative.

For example, $2^2 = 2 \cdot 2 = 4$

Similarly, $(-2)^2 = (-2)(-2) = \underline{\hspace{2cm}}$

4

Therefore, whether a deviation is positive or negative, when you square it, your answer will be positive/negative.

positive

53. In the table of data shown below, we have listed 3 observed values, their deviations from the mean, and the square of these deviations. Thus, X_1 has a value of 8 and a deviation from the mean of 3. The square of that deviation is 9 times 3, which equals 27.

8, 3

3, 3

9

Observation	Value	Deviation from 5	Squared Deviation
1	8	3	9
2	-2	-3	9
3	5	0	0

If you add (sum) all 3 observed values, you obtain the **total** of the observed values, which is 15. Dividing this total by 3 indicates that the mean of these three values is 5.

15

3

5

54. Adding up the deviations from the mean value of 5 will naturally yield an answer of 0, because the mean is defined as that particular reference value from which the deviations sum to 0.

0

0

Frequency distributions of 100 sample means based on samples drawn randomly from a large population. (The population has an approximately normal distribution, with a mean of 10 and a variance of 8.)

*Sample mean	Sample Size			
	2	5	25	100
5	1	0	0	0
6	2	0	0	0
7	8	1	0	0
8	11	10	1	0
9	19	23	15	3
10	20	32	67	93
11	16	21	17	4
12	13	9	0	0
13	6	3	0	0
14	3	1	0	0
15	1	0	0	0

* The mean of each sample was rounded-off to the nearest value shown in column one.

225.

11

In the first column of the table, we have listed possible values of the sample mean. The smallest value of the sample mean listed is _____ and the largest value listed is _____.

15

226.

Each sample mean was rounded off to the nearest value appearing in the first column. In other words, if the sample mean were $6\frac{3}{4}$, we would consider it equal to a sample mean of 7. If a sample mean were $6\frac{1}{4}$, we would consider it equal to a sample mean of _____.

6

55. To find the mean of a group of values, you first find their total, and then divide this total by the number of values. If you wanted to find the mean (average) squared deviation, you could first add all the squared deviations and then divide by the number of squared deviations. In our previous example (see the table in Frame 53), the total of the number of squared deviations equals _____ plus _____ plus _____, which equals _____. 9, 9, 0, 18

Since there were 3 observations in the collection of data, you should divide the total of the squared deviations by _____ to find the mean of these squared deviations. 3

Thus, the mean (average) of the squared deviations equals _____ divided by _____, which yields the answer of _____. 18, 3
6

56. Statisticians refer to the average of the squared deviations as the **variance**. Therefore, the **variance** of the previous collection of data is _____. 6

57. The **variance** is a statistic representing the variability of a collection of data. Values that are widely dispersed (spread out from the mean) have large deviations from the mean. Whether these deviations are positive or negative, they will result in large squared deviations. Therefore, saying that a collection has a large **variance** implies that the values tend to be _____ spread out from/ close to the mean. spread out from

58. A collection of data having observations of all the same value has the _____ possible variability. least
most/least

220. You could obtain random samples from the population of weekly mileages in the same manner in which you obtained random samples from the population of student opinions. Let's suppose you obtained 100 samples of size 2 in this manner. You could calculate the mean of each of these samples so that you would obtain _____ sample means.
221. We have already stated that we would represent the population mean by the symbol μ . We shall represent the sample mean by \bar{x} . Thus, if one of the samples of size 2 had a mean of 5, μ would equal _____ and \bar{x} would equal _____, since we just said that the population mean was 10.
222. Having collected 100 samples, we would have 100 _____ 's but only one μ / \bar{x} _____.
223. In a similar manner, we could obtain 100 samples of size 5 from the same population, calculate the mean of each sample, and the frequency distribution of these _____ sample means.
224. We could repeat this process to obtain 100 samples of size 25, and we could repeat it again to obtain 100 samples of size 100. The following table contains experimental sampling distributions similar to those you could expect to obtain for samples of size 2, size 5, size _____, and size _____.
- 25, 100

59. Suppose all the observations in a collection of data had the value 10. The mean of that collection of data would equal _____ and each observation would have a deviation from the mean equal to _____. 10
0

If each deviation equaled zero, each deviation squared would also equal zero. Since the **variance** of a collection of data is the typical size of the squared deviations, the variance of this collection of data would equal _____. 0

60. A collection of data having the least possible variability would have, therefore, a **variance** equal to _____. (It would also have a range equal to _____.) 0
0

61. Consider the collection of data shown below:

Observation	Value	Deviation From 6
1	8	
2	5	
3	5	
4	6	

The mean of this collection of data is 6. We have left room in the third column of the table to insert the deviation of each observation from the mean. The deviation of observation 1 from the mean equals _____ minus _____, which equals _____. 8, 6
2

62. Squaring the deviation of observation 1 from the mean equals _____ times _____, which equals _____. 2, 2, 4

In an earlier illustration, we considered 4 experimental sampling distributions, each of which was based on 100 samples obtained from a population of 10,000 student opinions. The major difference in these 4 experimental sampling distributions was the size of the samples (the number of opinions in each sample). Let's consider another illustration of how the sampling distribution is altered by increasing the size of the sample.

Let's suppose you obtained 100 samples by a random sampling procedure from a large population. Let's suppose this population has an approximately normal distribution (the special type of bell-shaped distribution we discussed earlier), with a population mean equal to 10 and a population variance of 8. The symbol μ is often used to represent the **parametric mean**. Thus, in this illustration μ would equal _____.

10

 σ^2

Since we indicated earlier that we would use the symbol σ^2 to represent the population variance, σ^2 would equal _____ in this illustration.

8

219.

Imagine that this population consists of the number of miles each of the 10,000 students estimated he traveled to and from school each week. Lets suppose the students were asked to report this mileage **rounded off to the closest mile** on their registration forms. In other words you would have a weekly mileage from each of the 10,000 students at the university. These 10,000 mileages would be the _____ from which we drew the samples.

63. We could find the deviation and the square of that deviation in a similar manner for each of the other observed values and summarize our work in the following table:

Observation	Value	Deviation From 6	Sq. of the Deviation
1	8	2	4
2	5	-1	1
3	5	-1	1
4	6	0	0

To find the average of the squared deviations (the variance), we would _____ all of the squared deviations and divide this total by _____.
add, (sum)
4

Therefore, the variance of the previous collection of data equals _____ divided by _____. Thus, 1.5 is the _____ of the previous collection of data.
6, 4
variance

64. The variance of the data in the following table equals _____ divided by _____. The variance, therefore, is _____.
100, 4
25

Observation	Value	Deviation from 10	Sq. of Deviation
1	15	5	25
2	5	-5	25
3	5	-5	25
4	15	5	25

65. Since the variance is the _____ of the squared deviations from the mean, the **formula** for the variance will be similar in some ways to the formula for the mean we considered earlier.
mean

66. To find the mean of a group of values, we first sum all the values and then divide this sum by the _____ of values.
number

heights as a sample of size _____ from the population of 10,000 student _____. The procedure used to obtain the sample would be a _____ sampling procedure.

10
heights
random

214. You could calculate any one of a number of statistics that you would describe this sample of 10 heights. For example, you could find the _____ of the sample by calculating the difference between the greatest height and the least height, in the sample, or you could add all the heights together and divide by 10. This would give you the _____ of the sample, since there were 10 heights in the sample.

range
mean

215. Or you could use the following formula

$$\frac{\sum (X - \bar{x})^2}{N}$$

to calculate the _____ of the sample.

variance

216. Suppose you obtained 100 samples of size 10 by a random sampling procedure from the previously described population. If you calculated the variance of each sample, you would have $\frac{100}{10}$ sample variances.

100

217. You could make a frequency distribution of these 100 sample variances. This distribution would be an experimental _____ of sample variances based on 100 samples of size 10 obtained by a random procedure from this particular population.

sampling
distribution

In a similar manner, to find the mean of a group of squared deviations, we first **sum** all the squared deviations and then **divide** this sum by the number of squared deviations. In other words, the total of all the squared deviations divided by the number of deviations equals the **mean** of the squared deviations, which particular mean we call the _____.

variance

67. Representing the mean of a group of values by \bar{x} and the value of a particular observation by X , we would represent the **deviation** of that observation from the mean as $(X - \underline{\hspace{1cm}})$. The expression $(X - \bar{x})^2$ would represent the square of the previous deviation.

 \bar{x}

68. If your collection of data consisted of three observations, you could represent the _____ of these three observations as:

sum (total)

$$X_1 + X_2 + X_3$$

69. Similarly, you could represent the sum of the _____ of these three observations from their mean as:

deviations

$$(X_1 - \bar{x}) + (X_2 - \bar{x}) + (X_3 - \bar{x})$$

Finally, you could represent the sum of the _____ deviations as:

squared

$$(X_1 - \bar{x})^2 + (X_2 - \bar{x})^2 + (X_3 - \bar{x})^2.$$

70. Just as we represented the **sum** of all the raw scores by $\sum X$, we could represent the _____ of the deviations from the mean by $\sum (X - \bar{x})$.

sum

211. Assuming that you felt the distributions of samples you obtained were representative of the distribution of

samples you would obtain in the future, you might state your **confidence** more explicitly as follows:

"I would expect to make an absolute

error of estimation greater than .1

about 48 times out of 100 with a sample

size of $\frac{2}{10}$ drawn randomly from the

same population, whereas I would only

expect to make an absolute error that

large about 32 times out of 100 if my

sample size were $\frac{2}{10}$."

10

212. A **confidence statement** of this sort is a very important

type of statistical statement. While you cannot say for

certain that increasing the sample size will increase

the accuracy of a single sample, you know that, in general,

your **confidence** in an estimate will be greater, the

is the size of the sample.

larger

larger/smaller

213. It would be possible to obtain an **experimental** sampling

distribution for any sample statistic. You would simply

obtain as many samples as you wished by some

particular sampling procedure, compute the value of a

particular sample statistic for each sample, and then

make a frequency distribution of these sample statistics.

For example, suppose you had recorded the **height** of

each of the 10,000 students at the university. You could

write each student's height on a slip of paper, mix all

of these slips together in a basket, and draw out 10

heights (10 slips of paper). You could view these 10

71. Regardless of what collection of data we are describing, however, we know that $\sum (X - \bar{x})$ will always equal _____, according to our definition of the mean. 0
72. Using the symbol \sum (the capital Greek letter _____) in the same way as we did when we wrote $\sum X$ and $\sum (X - \bar{x})$, we could represent the **total** of all the squared deviations by _____ $(X - \bar{x})^2$. \sum sigma
73. Since the variance is the **mean** of the squared deviations, we could represent the _____ of a collection of data as: variance

$$\frac{\sum (X - \bar{x})^2}{N}$$

74. Statisticians use the symbol σ^2 to represent the variance. Using this symbol, a formula for the variance could be written as follows:

$$\text{_____} = \frac{\sum (X - \bar{x})^2}{N} \quad \sigma^2$$

The symbol σ is the uncapitalized form of the Greek letter sigma. The summation symbol _____ was the capital Greek letter sigma. For this reason, the variance, represented by σ^2 , is often referred to as s _____ squared. \sum sigma

On all four graphs we have shaded columns corresponding to frequencies of samples which would have led to an absolute error of estimation greater than _____ in all four graphs.

.1

207. We have not shaded columns associated with sample proportions of .4, .5 and .6, all of which would lead to an error of estimation that was either 0, -.1, or +.1. Thus, the **unshaded** columns represent the frequency of samples leading to an **absolute error of estimation** of _____ or less.

.1

208. A comparison of these four distributions shows clearly that the **risk** of an absolute error of estimation greater than .1 was smallest for samples of size _____ and largest for samples of size _____.

100
2

209. In the light of this illustration, it appears that the larger the size of the sample, the more/less confident you can be that the sample proportion will be close to the parametric proportion—assuming, of course, that you accept the distribution of these 100 sample statistics as representative of what you would obtain if you continued sampling.

more

210. On the basis of these four experimental sampling distributions, you would probably expect to make an error of estimation greater than .1 **more** often if the sample size were large/small than if it were large/small.

small

large

75. We now have defined **two formulas**. The first formula is:

$$\bar{x} = \frac{\sum X}{N},$$

where \bar{x} (called the _____) is a statistic representing the c_____t _____ of a distribution.

mean
central tendency

The second formula is:

$$\sigma^2 = \frac{\sum (X - \bar{x})^2}{N},$$

where σ^2 (called the v _____) is a statistic representing the _____ of a distribution.

variance
variability

76. Let's try using the formula for both the mean and the variance on the collection of data shown in the following table.

Observation	Value	$(X - \bar{x})$	$(X - \bar{x})^2$
1	4		
2	6		
3	5		

Notice that we have added two extra columns to a table of raw data. In the **third** column of the table, we could list the _____ of each observation from the mean. In the **fourth** column we could list the _____ of each deviation from the mean.

deviation
square

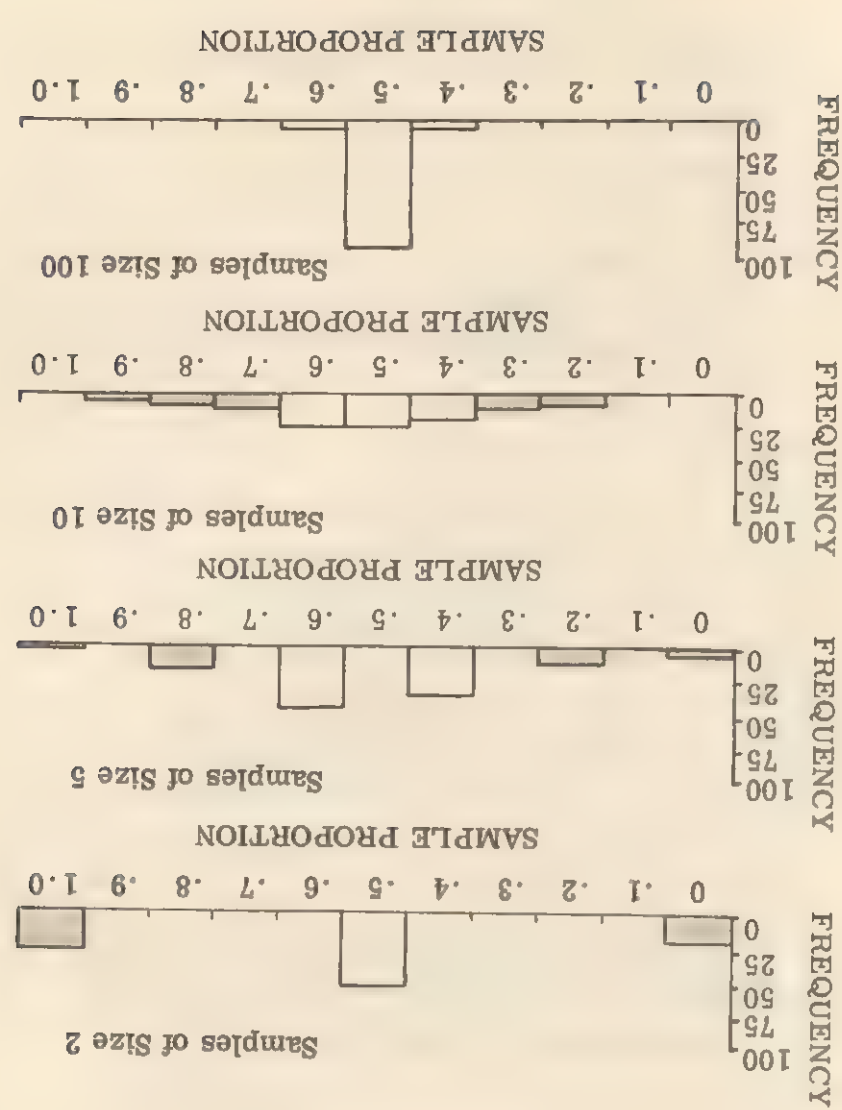
77. Before you can calculate the deviation of each value from the mean, you must first calculate the mean itself. Since the formula for the mean is _____, you must first find the _____ of all three values and then divide this result by _____ (since $N = 3$).

$$\frac{\sum X}{N} \quad \text{sum (total)}$$

3

The remaining two graphs indicate the experimental sampling distribution for samples of size 10 and samples of size 100. It is obvious that the **variance of sample proportions** for samples of size 10 is larger/smaller than the variance for samples of size 100, even though the **central tendency** of both distributions appears to be very close to the parametric proportion of ____.

A clear indication of how the **risk** of obtaining a highly unrepresentative sample diminished as the size of the sample was increased is provided by the following figure.



larger
smaller
5.

78. The sum of the observed values is _____. Dividing this sum by 3, you find the mean to be _____.

15
5

79. Accordingly, you could replace the headings on columns 3 and 4 in the previous table as follows:

Observation	Value	$(X - 5)$	$(X - 5)^2$
1	4		
2	6		
3	5		

Note that here we have replaced _____ found in the earlier table with its actual value, _____.

\bar{x}
5

Notice that since the mean is 5, the deviation of observation 1 from the mean equals _____, and the square of this deviation equals _____.

-1
1

The deviation of observation 2 equals _____, and the square of this deviation equals _____.

1
1

Finally, the deviation of observation 3 equals _____, and the square of this deviation equals _____.

0
0

80. We have summarized these answers in the following table:

Observation	Value	$(X - \bar{x})$	$(X - \bar{x})^2$
1	4	-1	1
2	6	+1	1
3	5	0	0

Graph A shows the distribution of sample proportions for samples of size 2. The bottom of the graph indicates the 11 sample proportions indicated in the first column of the previous table. Thus, the first column of the Graph A indicates the frequency with which samples having a sample proportion of 0 occurred in the 100 samples of size 2. The height of the column above the sample proportion of .5 in Graph A indicates the _____ of samples with sample statistics of _____ in the 100 samples of size 2.

frequency
.5

203.

Whereas only _____ of the 11 proportions listed along the bottom of Graph A are possible values of the sample proportion for samples of size 2, we have listed all 11 proportions in order to make it easy to compare the 4 experimental sampling distributions.

3

204.

The distribution in Graph B is that of samples of size _____. Once again, we listed all 11 values of the sample proportion, even though there are only _____ possible values for samples of this size.

5

6

The most frequently occurring sample of size 5 is indicated by the highest column in the graph. In other words, the most frequently occurring sample proportion for the samples of size 5 was a proportion of _____, which occurred _____ times.

.6

40

80. (Continued)

Remember, the formula for the variance,

$$\sigma^2 = \frac{\sum (X - \bar{x})^2}{N},$$

says to sum all of the squared deviations and then to

_____ this sum by the number of observations.

divide

The variance of the data in the previous table, therefore,

equals _____ divided by _____. This means $\sigma^2 =$ _____

2, 3, $\frac{2}{3}$

for this collection of three observed values.

81. Suppose the deviation from the mean of every value in a collection of data equaled 2 or -2. The square of each of the positive deviations would equal 2 times 2, or 4, and the square of every negative deviation would equal _____ times _____, which would also equal 4.

-2, -2

Since the square of every deviation from the mean would be 4, the typical or mean deviation would equal 4. We would, therefore, say that the **variance** of the distribution was _____.

4

82. Find the error in the following table.

Observation	Value	$(X - \bar{x})$	$(X - \bar{x})^2$
1	10	-5	25
2	4	-1	1
3	1	-4	16

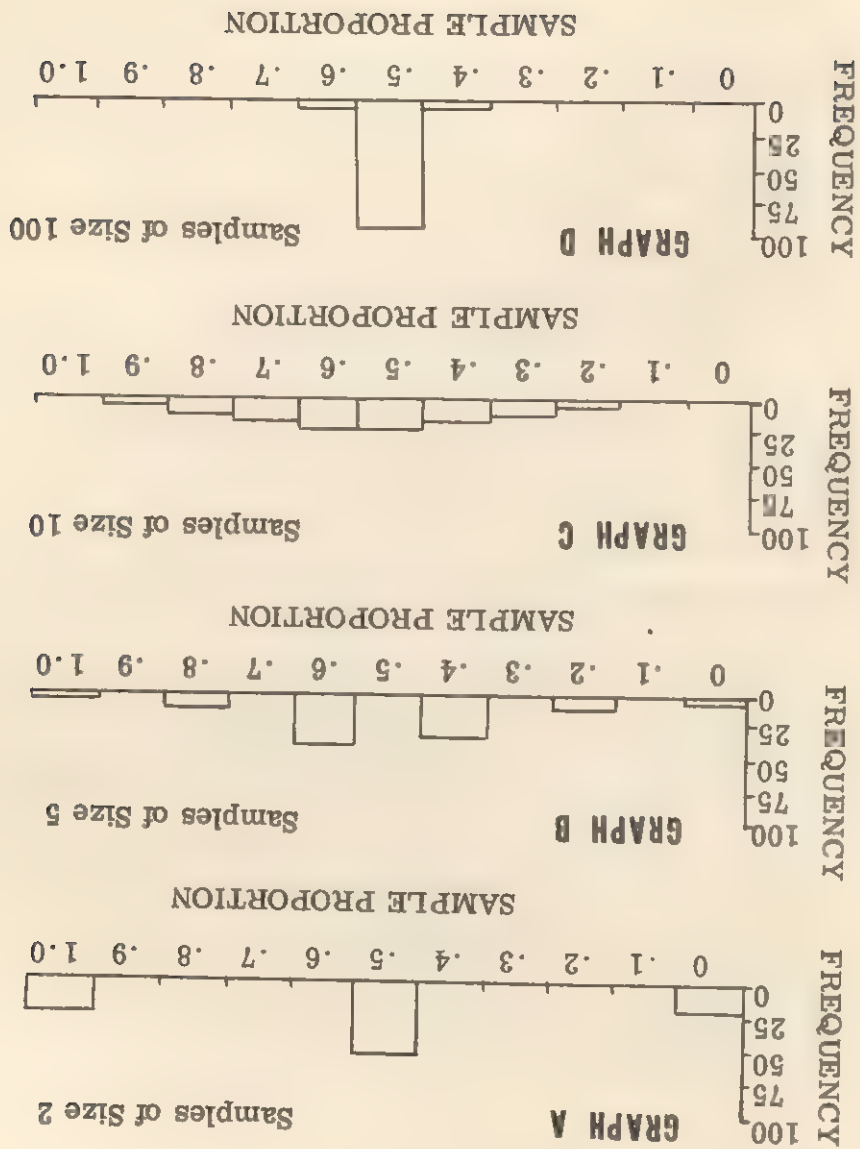
Notice that you would correct this error by changing _____ to _____, since the deviation of 10 from the reference value 5 is $\frac{+5}{-5}$.

-5, +5 (or simply 5)
+5

You could summarize the differences between the 4 experimental sampling distributions as follows:

The larger the size of the sample, the risk there appears to be greater/less of making a large error of estimation.

The differences between these 4 experimental sampling distributions are made even more apparent if we consider the graphs of these distributions.



83. Since the sum of the squared deviations equals _____ and since there are 3 observations, the variance of the previous data would equal _____ divided by _____. Therefore, $\sigma^2 =$ _____. 42
42, 3
14
84. If your data consisted of the values 3 and 7, the mean would be _____. The variance, therefore, would equal _____ divided by _____. In other words, $\sigma^2 =$ _____. 5
8, 2
4
85. Since $\bar{x} = \frac{\sum X}{N}$, the mean of 3 and 7 would equal $\frac{3+7}{2}$ or 5.

Similarly, if the mean equals 5, $\frac{\sum (X - \bar{x})^2}{N}$ would equal $\frac{(3-5)^2 + (7-5)^2}{2}$, which equals $\frac{(-2)^2 + (2)^2}{2} = \frac{8}{2}$ or _____. 4
86. Since the variance can be thought of as the typical size of a **squared** deviation, statisticians have found it useful to assign a special name to the **square root** of the variance. They call the square root of the variance the **standard deviation**. Therefore, the standard deviation of a distribution is a deviation which, if squared, would equal the _____ (even if none of the observed values actually has this particular deviation). variance
87. If the variance of your data were 9, an observation which had a positive deviation from the mean of _____ would have a squared deviation equal to the variance. 3

197. There were _____ samples of size 5 that would lead to an absolute estimation error greater than .2. (Remember, only values of p equal to **more than** .7 or **less than** .3 differ from the population proportion of .5 by more than .2.)

198. The 24 samples of size 5 that would lead to an absolute error of estimation greater than .2 are made up of the two samples having a sample proportion of 0, the 10 samples having a sample proportion of .2, the 11 samples having a sample proportion of .8, and the _____ sample(s) having had a sample proportion of _____.

199. While there were 24 samples of size 5 that would lead to an absolute error of estimation greater than .2, there were only _____ samples of size 10 that would have led to an error so large.

The only samples of size 10 in which the sample proportion differed from the parametric proportion by more than .2 were the one sample in which p equalled .9, the 4 samples in which p equalled .8, and the _____ samples in which p equalled _____.

200. Even more accurate estimates are apparently obtained from samples of size 100, since there were _____ samples of size 100 in which the sample proportion differed from the population proportion by more than .2.

In fact, there were only _____ samples of size 100 that did not have a sample proportion identical to the population proportion. (Remember, the sample proportions were rounded off to the nearest $\frac{1}{10}$.)

88. In other words, if the variance of your data were 9, the **standard deviation** would be _____, since $(3)^2 = 9$. 3

If the variance of your data were 25, the standard deviation would equal _____, since $(\quad)^2 = 25$. 5, 5

89. Just as a group of values may have a mean that is not equal to any of the values, it is not necessary for any particular value in a collection of data to have a deviation from the mean exactly equal to a standard deviation. Therefore, whereas the standard deviation can be thought of as the typical size of a deviation in same collection of data, it _____ necessary for any value in the data to actually have this deviation. is not
is/is not

90. To summarize, the **variance** represented by the symbol _____ is the typical or average squared deviation. σ^2
Any squared deviation equal to this average squared deviation would be called the _____ standard deviation
Remember, it _____ necessary that any value in the data actually have this particular deviation. is not
is/is not

91. The **variance** is equal to the **square** of the **standard deviation**. Therefore, the standard deviation is equal to the _____ of the variance. square root

Since the variance is represented by σ^2 , the standard deviation could be represented by σ . But σ^2 is simply σ . Thus, the **variance** is represented by $\frac{\sigma}{\sigma^2}$ σ^2
and the **standard deviation** by $\frac{\sigma}{\sigma^2}$. σ

perfect estimate of the population parameter, most estimates would be pretty close to the population value. While you would have made 48 absolute estimation errors greater than .4 with the samples of size 2, there were only _____ samples of size 5 which would have led to an estimation error that large.

195.

Among the 100 samples of size 5, there are only two samples with sample proportions of 0 and one sample with sample proportions of 1. Increasing the sample size to 5, therefore, appeared to reduce/increase the risk of making an absolute error of estimate greater than .4.

196.

(Once again, we repeat the table shown earlier for your convenience.)

TABLE A

Distribution of 100 samples drawn at random from a large population where p equals .5.

Sample		Sample Size			
Proportion		2	5	10	*100
0	23	2	0	0	0
.1	-	-	0	0	0
.2	-	-	10	5	0
.3	-	-	-	12	0
.4	-	-	36	21	2
.5	52	-	-	25	95
.6	-	-	40	24	3
.7	-	-	-	10	0
.8	-	-	11	4	0
.9	-	-	-	1	0
1.0	25	1	-	0	0

* Sample proportions rounded off to the nearest one-tenth.

92. By using σ^2 to represent the _____ and σ to represent the _____, we emphasize the fact that the variance is simply the _____ of the standard deviation. In other words, we emphasize that the _____ of the variance equals the standard deviation by letting _____ represent the standard deviation and _____ represent the variance.
93. Suppose all the values in a collection of data deviated from the mean by either +3 or -3. The **variance** of that collection of data would equal _____ and the standard deviation would equal _____.

In other words, σ^2 equals $\frac{\quad}{9/3}$ and σ equals $\frac{\quad}{9/3}$.
94. We have considered three statistics used to represent the central tendency of a collection of data. These three statistics are the _____, the _____, and the _____.
95. The _____ is the most frequently occurring value in a collection of data. The _____ is a value which would divide a list of the ranked data into two equal parts. The _____ is that particular reference value from which the sum of the deviations equals 0.
96. We have also considered three statistics used to represent the variability or dispersion of a collection of data. They are the _____, the _____ and its square root, called the _____.
- variance
standard deviation

square
square root

 σ, σ^2

9
3

9, 3

mean, median
mode

mode
median

mean

range, variance
standard deviation

191. Considering only the distribution based on samples of size 2, notice that _____ of the 100 samples had a sample proportion identical with the parametric proportion of .5.

52

192. If you felt the 100 samples of size 2 obtained were representative of the type of samples you would obtain through repeated sampling, you might make the following statement about samples of size 2 drawn from a population having a value of \tilde{p} equal to .5: If you were estimating \tilde{p} on the basis of a sample of size 2 from a population where \tilde{p} was equal to .5, you would expect to make a perfect estimate about _____ times out of every 100 samples.

52

193. On the other hand, you could also make this following statement: You could expect to make an absolute estimation error greater than .4 about _____ times out of 100.

48

You would make an absolute estimation error greater than .4 if you obtained a sample proportion of either 0 or 1. Since there were 23 samples with sample proportions of 0 and _____ samples with sample proportions of 1 in the 100 samples of size 2 obtained, you would expect to make an estimation error that large about 48 times out of 100 (assuming, of course, you felt the previous 100 samples were representative of those you would expect to obtain in the future).

25

194. When you increased the sample size to 5, it was impossible to obtain a sample proportion of exactly .5. Thus, the frequency of sample proportions of .5 among samples of size 5 was necessarily _____. On the other hand, while samples of size 5 would never yield a

0

97. The _____ is the difference between the largest and smallest observed value. The _____ is the typical or average of the squared deviations from the mean of that collection of data. The standard deviation is a deviation which when squared will equal the _____.

range
variance

variance

98. We have also found a way to write rules for calculating the mean and the variance. To find the typical value or mean, we _____ all the values and _____ by the number of values. The formula for the mean is written, therefore, as:

add (sum), divide

$$\bar{x} = \underline{\hspace{2cm}}$$

$$\frac{\sum x}{N}$$

99. To find the typical squared deviation (the variance), we sum all of the _____ and divide by the number of values. The formula for the variance, therefore, is written as:

squared deviations

$$\sigma^2 = \underline{\hspace{2cm}}$$

$$\frac{\sum (x - \bar{x})^2}{N}$$

The **standard deviation** is simply the _____ of this variance.

square root

100. The statistics which describe the _____ of the distribution represent in one way or another the typical value to be found in that distribution. A statistic representing the _____ of the distribution describes the degree to which the observed values are spread out or dispersed.

central
tendency

variability

188.

The most frequently occurring sample proportion for samples of size 2 was _____, and the most frequently occurring sample proportion for samples of size 10

.5

189.

For your convenience, the following table is a duplicate of the one shown earlier.

TABLE A

Distribution of 100 samples drawn at random from a large population where p equals .5.

Sample Proportion	Sample Size			Frequency
	2	5	10	
0	23	2	0	0
.1	-	-	0	0
.2	-	10	5	0
.3	-	-	12	0
.4	-	36	21	2
.5	52	-	25	95
.6	-	40	24	3
.7	-	-	10	0
.8	-	11	4	0
.9	-	-	1	0
1.0	25	1	0	0

* Sample proportions rounded off to the nearest one-tenth.

In terms of central tendency, in fact, all four sampling distributions seem to be clustered around the parametric proportion of _____.

.5

190.

An important feature of these four distributions is the fact that the sample proportions were *less* variable in samples of size _____ than they were in the samples of size _____.

100

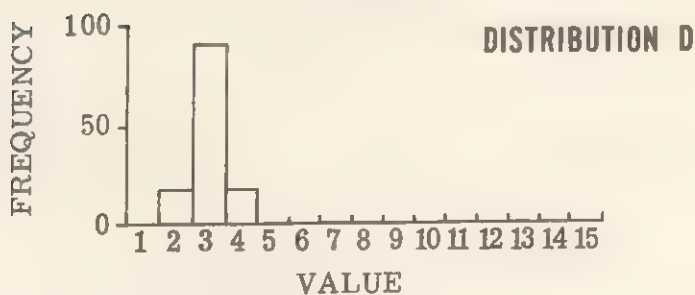
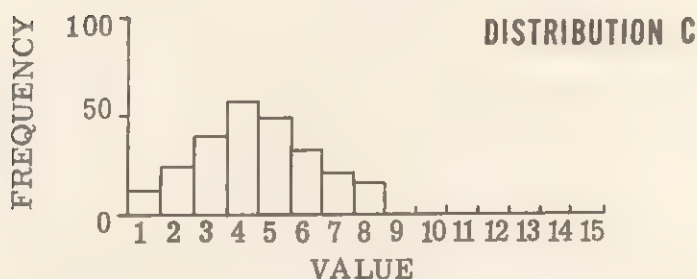
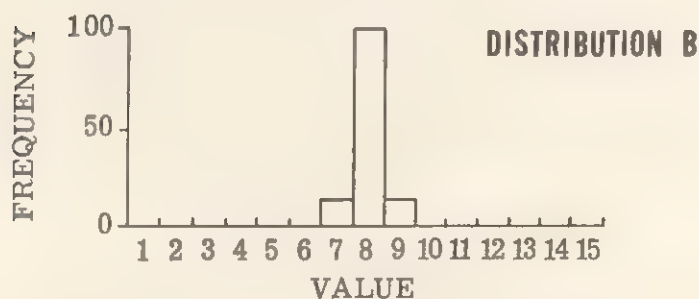
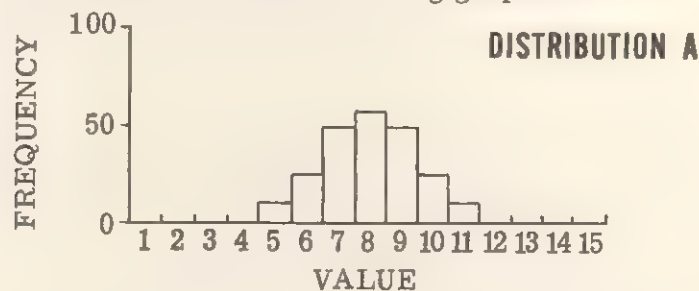
of size $\frac{2}{100}$.

2

101.

Statistics describing central tendency do **not** tell you anything about the variability of a distribution.

Statistics describing variability do **not** tell you anything about central tendency. For example, consider the four distributions shown in the following graphs.



Notice that distributions A and B are similar in terms of their central tendency/variability, yet quite different in terms of their central tendency/variability.

central tendency

variability

184. If 34 of the 100 people had a favorable opinion, the actual sample proportion would be .34. This sample proportion would be between (less than one and greater than the other) the proportion .3 and the proportion $\frac{.2}{.4}$ that are shown in the first column of the table.

Since we rounded off the true value of p to the nearest $\frac{1}{10}$ when we calculated the frequency shown in the table, we considered a sample proportion of $34/100$ to be a proportion of .3 (because $34/100$ is closer to $\frac{.3}{.4}$ than it is to $\frac{.3}{.4}$).

185. A problem arises, however, when a sample proportion of $35/100$ is obtained, since $35/100$ is just as close to .3 as it is to .4. We simply made an arbitrary rule that if the sample proportion were exactly halfway between two of the proportions shown in the first column of the table, we would group it with the **smaller** proportion. In other words, we would consider a sample proportion of $35/100$ to be equal to a proportion of .3 and a sample of $45/100$ to be equal to $\frac{.4}{.5}$.

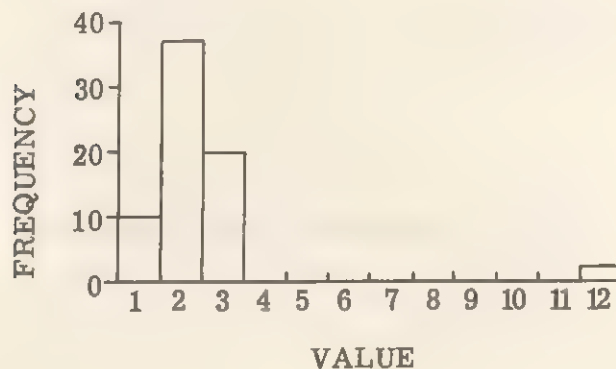
186. All eleven proportions shown in the first column were **possible** values of the sample proportion for samples of 100. According to the previous table, however, only three **observed** values of the sample proportion (rounded off to the nearest one-tenth) were .4, .5, and .6.

187. There were two samples of size 100 that had a sample proportion of .4 — 95 samples of this size having a sample proportion of _____ and _____ samples having a sample proportion of .6.

102. On the other hand, Distribution A is similar to
 Distribution C in terms of central tendency/variability, variability
 whereas the mean of Distribution A appears to be
larger than/equal to the mean of Distribution C. larger than

103. Distribution B and D are similar in terms of
central tendency/variability. variability

104. The choice of one statistic over another to represent
 same feature of a distribution depends upon the particular
 feature you wish to represent. For example, consider
 the distribution shown below. Except for a few extreme
 (unusually large) values, most of the values are grouped
 around the value 2/5. 2



If you represented the central tendency of the preceding
 data by the mean, the extreme values would tend to pull
 the mean away from the value 2. On the other hand, if
 you used the **mode** to represent the central tendency of
 the distribution, its value would be 2. In other 2
 words, the mode **might** be a better way of representing
 the central tendency in terms of this unusual distribution
 since the mode is / is not influenced by the few is not
 extreme values found in the distribution.

180.

There are no dashes in the column containing the

distribution based on samples of size 10. This is

because every sample proportion listed in the first

column of the table is a possible value of the

sample proportion for samples of size 10.

181.

The number in the same row as the proportion .2 and in

the column headed by sample size 10 is _____. This

indicates that exactly _____ samples of size 10 had a

sample proportion of .2.

182.

The column on the far right of the table is headed by a

sample size of _____.

100

In a sample of 100, the smallest number of favorable

opinions would be 0 and the largest number would be 100.

Since there is a particular sample proportion

corresponding to each possible number of favorable

opinions, there are exactly _____ possible values of

101

the sample proportion with a sample of 100 opinions.

183.

In order to compare the sample distribution for size 100

with the sample distributions based on smaller samples,

we have **grouped** the sample proportions in the samples

of size 100 by **rounding off** each of sample proportions

to the nearest $\frac{1}{10}$.

In other words, while a sample proportion of .02 would

occur if two of the 100 opinions were favorable, we

would consider that sample proportion to be 0, since

.02 is closer to $\frac{0}{10}$ than it is to $\frac{0.1}{10}$.

0, .1

REVIEW II

FILL IN THE BLANKS:

1. The most frequently occurring value in the distribution is called the _____.

mode

2.

STUDENT	NUMBER OF ERRORS
A	3
B	6
C	2
D	3
E	11
F	8
G	3

In the table above, pick out the modal _____ number of errors.

3

3. A value in a collection of data which is smaller than half of the other observed values and larger than the remaining values is called the _____.

median

4. Find the median in the following collection of data:

14, 11, 8, 4, 2 _____

8

5. Find the median in the following set of numbers:

9, 8, 6, 4 _____

7

174. Therefore, you could say that of the _____ samples of size 2, there were _____ which had a sample proportion of 0, _____ which had a sample proportion of .5, and _____ which had a sample proportion of 1. These frequencies define an experimental sampling distribution of p for 100 samples of size 2.
175. In the next column (headed by a sample size of 5), there are only _____ dashes. This indicates that of the eleven proportions listed in the first column only _____ could occur in samples of size 5.
176. The frequencies 2, 10, 36, _____, _____, and _____ listed in the column for sample size 5 indicate the distribution of sample proportions for the 100 samples of size _____. 5
177. By using the row headings to identify each value of the sample proportion, therefore, you could determine that exactly 2 of the 100 samples of size 5 had a sample proportion of 0 (no favorable opinions). Similarly, there were exactly 10 samples of size 5 that had a sample proportion of _____. 2
178. There were 36 samples of size 5 that had a sample proportion of _____, and _____ samples of size 5 that had a sample proportion of .6. .4, 40
179. The sum of the frequencies in the second column equals _____ because there were exactly 100 sample statistics in the distribution. 100

MULTIPLE CHOICE:

6. It is possible for two collections of data to have different means and:
- a. radial variability.
 - b. diametric variability.
 - c. the same variability.
 - d. none of the above
- _____ c
7. The degree to which the observed values are spread out or are dispersed from the mean of a collection of data can be referred to as the:
- a. mean.
 - b. variability of that collection of data.
 - c. median.
 - d. none of the above
- _____ b
8. The average of the squared deviations from the mean is called the:
- a. standard deviation.
 - b. variance.
 - c. mean.
 - d. none of the above
- _____ b
9. The square root of the variance is called the:
- a. mean.
 - b. median.
 - c. standard deviation.
 - d. none of the above
- _____ c

170.	At the top of each of the 4 columns is a number indicating the size of the sample on which that particular distribution was based. The first of these headings is 2, indicating that the distribution shown in that column is based on samples of size 2. The second heading is _____, indicating that the distribution in that column is based on samples of size _____. 5	5
171.	The last two columns indicate distributions of sample statistics based on samples of size _____ and samples of size _____, respectively. 10 100	
172.	The proportions listed in the column headed "Sample Proportion" represent different values of the sample statistic. For example, the second column contains the distribution based on samples of size 2. If you recall, there are only _____ possible values of the sample proportion if the sample size is 2. We have indicated this by placing a dash in each row of the second column corresponding to a sample proportion that _____ could not occur with a sample of size 2. 3	
173.	The frequencies 23, 52, and 25 appearing in the second column identify the distribution of sample statistics based on samples of size 2. There were 23 samples of size 2 in which there were no favorable opinions. This is why 23 appears in the same row as the sample proportion _____ (shown in the first column). 0 There were 52 samples of size 2 in which the sample proportion was _____, since 52 occurs in the same row as a sample proportion of .5. .5 Finally, there were _____ samples of size 2 in which all of the opinions were favorable (p equal to 1). 25	25

10. If the variance of your data were 36, the standard deviation would be:

- a. 9.
- b. 72.
- c. 4.
- d. none of the above

d

11. The symbol representing the standard deviation is:

- a. \sum .
- b. σ .
- c. σ^2 .
- d. none of the above

b

TRUE OR FALSE:

12.

a negative deviation from the reference value.

false

13. Let us assume that the reference value is 8 and the observation is 6. In this case, the deviation is 2.

false

14. If we add all the deviations of a group of observed values from a particular reference value, and the answer is zero, that reference value is the mean of those values.

true

15. The difference between the largest and smallest value in a collection of data is called the median.

false

The following table presents four experimental sampling distributions similar to those that would have been obtained using the procedure just described.

TABLE A

Distribution of 100 samples drawn at random from a large population where \tilde{p} equals .5

Sample Proportion	Sample Size			
	2	5	10	*100
0	23	2	0	0
.1	-	-	0	0
.2	-	10	5	0
.3	-	-	12	0
.4	-	36	21	2
.5	52	-	25	95
.6	-	40	24	3
.7	-	-	10	0
.8	-	11	4	0
.9	-	-	1	0
1.0	25	1	0	0

* Sample proportions rounded off to the nearest one-tenth.

In the first column of the table, we have listed 11 values of the sample proportion, starting with a sample proportion of _____ (which indicates that none of the opinions were favorable) and ending with a sample proportion of _____ (which indicates that all of the opinions were favorable).

The 4 remaining columns of the table contain the distributions (frequencies) based on each of the 4 different sample sizes. The words _____ printed at the top of the table refer to these four possible sample sizes.

16. If your data consisted of the values 73, 22, 14, 91,
and 11, the range would be 47. _____ false
17. It is possible for two collections of data to have the
same mean, but different variability. _____ true

163.

It would also be possible to obtain 100 samples consisting of 100 opinions each. To do this you would merely draw _____ (slips of paper) from the basket each time you wished to collect a sample, remixing these slips in the basket before you drew the next sample.

164.

If there were exactly 10 favorable opinions in the sample of 100, the value of the sample proportion (p) would be

.1

165.

To summarize, then, we have considered a way of obtaining 4 experimental sampling distributions. Each distribution consists of the _____ for each possible value of the sample proportion that occurred in 100 samples.

166.

The major difference in these 4 experimental sampling distributions is the number of opinions included in each sample. We obtained one distribution for samples of size 2, another for samples of size _____, another for samples of size _____, and a fourth distribution for samples of size _____.

167.

While the sample size on which each of these four distributions is based is different, the total of the frequencies in each distribution equals _____, since each distribution is based on _____ samples.

100

100

100

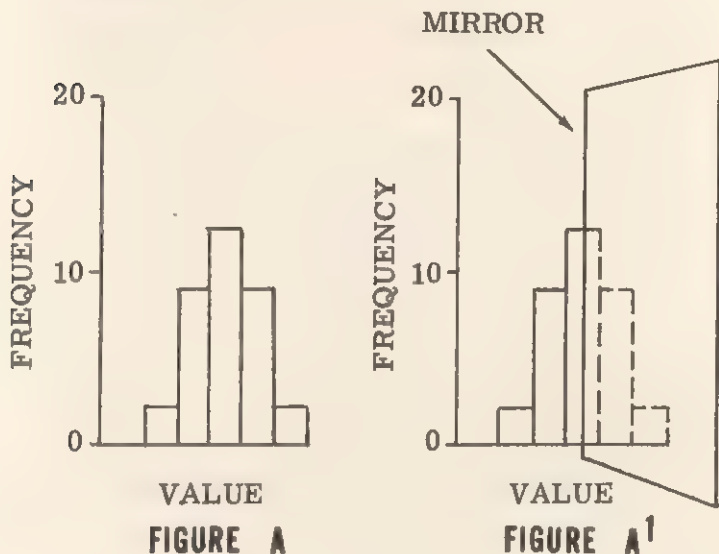
10

5

frequencies

Section V: Types of Distributions

1. In addition to describing the **central tendency** and **variability** of a distribution, it is often useful to say something else about the general shape of the distribution. For example, consider first the distribution shown in Figure A below. Now look at Figure A¹. In Figure A, we have shown how half of Figure A would look reflected in a mirror.



Notice how the reflection of the **left** half of the distribution has exactly the same shape as the **right** half of the distribution. (The right half is behind the mirror in Figure A¹.) In other words, if we cut the distribution shown in Figure A in half, along the line where we placed the mirror in Figure A¹, we would have divided the distribution into two parts with the same shape but facing in opposite directions.

157. Notice that each sample of 5 could contain _____, _____, _____, _____, or _____ favorable opinions.
0, 1, 2, 3, 4, 5
158. Therefore, there are only _____ possible values of the sample proportion: 0, .2, _____, _____, and _____.
6
.4, .6, .8, 1
159. If you determined the frequencies with which each of the possible values of the sample proportion occurred in the 100 samples, the total of all of the frequencies would equal _____.
100
160. In the same manner as you obtained 100 samples of size 2 and 100 samples of size 5, it would be possible to obtain 100 samples of size 10. Instead of drawing 2 options for each sample or 5 options for each sample, you would draw _____ options for each of the 100 samples.
10
161. The fewest possible favorable opinions in each sample of 10 would be 0. The largest number of favorable opinions would be _____.
10
162. Therefore, there are _____ possible values of the sample proportion: _____, _____, _____, _____, and _____.
0, .1, .2, .3, .4, .5
.6, .7, .8, .9, 1
- Samples of size 5 could not yield a sample proportion of .1, whereas a sample of size $\frac{10}{2}$ can have p equal to .1.
10

1. (Continued)

In comparison, consider the distribution shown in Figure B below. In Figure B¹ the reflection in the mirror does/does not have the same shape as that part of the distribution behind the mirror.

does not



Therefore, of the previous distributions A and B, Distribution A/B can be divided in half so that the

A

right half looks like a "mirror image" of the left half, whereas it would not be possible to divide the other distribution in such a manner.

152. Since the sample proportion (p) is simply the number of favorable opinions divided by the number of opinions in the sample, there are only 3 possible values of the sample proportion: _____, _____, and _____.
0, .5, 1

153. It _____ be possible to obtain a sample _____ would/would not _____ would not

154. If you counted the number of samples (out of the 100 samples) that had a sample proportion of 0, a sample proportion of .5, or a sample proportion of 1, those frequencies would have to total _____.
100

155. The list of 3 frequencies indicating how often each of the 3 possible values of the sample proportion occurred in the 100 samples defines a _____ of _____ sample proportions.

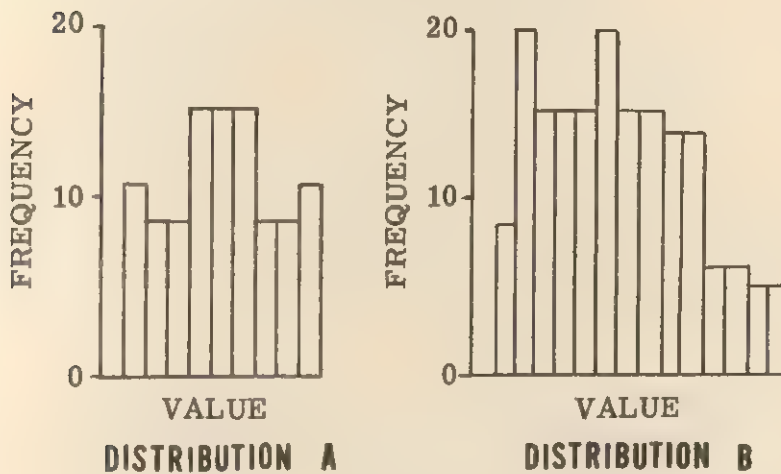
In other words, those 3 frequencies define a(n) _____ s _____ experimental/theoretical _____ of 100 sample proportions. _____ experimental sampling _____ distribution

156. We now have an experimental sampling distribution of 100 sample statistics. Each sample statistic is the proportion of favorable opinions in a sample of size 2. It would be interesting to compare the experimental sampling distribution based on samples of size 2 with a similar sampling distribution based on larger samples. For example, you could obtain 100 additional samples in the same manner, this time including 5 opinions in each sample. These 100 samples of size _____ would provide you with _____ new sample proportions. _____ 5 _____ 100

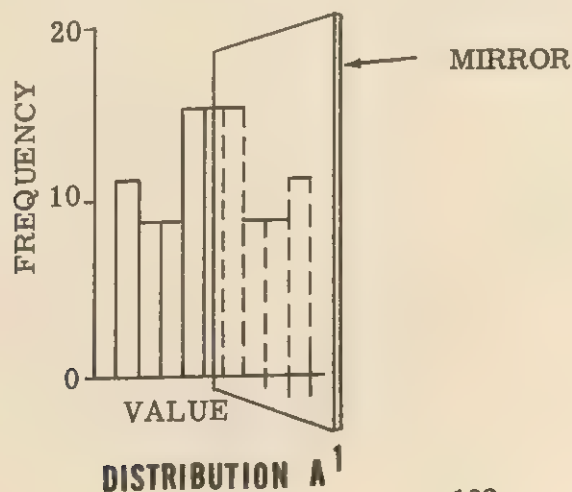
2. A distribution which can be divided so that its left half appears to be a mirror image of its right half is called a **symmetrical** distribution. If a distribution cannot be so divided, it is called an **asymmetrical** distribution. Thus, of the two distributions shown below, Distribution A/B is symmetrical, whereas the other distribution is _____.

A

asymmetrical



Distribution A would be called **symmetrical** since you could place a mirror so that the reflection in the mirror had exactly the same shape as that part of the distribution hidden behind the mirror. (See the illustration below.) In other words, the right half of the distribution is a "mirror image" of the left half of the distribution.



146. Since p equals .5, we know that exactly _____ of

the 10,000 students have a favorable opinion concerning the overseas campus.

147. There must be exactly 5,000 students in favor of the overseas campus since \hat{p} equals .5 and 5,000 divided by _____ equals .5.

10,000

148. To begin with, imagine that 100 samples of 2 opinions each were drawn by the previously described **random** procedure from the population of 10,000 student opinions.

In other words, suppose the slips containing the 10,000 opinions were mixed in the basket and exactly _____

2

slips were drawn out each time a sample was obtained. Further imagine that exactly _____ samples of two

100

opinions each were obtained in this manner. (Assume the slips taken for one sample were remixed in the basket before the next sample was drawn.)

149. If we represented the proportion of favorable opinions in the first sample by p_1 and the proportion of favorable opinions in the second sample by p_2 , the last

sample proportion we obtained would be represented by _____.

p_{100}

150. Each of these 100 sample proportions would have a particular value corresponding to the proportion of

responses in each of the 100 samples of size _____.

favorable, 2

151. Notice that the **number** of favorable opinions in a sample of size 2 could only equal _____, or _____.

0, 1, 2

2. (Continued)

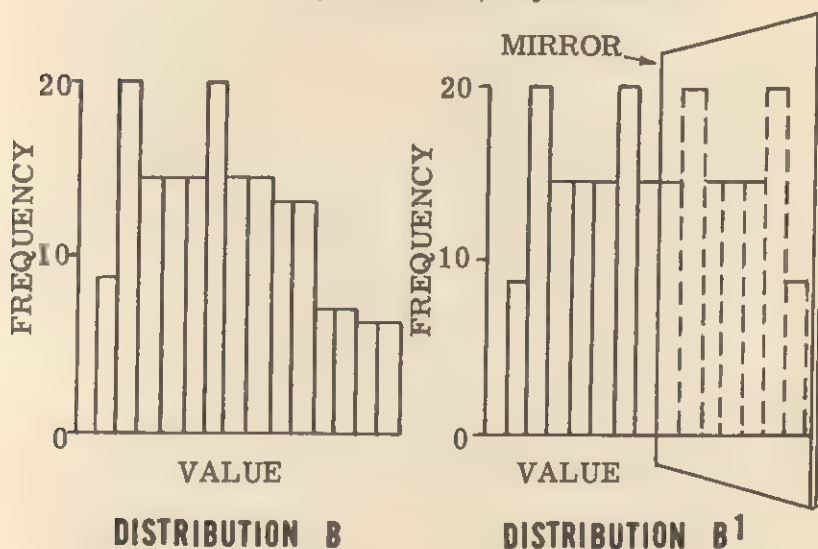
On the other hand, the reflection of the left half of Distribution B (Distribution B¹) does/does not have the

does not

same shape as that part of the distribution hidden behind the mirror. We would say, therefore, that

Distribution B was symmetrical/asymmetrical.

asymmetrical



3. Of the four distributions shown below, Distributions _____ and _____ would be described as **symmetrical**, while the other two distributions would be described as **asymmetrical**.

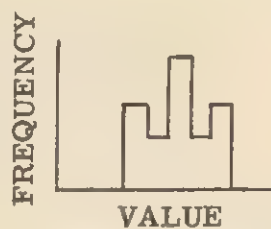
B, D



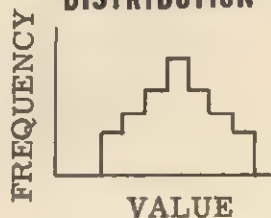
DISTRIBUTION A



DISTRIBUTION C



DISTRIBUTION B



DISTRIBUTION D

Sampling distributions based on an actual collection of samples (such as the ones we considered in the preceding illustrations) are often referred to as **experimental**

sampling distributions. A sampling distribution based solely on logical considerations, therefore, would be called $a(n)$ experimental/theoretical sampling

distribution, whereas a sampling distribution based on a group of samples you had actually collected would be called $a(n)$ experimental/theoretical sampling distribution.

144.

An important feature of sampling distributions is the relationship between the **variability** of the sampling

distribution and the **size** of the samples. In general, the larger the size of the sample, the less the sample

statistic will tend to vary from sample to sample. In other words, as you increase the size of the sample,

you tend to increase/decrease the variance of the sampling distribution.

145.

In order to illustrate the relationship between the size of the sample and the variability of the sampling distribution, let's consider the following example.

In the previous illustration, it was found that the proportion of the 10,000 students who were in favor of the overseas campus was exactly .7. For the present illustration, let's suppose that the true population

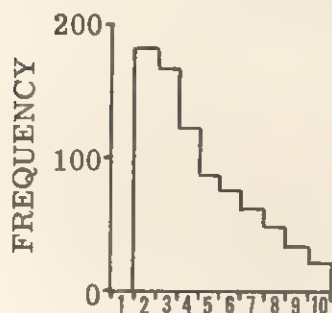
proportion of students in favor of the overseas campus was exactly .5. Thus, \tilde{p} now equals _____ instead of .7, as was the case in the previous example.

.5

4. In some distributions, the scores seem to be piled up towards one end of the distribution. For example, in Distribution A/B (shown below) most of the observed

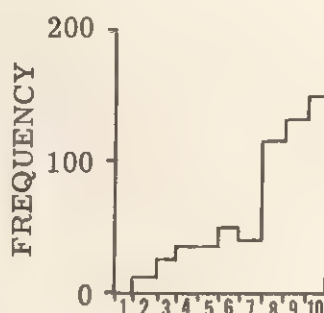
A

values occurred near the **low-valued** end of the distribution, with only a few values occurring near the upper end of the distribution.



VALUE

DISTRIBUTION A



VALUE

DISTRIBUTION B

5. The scores in Distribution B tend to be piled up near the high/low values, with only a few observed values down near the low end of the distribution.

high

6. Both of the distributions shown above would be referred to as asymmetrical/symmetrical distributions.

asymmetrical

7. Asymmetrical distributions in which most of the scores are piled up near one end could be regarded as "lopsided."

139. We already noted that 35 out of the 100 samples of size 5 would have led to an absolute error of estimation greater than .2. In order to make an absolute error of estimation greater than .2, you would have to obtain a sample of 10 that had fewer than favorable opinions or more than favorable opinions. (Remember, \tilde{p} equals .7.)

5
9

140. While there were 35 samples of size 5 that would lead to an error of estimation greater than .2, there were only _____ such samples of size 10 (according to the previous table).

5

141. The only observed samples of size 10 in which the absolute difference between \tilde{p} and p was greater than .2 were the 5 samples in which p equalled _____.

.4

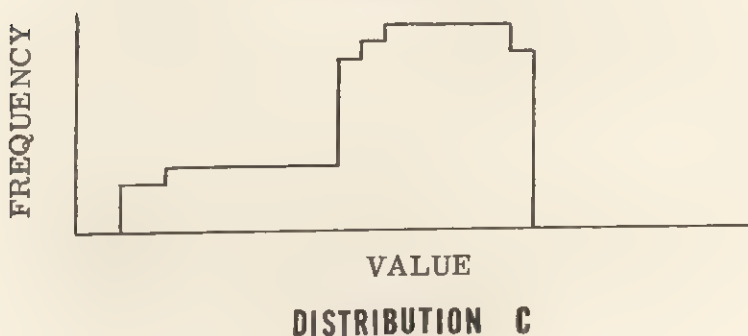
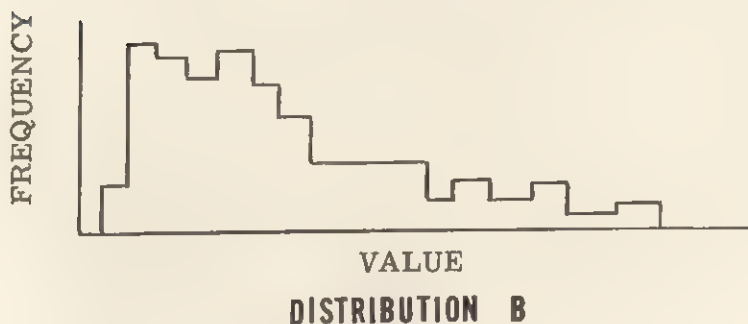
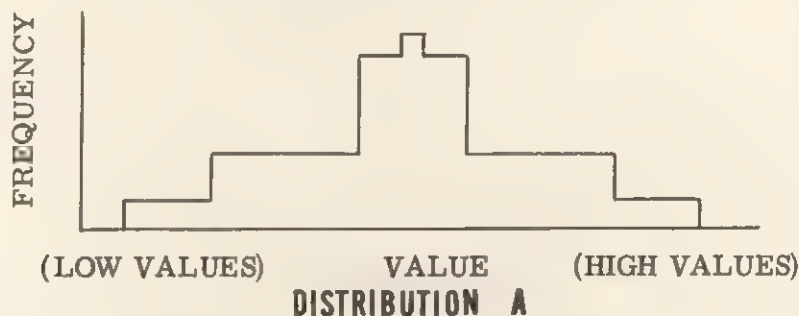
142. The sampling distribution for samples of size 5 and samples of size 10 were obtained in the illustrations by actually collecting these samples from a particular population by a particular procedure. Later, you will see how it is possible to **predict**, through logical considerations, the sampling distribution of a sample statistic obtained by a **random sampling procedure**. These **predicted** sampling distributions are often referred to as **theoretical** sampling distributions because they are based on logical considerations rather than on samples that have actually been collected.

Thus, a sampling distribution based on an actual collection of samples _____ would/would not be an example of a theoretical sampling distribution.

would not

Distributions of this sort are said to be **skewed**. If a skewed distribution has most of the observations piled up near its **low** values, the distribution is said to be positively skewed. If the distribution has most of the observations piled up near the **high** values, the distribution is said to be **negatively** skewed. Thus, of the three distributions shown below, Distribution ____ is **positively skewed**, whereas Distribution ____ is **negatively skewed**. Distribution ____, however, is neither positively nor negatively skewed, since this distribution is symmetrical.

B
C
A



In the illustration we just considered, it was suggested that a larger sample size would lead to fewer unrepresentative samples. The following table summarizes the distribution of sample proportions obtained when 100 samples of size 10 were collected by the **random sampling procedure** described earlier. The only difference in the procedure for collecting samples of size 10 and the procedure for collecting samples of size 5 is that you draw _____, rather than 5, slips of paper from the basket for each sample.

Number of favorable opinions in a sample of 10	Proportion of favorable opinions in the sample	Frequency of such samples
0	0	0
1	.1	0
2	.2	0
3	.3	0
4	.4	5
5	.5	10
6	.6	20
7	.7	40
8	.8	25
9	.9	0
10	1.0	0

137.

Notice that when samples of 10 were collected, there were _____ possible values of the sample proportion.

11

138.

Each of these 11 possible sample proportions corresponds to a particular number of favorable opinions in a sample of 10. (This number is shown in the first column of the preceding table.) Therefore, if there were 6 favorable opinions in the sample of 10, the sample proportion

.6

would be _____.

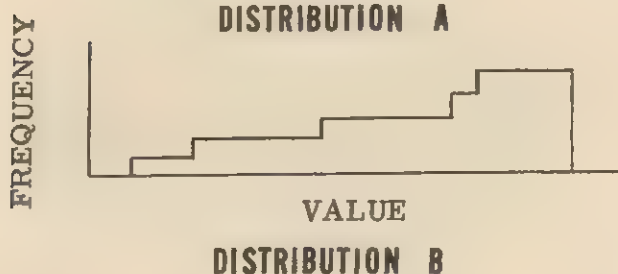
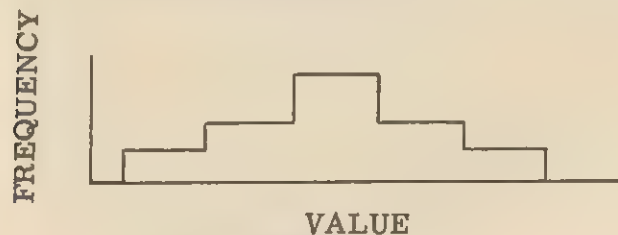
8. Remember that a skewed/symmetrical distribution is skewed regarded as "lopsided." A skewed/symmetrical symmetrical distribution, however, could be regarded as perfectly balanced at its center, so balanced that its left half is a "mirror image" of its right half.

9. We can think of any collection of data as a record of the observed _____ of a variable. values

10. Any collection of data can be described in terms of the various frequencies with which the different values occur in the data. This group of frequencies is referred to as the _____ of the data. distribution

Often the difference between two distributions can be made apparent by drawing a picture of the distribution in the form of a frequency graph. Graphs of this sort make the "shape" of the distribution apparent.

For example, we could describe Distribution A/B (shown A below) as a symmetrical distribution and Distribution A/B as a positively/negatively skewed distribution. B, negatively



Knowing something about the distribution of sample statistics gives you some basis for deciding whether a sample is adequate for your purposes. According to the previous graph, only 35 of the 100 samples you obtained would have led to an absolute estimation error greater than .2. Or, to put it differently, only 35 sample statistics differed from the population statistic by more than _____.

.2

If you believed that the distribution of sample statistics you had obtained was representative of the distribution you could expect in the future, you might make the following sort of statement:

"If I obtained a sample of 5 opinions from this population by this random sampling procedure, I would expect to make an error of estimation greater than .2 only about _____ times out of 100."

35

135.

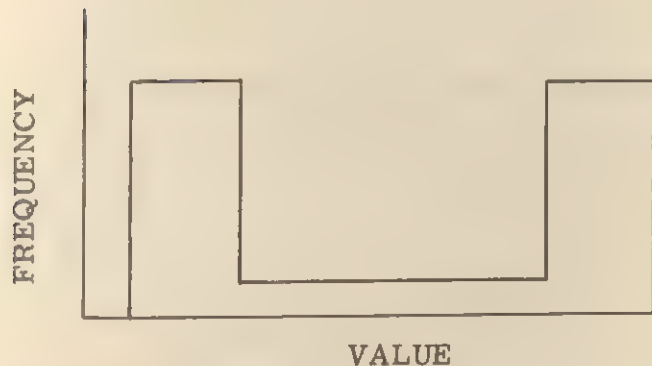
The previous statement is an example of one of the most important types of statistical reasoning. While you are not absolutely certain that your sample will give you a representative picture of the population, you can say something about the likelihood of making errors of estimation of any particular size. In other words, you can say something about your **confidence** in an estimate based on a single sample. A **confidence statement** of this sort is based upon your knowledge of the distribution of sample statistics obtained by a particular sampling procedure. In other words, it is based upon your knowledge of the _____ d _____ of the sample statistics.

11. You would describe Distribution A (above) as **symmetrical** since its left half is simply the reverse of its right half. In other words, the left half of Distribution A mirrors its right half. Distribution B, however, is **not** symmetrical. Distribution B is "lopsided," with most of the observed values piled up near one end. Lopsided or **skewed** distributions are described as positively skewed when the piling up occurs near the _____-valued end of the distribution, and negatively skewed when the piling up occurs near the _____-valued end.

high

low

12.



Most of the observed values in the above graph are "piled up" near the _____ of the distribution, with very few observed values near the _____ of the distribution.

ends

center

13. The previous distribution, however, _____ would be described as symmetrical, since its left side _____ mirror its right side. _____ does/does not

would

does

Since there are only 5 opinions in each sample, there were only _____ possible values of the sample statistic: $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5},$ or $\frac{5}{5}$.

6

132.

We noted previously that most of the samples were reasonably representative of the population proportion p equal to .7. (Samples with p equal to .6 or .8 were quite frequent.) There were, however, an appreciable number of samples in which p equalled .2, .4, or 1. Samples of this sort would give a(n) _____ impression of the representative/unrepresentative population, since p equalled _____. .7

133.

In order to decide whether to use a sample rather than to collect data on the whole population, it would be necessary to contrast the **advantages** of only obtaining a sample rather than a complete collection of data against the **risk** of obtaining an unrepresentative sample, one that would give you a distorted picture of the population. If it were very difficult to obtain a complete collection of data on the whole population and if you only needed a **rough estimate** of the population statistic, it might be adequate to base your estimate on a _____. On the other hand, if it were not too expensive or too difficult to obtain complete information about the population and if you needed very accurate information about the population statistic, you probably would/would not estimate p from p .

sample

would not

14. As a psychologist, you will encounter distributions with many different shapes. You will find, however, that certain types of distributions are encountered more often than others. For example, a very common type of distribution is one in which most of the values are piled up near the mean, with fewer and fewer values occurring farther from the mean.

In other words, values similar to the mean would have the larger/smaller frequencies, whereas values

larger

farther away from the mean would have larger/smaller frequencies.

smaller

15. Distribution A/B (below) would be an example of the common type of distribution we just described.

A

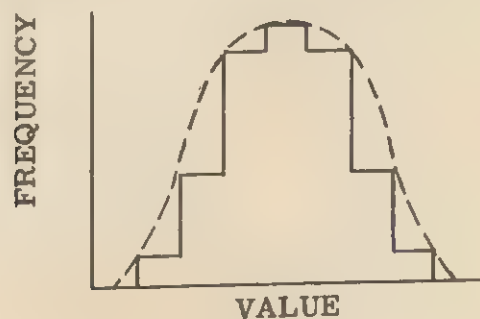


VALUE
DISTRIBUTION A



VALUE
DISTRIBUTION B

16. This commonly encountered type of distribution is often described as "bell-shaped" because its shape is similar to a bell's.



129.

According to Table One, the proportion of favorable

options in the second sample was _____. We could

continue listing the proportion of favorable opinions in

each successive sample (as indicated by the column of

dots in this table) all the way to the 100th sample. The

actual proportion of favorable opinions in the last sample,

p_{100} is equal to _____.

.8

130.

We are interested in a particular sample statistic: the

proportion of **favorable** opinions in each sample.

Sometimes this sample statistic had a large value, such

as .8 or .9, and sometimes it had a low value, such as

.5 or .2. Thus, the value of the sample statistic **varied**

from one sample to the next. You saw how it was

possible to calculate a **frequency distribution** for the

observed values of the sample statistic (just as you would

calculate a frequency distribution for the data in a table

of raw data). The distribution of the sample statistic p

is simply a list of the frequencies of each of the possible

values of this s _____

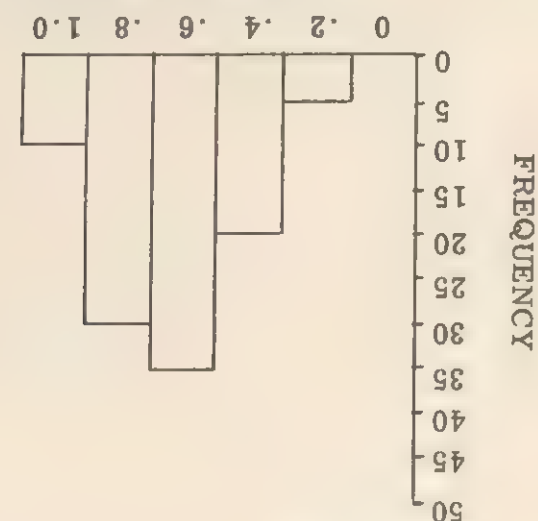
sample statistic

131.

The following graph indicates the frequency distribution

of the sample statistic p obtained from each of the 100

samples of 5 opinions each.



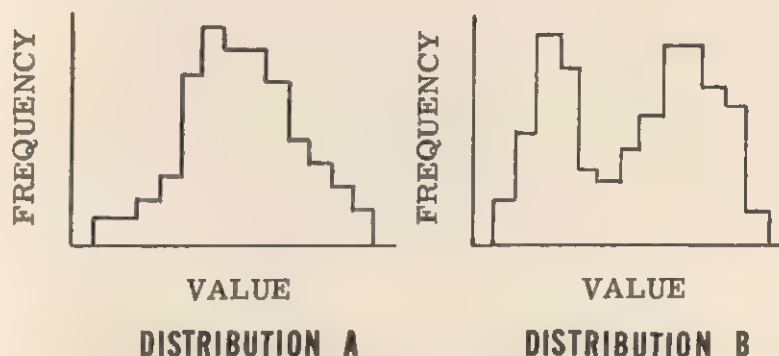
(PROPORTION OF FAVORABLE OPINIONS IN SAMPLES OF FIVE)

17. Although the shape of the previous distribution is **not** identical to the shape of a bell, it is convenient to describe this kind of distribution as **approximately** "bell-shaped." You could describe the difference between the two distributions below by saying that Distribution A / B

A

is approximately "bell-shaped," whereas Distribution A / B is not.

B



18. Let's consider an example of a "bell-shaped" distribution. Suppose you filled a glass jar with 20 marbles and then asked a large number of students to estimate how many marbles there were in the jar. Some of the estimates would be too high and others would be too low. You would expect, however, that most of the estimates would be fairly close to the actual number of marbles in the jar. Occasionally, you would obtain some poor estimates, such as 15 or 25. On the other hand, you would expect estimates near twenty / twenty-five to be more frequent than estimates near twenty / twenty-five.

twenty

twenty-five

126.

Since we obtained 100 samples from the population, the proportion of favorable opinions in the last sample obtained could be represented by the symbol p_{100} .

127.

Study the two tables shown below.

Table One	
P	Value
p_1	.2
p_2	.5
p_3	.7
.	.
.	.
.	.
p_{99}	.3
p_{100}	.8

Table Two	
Observation	Value
x_1	5
x_2	10
x_3	25
.	.
.	.
.	.
x_{99}	15
x_{100}	8

128.

The table on the right (Table Two) is a table of x values similar to the ones you saw earlier. We have indicated that the complete table contains n observations of a numerical variable. (We have simply used a series of 3 dots to indicate that the complete list would actually take up much more room.)

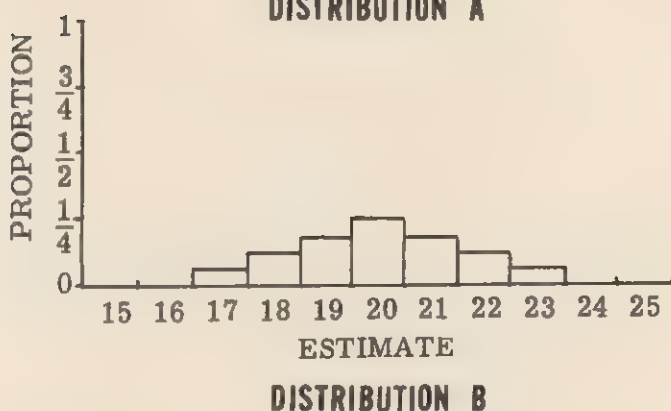
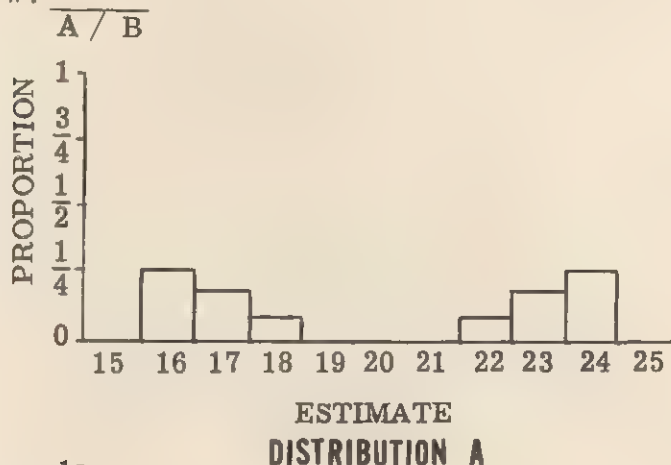
first

The table on the left (Table One) is a very similar to a table of raw data. This is the sort of table in which we could list the proportion of favorable opinions in each of the 100 samples obtained. Thus, p_1 identifies the proportion of favorable opinions in the n th sample. The actual p_1 was equal to .2 (as indicated by the entry in the first row and second column). The proportion of favorable opinions in the first sample was _____.

.2

19. You would probably find the distribution of these estimates similar to which of the distributions shown below?

B



20. Both of the above graphs would be called _____
absolute/relative
frequency distributions, since they show the p _____
of subjects who estimated each of the possible values.

relative

proportion

21. If Distribution A (above) had been the distribution of your data, the subjects would have been acting very strangely. According to Distribution A, many of the subjects over-estimated and many of the subjects under-estimated but very _____ of the subjects made estimates close to the true number of marbles in the jar. (Remember, there were 20 marbles in the jar.)

few

22. According to the Distribution B, (above) none of the subjects' estimates was greater than _____
or less than _____.

23

17

121. The actual number of students in favor of the overseas branch in the population was .7. Thus, you could say that the population proportion of favorable opinions was

122.

In order to distinguish the proportion of favorable opinions in the **population** from the proportion of favorable opinions in a **sample**, we will represent the population proportion by the symbol p . Thus, in the present illustration, p equals _____.

123.

The little wavy line above the small letter p is called a "tilde" (pronounced til-da, so that it rhymes with Hilda). In general, whenever you see a tilde placed over a statistic, that statistic describes the **population**. You would know, therefore, that $\frac{\sigma^2}{2}$ represented the variance of a population rather than the variance of a sample.

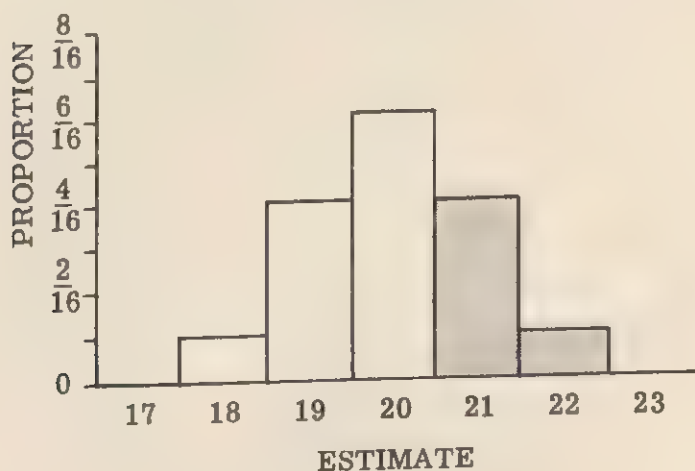
124.

The symbol p without a tilde will be used to represent the proportion of favorable opinions in a **sample**. If you obtained a sample in which .3 of the opinions were favorable from a population in which .7 of the opinions were favorable, \tilde{p} would equal _____ and p would equal _____.

125.

It will also be useful to have a symbol identifying the proportion of favorable opinions in a particular sample. For example, p_1 could represent the proportion of favorable opinions in the **first** sample. The symbol p_2 could represent the proportions of favorable opinions in the **second** sample. The symbol p_3 could represent the proportion of _____ opinions in the _____ sample, and so on.

23. You could describe the variability of the estimates in B by saying that "the range of the estimates was ____." 6
In other words, the **range** equals ____ minus ____ . 23, 17
24. The proportion of students who estimated that there were **more than** 21 marbles in the jar is simply the proportion who estimated that there were **22 or more**. If $\frac{2}{64}$ of the students estimated 22 marbles, $\frac{1}{64}$ of the students estimated 22 marbles, and none of the students estimated more than 23 marbles, you could say ____ of the students estimated there were more than 21 marbles in the jar. $\frac{3}{64}$
25. The proportion of students who estimated more than 20 marbles is simply the proportion of students who estimated there were 21 marbles, plus the proportion who estimated there were ____ marbles, plus the proportion who estimated there were ____ marbles. 22
23
26. Imagine that the distribution of estimates was as follows: (Notice that we have shaded the columns representing subjects who estimated **more than** ____ marbles.) 20



biased

117. Rather than obtaining a biased/random sample, in which

some opinions in the population would have more

opportunity to be included than would others, it was

suggested that a biased/random sampling procedure be

used, thereby providing every student's opinion the same opportunity to be included in the sample.

118.

One possible random/biased procedure for obtaining a

random

sample would be to place each of the student opinions on a separate slip of paper, thoroughly mixing all of these slips of paper together in a basket, and drawing out as many opinions as you wished to include in the sample.

119.

To determine how likely it was that a single sample

obtained in this manner would be representative of the population, 100 samples were actually obtained by this procedure. Each of the 100 samples consisted of 5

opinions. These 5 opinions (the slips of paper on which the opinions were written) were/were not mixed again

were

with the other slips of paper in the basket before the next sample was obtained (in order that all opinions would have the same opportunity of being included in each sample).

120.

The slips of paper drawn for 1 sample had to be replaced before obtaining the next sample, since each opinion in the population had to have the same opportunity of being included in each sample if this was to be a random sampling procedure.

same

27. We pointed out earlier that the sum of all the proportions in a proportional (relative) frequency distribution has to equal _____. Since the height of each column represents the proportion of students who estimated a particular value, the total height of all the columns added together must equal _____.

1

1

28. According to the preceding graph, $\frac{1}{16}$ of the students estimated 18, $\frac{4}{16}$ estimated 19, _____ estimated 20, _____ estimated 21, whereas the remaining $\frac{1}{16}$ of the students estimated _____.

$\frac{6}{16}$ $\frac{4}{16}$

22

Thus, the total of the proportions represented by all the columns is _____/16, which equals _____.

16, 1

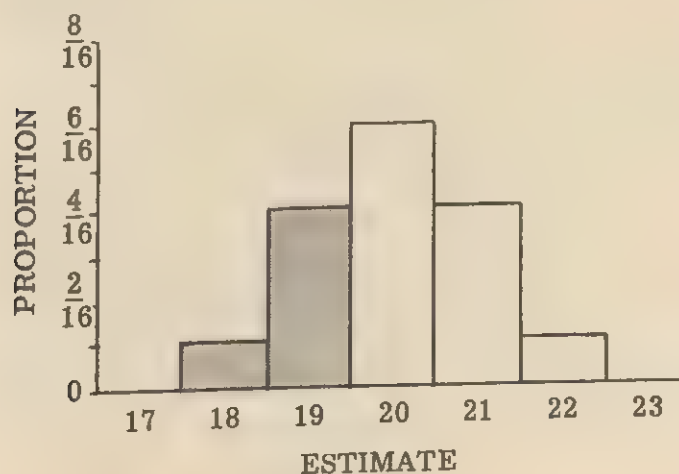
29. If we added together the heights of the two shaded columns in the previous distribution, they would form a column whose height was equal to the proportion of students who estimated **more than** _____ marbles. Notice that the proportion of students estimating **more than 20** equals _____ plus _____, or $\frac{5}{16}$.

20

$\frac{4}{16}$ $\frac{1}{16}$

30. In the following distribution we have shaded the columns representing students who estimated _____ than 20 marbles.

fewer



Only 5 samples of size 10 were obtained that would have led to an absolute error of estimate greater than .2, whereas _____ samples of size 10 were obtained that would have led to an absolute error greater than .2.

114.

This illustrates a very important characteristic of random samples. The $\frac{\text{larger/smaller}}{\text{the size of the}}$ samples, the less frequently would you expect to obtain samples leading to large absolute errors of estimation.

Another way of saying the same thing is to say that the larger the size of the sample, the more closely grouped will the sample statistics tend to be about the true

parametric value. In other words, the larger the sample size, the $\frac{\text{larger/smaller}}{\text{will tend to be the variability}}$

in your sample statistic.

115.

Let's review the important features of the previous illustrations. A complete collection of data was

specified: the opinions (yes or no) of 10,000 students at the university concerning the advisability of opening a foreign branch. This complete collection of data

consisted, therefore, of _____ observations of a variable we could call **opinion**.

116.

After this complete collection of data had been obtained, it was suggested that a satisfactory estimate of the student opinions might be made by simply obtaining **part** of the complete collection. In other words, instead of obtaining

all $\frac{\text{(number)}}{\text{observations in the population}}$, it might

30. (Continued)

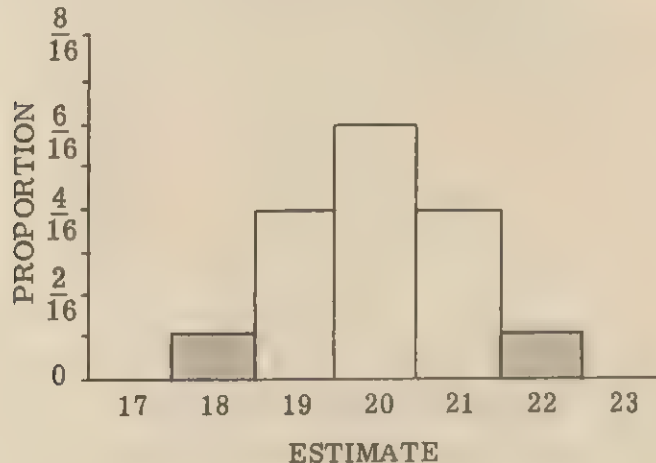
A column as high as the two shaded columns combined would represent the proportion of students who estimated fewer than _____ marbles. This proportion would be ____.

20, $\frac{5}{16}$

31. In the following distribution we have shaded columns corresponding to people who estimated **fewer** than _____ or **more** than _____ marbles.

19

21



32. A subject who estimated 22 marbles would have made an **error** of 2 since there are actually 20 marbles in the jar. A subject who estimated 18 marbles would also have been in error by 2. The estimates indicated by the shaded columns in the previous distribution represent subjects who made an error of more than _____.

2

33. If the **mean** of the distribution of estimates was 20, an estimate of 22 would correspond to a positive deviation from the mean of _____.

2

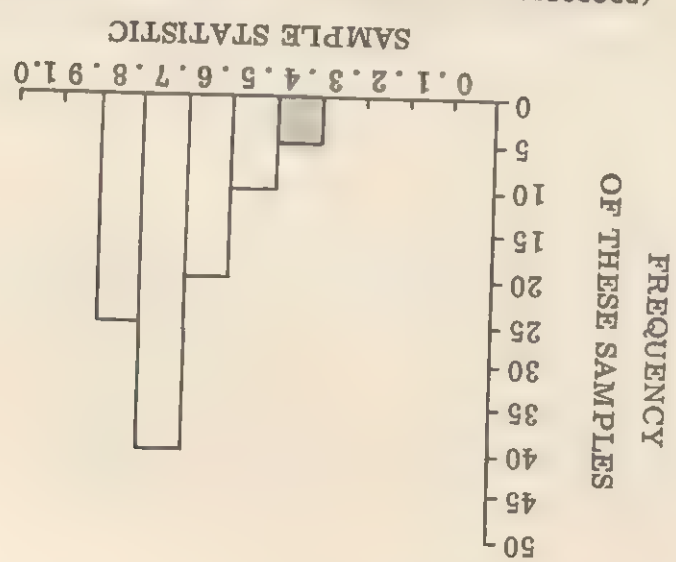
Similarly an estimate of _____ would correspond to a negative deviation of -2.

18

112.

In the following graph, we have shaded those columns which represent samples that would lead to an absolute error of estimate **greater than** _____. (Remember, the population proportion was .7.)

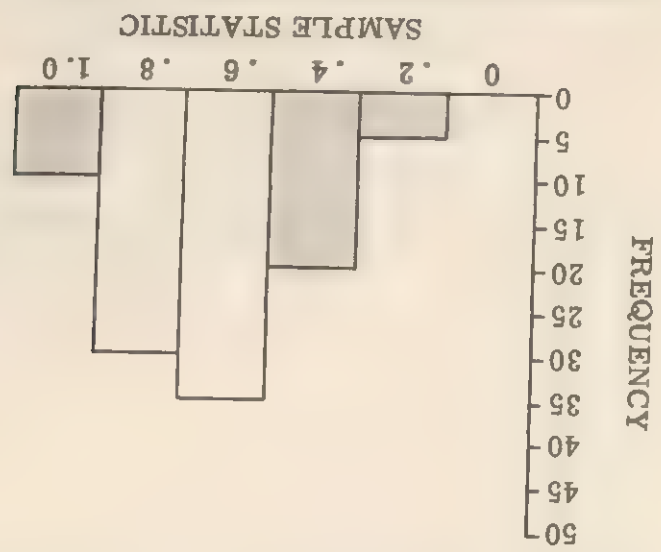
. 2



(PROPORTION OF FAVORABLE OPINIONS IN A SAMPLE OF TEN)

113.

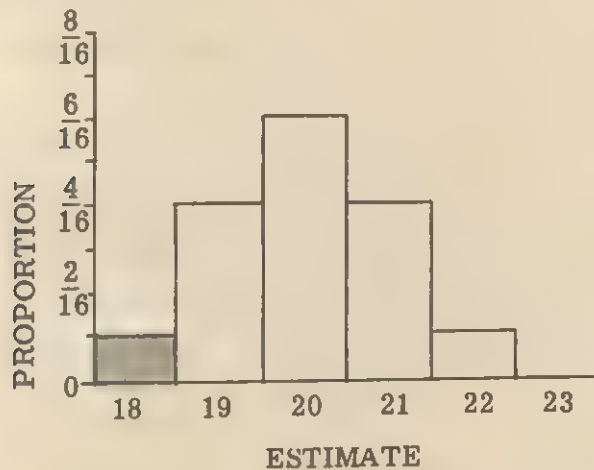
By increasing the size of the samples, we have apparently reduced the risk of obtaining a sample leading to an absolute error of estimate greater than .2. This fact is made most apparent by comparing the sampling distribution for samples of size 10 to the sampling distribution for samples of size 5.



(PROPORTION OF FAVORABLE OPINIONS IN SAMPLES OF FIVE)

34. The shaded columns in the previous distribution indicate the proportion of subjects whose estimates had either a positive deviation from the mean of _____ 2
or more, or a negative deviation of _____ **or more**. -2
 (We use the words "or more" to indicate an estimate even farther away from the mean in either a positive or negative direction.)

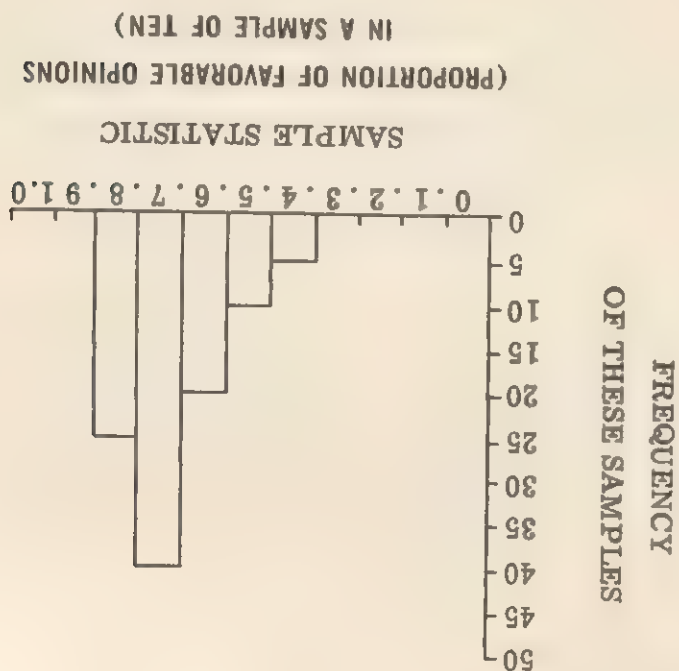
35. The shaded column in the following distribution represents the proportion of estimates that had a _____ deviation from the mean estimate of _____ negative
 positive/negative -2



36. We will use the phrase **absolute deviation** or **absolute error** when we are interested only in the distance between a value and the mean and not in whether the deviation is positive or negative.

In other words, when we are interested only in the difference between a value and the mean and not in whether this difference is positive or negative, we will use the phrase _____ or _____
 _____.

absolute deviation
 absolute
 error



This graph illustrates the same series of samples as is summarized in the previous table. Notice how most of the sample proportions are grouped around the true proportion of .7.

111. Sample proportions of 0, .1, .2, .3, .4, .5, .6, .7, .8, .9, or 1 occurred in the 100 samples.

37. A value that deviates from the mean by +2 is the same **distance** away from the mean as a value having a deviation of -2. The value ten deviates from the value of eight by +2 (or simply 2). The value 6 deviates from 8 by -2. Both 6 and 10, however, are the same distance away from the value _____. This is what is meant when we say that the value 10 and the value 6 have the same **absolute deviation** from 8.

8

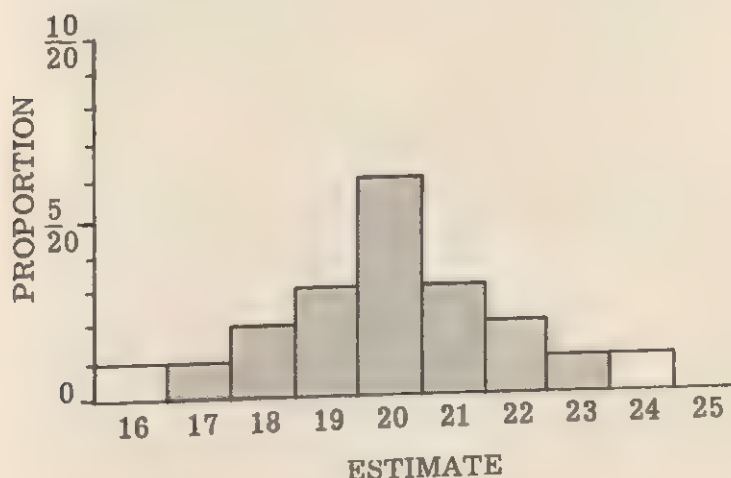
38. We just used an illustration in which 100 students attempted to estimate the number of marbles in a jar containing 20 marbles. An estimate of 22 would have been just as accurate as an estimate of 18, since both estimates would have been in error by _____.

2

39. If the mean of the estimates was 20, an estimate of 22 would represent a positive deviation of 2 and an estimate of 18 would represent a negative deviation of -2. Therefore, both an estimate of 22 and an estimate of 18 would represent the same _____ from the mean.

absolute deviation
(error)

40. Suppose the distribution of student estimates was as follows:



104. Notice that samples in which exactly _____ of the 10 subjects had favorable opinions occurred more frequently than did any other type of sample.
105. Considering the **sample proportion** as an **estimate** of the population proportion, there would have been exactly _____ samples which would have led to a perfect estimate of the population proportion.
106. Sample proportions of _____, or _____, would have led to an absolute estimation error of .1 or less. The frequencies of these three types of samples were _____, and _____ respectively.
107. Adding these three frequencies together indicates that out of the 100 samples there were exactly _____ samples which would result in an absolute error of estimate of .1 or less.
108. Similarly, exactly _____ of the 100 samples would have led to an absolute error of estimate of .2 or less.
109. The only samples **actually obtained** that would have led to an absolute error of estimation **greater than .2** were those samples in which _____ of the subjects had a favorable opinion.
- Thus, _____ of the 100 samples would have resulted in a sample statistic of .4 and would thereby have led to an absolute error of estimation greater than _____.

40. (Continued)

The mean of this distribution is 20. Therefore, an estimate of 24 would represent a positive deviation of _____, and an estimate of 16 would represent a negative deviation of _____.

4
-4

41. On the previous distribution graph, the shaded columns represent estimates having an absolute deviation of **less** than _____.

4

42. The unshaded columns in the previous graph indicate the proportion of estimates having an absolute deviation from the mean of **more** than _____.

4

43. In a bell-shaped distribution, large absolute deviations are _____ frequent than small absolute deviations.
more/less

less

44. The **small** absolute deviations are **more** frequent because most of the observed values are clustered around the mean in a bell-shaped distribution. Estimates much larger or much smaller than the mean have a _____ absolute deviation and are less frequent
large/ small
than those values with _____ absolute deviations.
large/ small

large

small

45. Students who estimated 25 marbles are **unusual** in the sense that few students made estimates that large. Similarly, students estimating only 15 marbles are also unusual, since there were very few estimates that small. In other words, the _____ the absolute deviation
larger/ smaller
of a student's estimate from the mean, the more **unusual** was his estimate.

larger

If you actually carried out a sampling procedure of this sort, you would very likely obtain data similar to that summarized in the following table.

Number of favorable opinions in a sample of 10	Proportion favorable opinions in the sample	Frequency of such samples
0	0	0
1	.1	0
2	.2	0
3	.3	0
4	.4	5
5	.5	10
6	.6	20
7	.7	40
8	.8	25
9	.9	0
10	1.0	0

The first column of the previous table indicates the 11 possible types of samples you could obtain, where each sample is identified by the number of favorable opinions it contains.

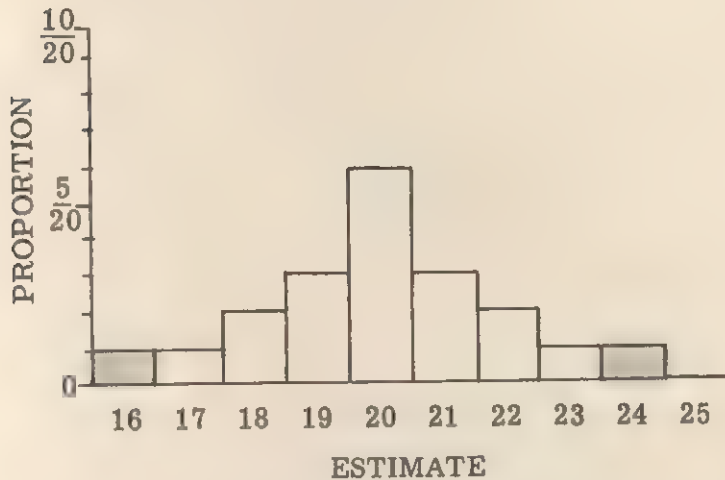
In the second column of the previous table, we have indicated the sample proportion corresponding to each of these 11 possible types of samples. In other words, if there were 7 favorable opinions in a sample of 10, the proportion of favorable opinions in that sample would equal $\frac{7}{10}$.

Finally, in the third column of the table, we have indicated how often each type of sample actually occurred in the 100 samples that were obtained. According to the table, there were exactly _____ samples in which all of the subjects had unfavorable opinions.

There were _____ samples out of the 100 samples in which exactly 4 of the 10 subjects had favorable opinions.

46. In the distribution shown below, we have shaded the columns representing the proportions of students who made either unusually _____ or unusually _____ estimates.

large
small



47. An estimate representing a large absolute deviation from the true number of marbles in the jar would be considered a _____ estimate.

poor

48. Suppose you conducted the marble estimation experiment with two groups of students, one group of 8-year-old students and another group of 18-year-old students. While you would expect the 18-year-old students to make some errors in their estimates, you would expect them to be more accurate than you would the younger students.

In other words, although the older students would make some mistakes, you would expect the absolute deviations of their estimates from the true number of marbles to be generally _____ than those of the younger students.

smaller

The smallest number of favorable opinions you could have in any sample would be 0, whereas the largest number of favorable opinions would be _____. Thus, the 11 possible sample proportions would be:

0	_____
.1	_____
.2	_____
.3	_____
.4	_____
.5	_____
.6	_____
.7	_____
.8	_____
.9	_____
1	_____

100. Thus, with a sample of 10, it _____ be _____

possible to obtain a sample statistic which was identical to the population statistic of .7.

101. Exactly _____ out of the 10 opinions samples would have to be favorable in order for your estimate to be identical to the true population proportion.

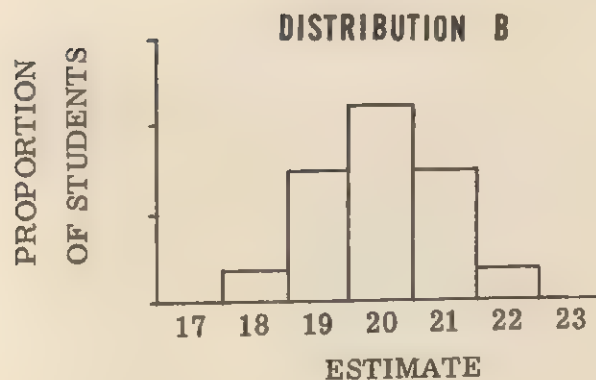
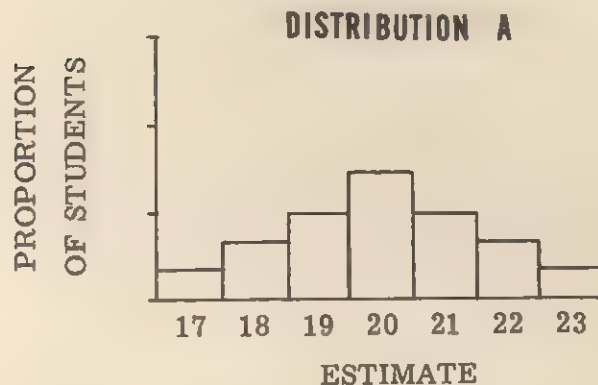
7

Accordingly, if the following distributions represented the distributions of estimates from the two groups,

Distribution $\frac{\text{A}}{\text{B}}$ is probably the distribution of

A

estimates for the **younger** group.



49. Distribution B represents the older students' estimates. It is clear that the older students tended to make more accurate estimates than did the younger students. The older students' estimates (Distribution B) tended to be closer to the true number of marbles than were the estimates of the younger students. You could say that the variability of the $\frac{\text{younger}}{\text{older}}$ students' estimates was greater than that of the other group.

younger

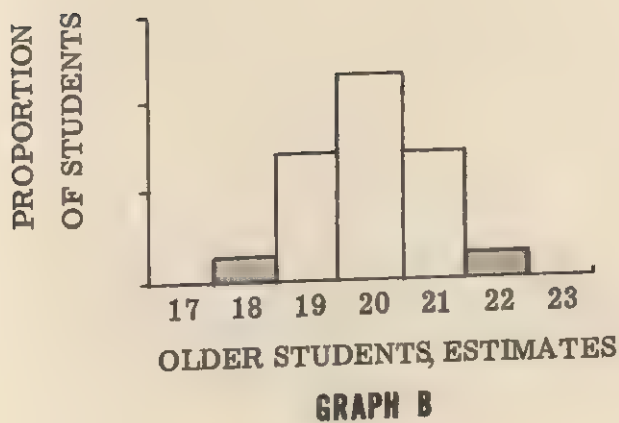
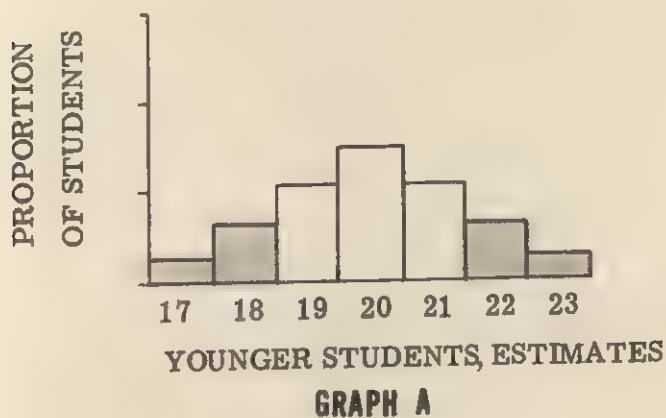
population proportion. He might then suggest that instead of taking samples of _____, as you did number 5
it would be more time-consuming to obtain a sample of 10 than it would a sample of 5, it would still be easier than measuring the opinion of every person in the population.

96. Suppose you repeated the previous sampling procedure, this time obtaining samples of 10 rather than 5. In other words, you listed all of the students' names on slips of paper and placed these in a basket. After thoroughly mixing the slips in the basket, however, you would draw out _____ of these slips to form a sample of _____ rather than a sample of size 5.

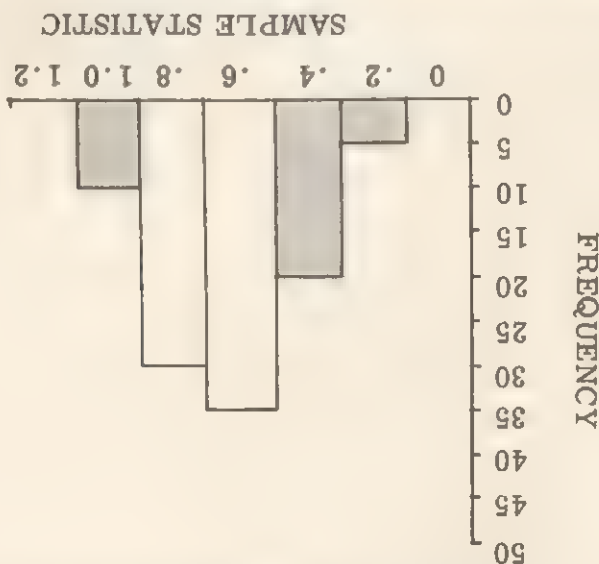
97. Before obtaining the next sample, you would first replace the 10 slips you took out for the first sample. Then, after once more mixing up the slips, you would take out another sample of _____.

98. In this manner, you could draw one sample of 10 after another, until you had exactly 100 samples, just as you did for the samples of 5. This time, however, your sample size is _____, whereas previously it was _____.

50. We could represent the difference in _____ variability
between the two distributions by the range. The range
of Distribution A is _____ and the range of 6
Distribution B is _____. 4
51. Estimates of either _____ or _____ would have an 18, 22
absolute deviation of 2 from the true number of marbles
(20), since the first estimate would be 2 fewer than the
actual value and the second estimate would be 2 greater
than the actual value.
52. The previous distributions are reproduced below. This
time we have shaded the columns representing estimates
whose absolute deviations from the true value of 20 were
_____ or more. 2



For example, look at the following graph. It is the same one we presented earlier; in this graph, however the $\frac{\text{shaded}}{\text{unshaded}}$ columns represent the samples that would lead to an absolute estimation error **greater** than .1. (Remember, the population proportion was .7.)



(PROPORTION OF FAVORABLE OPINIONS IN SAMPLES OF FIVE)

95.

While most of the samples of 5 you obtained would give you an estimate of the population proportion that differed from the true value by only .1, you still obtained a reasonable number of samples that would lead to larger error. You might be willing to accept the risk of making a poor estimate, since there was very little effort required in obtaining a sample of 5 compared to determining the attitude of every student in the population. You might tell your statistician friend, however, that all, of the students their opinion, you are not willing to accept the risk of obtaining one of the relatively infrequent samples which would lead to a poor estimate of the

53. One way of comparing the **accuracy** of the younger and older students' estimates is to compare the proportion of students in each group whose estimates differed from the true value by 2 or more. The proportion of students who made errors **greater than one** is indicated by the _____ columns in the previous graphs. shaded
shade/unshaded
54. Estimates whose absolute deviation from the true value of 20 were greater than one occurred more often in the group of _____ students than they did in the other younger
younger/older group.
55. Among the younger students, an estimate of 22 was _____ **unusual** than an estimate of 21, since the more
more/less proportion of younger students who estimated 22 was _____ than the proportion of younger students smaller
greater/smaller who estimated 21.
56. An estimate of 22, however, was _____ **unusual** for less
more/less younger students than it was for older students, since the proportion of younger students who made estimates of 22 is _____ than the proportion of older larger
larger/smaller students who made estimates of 22.
57. Compared only with the rest of the people **in his own age group**, a young student who estimated 22 would not have performed quite so poorly as an older student who had estimated 22, since an estimate of 22 was more common (occurred more frequently) in the _____ group younger
younger/older than it was in the other group.

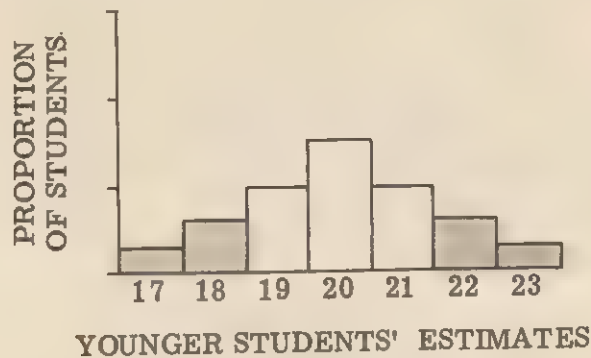
91. We shall use the phrase **absolute estimation error** to represent the difference between your estimate and the true parametric values, **disregarding which of the two was larger**. In other words, both a sample proportion of .6 and a sample proportion of .8 would lead to an **absolute estimation error** of .1. Since $\hat{p} = .7$ both sample proportions differ from the true value by .1, even though the sample proportion of .6 is _____ than the true value by .1 and the sample proportion of .8 is _____ than the true value by .1.
92. It is perfectly clear from the previous table that large absolute estimation errors are _____ frequent _____ less
93. In other words, large absolute estimation errors (whether they be the result of overestimating the population parameter or of underestimating the population parameter) are **unusual** compared to estimates that are closer to the population parameter. In fact, an absolute estimation error of .5 occurred _____ times out of the 100 samples.
94. To review, at the statistician's suggestion you considered the sampling distribution of samples of 5 opinions, each collected by a random procedure from a population of 10,000. The **experimental** sampling distribution you obtained suggested that most samples obtained in this way would probably give you a fairly good estimate of the population proportion. However, there were an appreciable number of samples which would have led to an absolute error greater than .1.

58. Suppose you decided to give a prize to all the students whose estimates were within one marble of the true value. Or, to put it differently, suppose you decided to give a prize to all the students whose estimates had an absolute deviation from the true value of _____ or less.

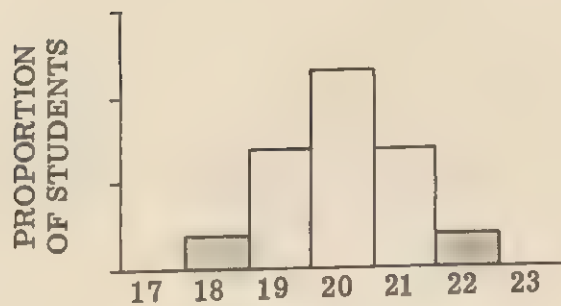
1

59. The _____ columns in the following graphs indicate the proportions of students in each age group who would receive a prize.

unshaded



GRAPH A



GRAPH B

87. The following table contains the same information as did the previous graph. In addition, we have indicated the error of estimate which would occur for each of the six possible kinds of samples. A sample in which none of the opinions were in favor would lead to an estimate of the population value that would be in error by $-.7$.

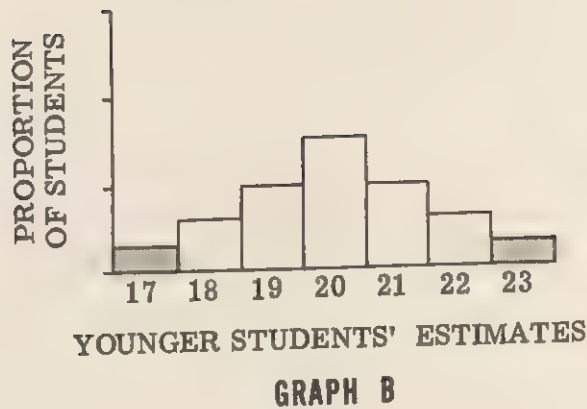
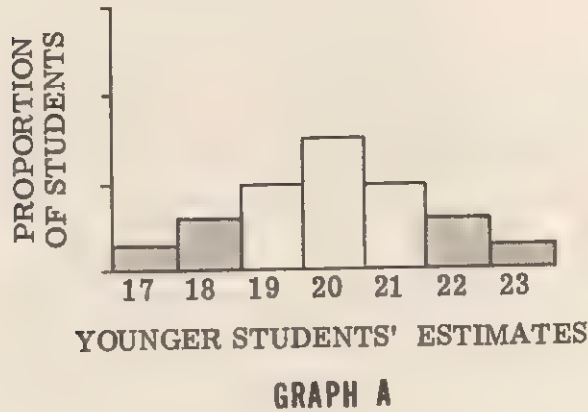
Frequency of Opinions in Favor	Frequency of Such Samples	Error of Estimate
0	0	$-.7$
.2	5	$-.5$
.4	20	$-.3$
.6	35	$-.1$
.8	30	.1
1.0	10	.3

88. Proportions in which 3 out of the 5 opinions were favorable give you a sample proportion equal to _____. This sample proportion would lead you to make an estimate which was in error by _____.
89. Samples with 4 of the 5 opinions in favor give a sample proportion equal to .8. Since .8 is greater than the parametric proportion, those samples would give you an estimate which would be in error by _____.
90. If all 5 of your sample opinions were favorable, your estimate of the population proportion would be _____. This would represent an error of estimate of _____.

60.

If you only gave a prize to those younger students whose estimates were within 1 of the actual value, the unshaded columns in Graph A / B (below) would indicate the proportion of younger students who received a prize.

A



61.

Since the older students were more accurate in estimating the number of marbles, fewer/ more of their estimates would meet the requirements for a prize than would those of the younger students because more/ less of the older students had estimates close enough to the true value.

more

more

82. According to the previous sampling distribution, it would/would not be possible to obtain a sample with .2 of the opinions in favor, but it would/would not be unusual to obtain a sample of this sort.
83. Of the 100 samples (of 5 opinions each) that were obtained, there were only _____ samples which would have led you to make an estimate that differed by **more than .3** from the true population proportion.
84. Since the population proportion was .7, only proportions _____ would differ from this true proportion by more than .3
85. If you used a sample statistic of .4 as your estimate of the population statistic, the difference between your estimate and the population proportion would be .7 minus .4, which would equal .3. The question was: What sample statistic would differ from the true population value by **more than .3**? The only possible values of the sample statistic (in samples of 5) that would lead to estimates differing by **more than .3** from the true value are samples having **no** favorable opinions or **one** favorable opinion. In other words, sample proportions of _____ or _____ would differ from the true population proportion of .7 by more than .3.
86. According to the previous graph, there were no samples in which all the subjects were opposed to the proposal, and there were only _____ samples in which one of the subjects was in favor of the proposal.

5

0, .2

.4

5

would

would

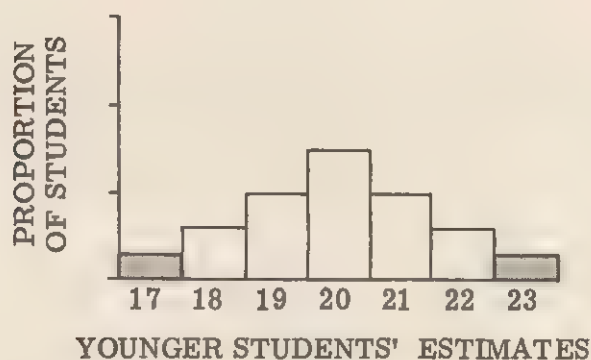
62. The unshaded columns in the following graphs indicate the proportion of older students who would receive a prize if we only gave a prize to students who made errors of _____ or less.

1

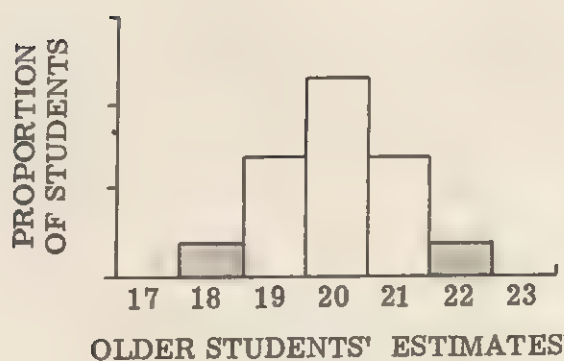
The other graph indicates the proportion of younger students would receive a prize if we gave a prize to all younger students who made an error of _____ or less

in their estimates.

2



GRAPH A



GRAPH B

78.

The preceding sampling distribution suggests that samples obtained in this manner tend to have sample statistics quite similar to the population statistic. This is obvious on the previous graph, since most of the samples had proportions equal to .6 or .8, whereas samples with proportions equal to .2 or 1 were much frequent.

more/less

less

79.

Note that the sampling distribution is roughly bell-shaped, with most of the sample proportions clustered around the true population value of $\frac{.4}{.7}$.

.7

80.

This sampling distribution suggests that if you used the same sampling procedure to obtain a single sample and if you used the sample statistic to estimate the population statistic, you would run little risk of making an estimate of the population proportion equal to .2, since a sample proportion equal to .2 was quite unusual (very few/many of the samples you obtained had a proportion equal to .2).

few

81.

Samples with proportions equal to .6 or .8 were not unusual. In fact, they were quite typical, since the frequencies of samples in which .6 or .8 of the students were in favor of the overseas campus are relatively compared to the other possible values of the

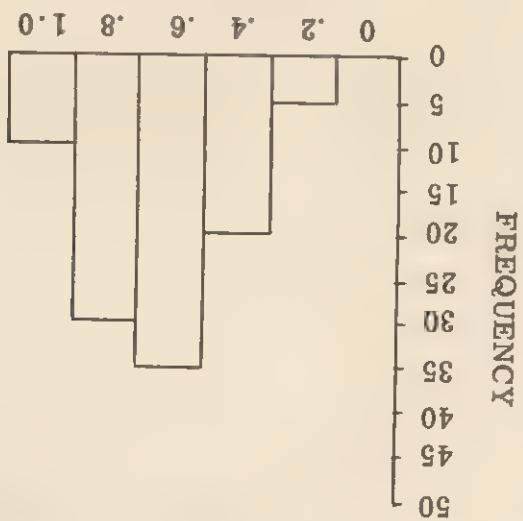
high

sample statistic.

- The proportion of younger students obtaining prizes and the proportion of older students obtaining prizes
 _____ be approximately the same if we used _____ would
 would/ would not
 these two rules for awarding prizes.
63. We could conclude from these considerations that a
 _____ student whose estimate was within 1 _____ younger
 younger/ older
 of the actual value was doing just about as well in
relation to the rest of the people in his age group as was
 a(n) _____ student whose estimate was within 2 _____ older
 younger/ older
 of the actual value.
64. By considering the difference in variability between the two groups, we established rules for giving prizes whereby approximately the same proportion of students received prizes in each group. Since large errors (absolute deviations) were more frequent in the younger group, a _____ student would not be required to be _____ younger
 younger/ older
 quite as accurate in order to win a prize as would a student in the other age group.
65. The preceding example illustrates why it is often useful to consider the **variability** of a distribution when you are evaluating a particular observed value. For example, imagine you were teaching a course in psychology. You gave your students two examinations during the semester. Suppose there were 10 questions on each test and the student received either a score of 1 or a score of 0 on each question. The possible total score on each test would be somewhere between _____ and _____.

0
10

75. The previous table is similar to a frequency distribution table. It is also possible to draw a frequency distribution graph (similar to the ones we have seen earlier) indicating the distribution of these sample statistics.



(PROPORTION OF FAVORABLE OPINIONS IN SAMPLES OF FIVE)
 SAMPLE STATISTIC
 Notice that there were more samples in which the proportion of students in favor of the proposal equaled _____ than any other possible value of the sample statistic.

76.

The **highest** column on the graph represents the frequency of samples in which the sample statistic (proportion of opinions in favor) equaled _____.

77.

The distribution shown in both the previous table and the previous graph is that of a sample statistic. Thus, the distribution could be described as a _____ distribution.

sampling

65. (Continued)

Suppose the results of these tests were those shown in the following frequency tables.

Examination A		Examination B	
Score	Frequency	Score	Frequency
0	0	0	0
1	0	1	0
2	0	2	1
3	1	3	2
4	4	4	3
5	5	5	4
6	5	6	4
7	4	7	3
8	1	8	2
9	0	9	1
10	0	10	0

According to these data, there were _____ students in the class.

20

66. There must have been 20 students in the class because the sum of the _____ in each frequency table is 20.

frequencies

67. The **variability** of the scores on Test A / B appears to be slightly larger than the variability on the other test. The **range** is one statistic which would represent this difference in variability, since the range was _____ on Test A and _____ on Test B.

A

5

7

68. It also appears that σ^2 was larger on Test A / B than on the other test.

A

Notice that this table is very similar to a frequency table. There are _____ possible values of a variable called "proportion of opinions in favor."

6

The frequencies in the second column of the table indicate how many samples with each possible value of the sample statistic were obtained. The sum of these frequencies indicates that exactly _____ samples were obtained.

100

73. Notice that there were **no** samples in which all of the students were opposed to the overseas campus. However, there were _____ samples in which all of the students were in favor of the overseas campus.

10

74. Since there were 5 opinions in each sample, it is impossible to obtain a sample proportion equal to the true population proportion of .7. However, in 35 of the samples there were exactly _____ students in favor of the proposal, and in 30 of the samples exactly _____ of the students were in favor of the proposal. In other words, 35 samples had a sample proportion of _____ and 30 samples had a sample proportion of _____.

.8

.6

4

3

69. The preceding absolute frequency distributions could be converted to **relative** frequency distributions by dividing each frequency by _____ in order to convert it to a _____.

20
proportion

The two proportional distributions which you would obtain in this manner are shown in the following tables.

Examination A		Examination B	
Score	Frequency	Score	Frequency
0	0	0	0
1	0	1	0
2	0	2	1/20
3	1/20	3	2/20
4	4/20	4	3/20
5	5/20	5	4/20
6	5/20	6	4/20
7	4/20	7	3/20
8	1/20	8	2/20
9	0	9	1/20
10	0	10	0

Notice that $\frac{1}{20}$ of the students received scores of **3 or lower** on Examination A, whereas _____/20 of the students received scores of **3 or lower** on Examination B.

3

70. The **mean** of both previous distributions (Examination A and B) is $5\frac{1}{2}$. A score of 5 would have a negative deviation of $-\frac{1}{2}$ from the mean and a score of 6 would have a positive deviation of _____ from the mean.

$\frac{1}{2}$

71. The only two scores which have an absolute deviation of $\frac{1}{2}$ or less (from the mean of $5\frac{1}{2}$) are _____ and _____.

5, 6

72. One-half ($\frac{10}{20}$) of all 20 students received scores whose absolute deviation from the mean was less than $\frac{1}{2}$ on Examination _____.

A

Suppose you agreed to this demonstration and, with the aid of your secretary, made the necessary arrangements to obtain the samples in this manner. If you decided to obtain samples of 5 opinions, you would place the slips of paper indicating student opinions in a large basket, mix the basket thoroughly, and draw out _____ slips of paper.

Before proceeding to obtain the next sample, you would have to replace the first 5 slips of paper you had taken out. Otherwise, the students whose opinions were represented in the first sample would not have an equal opportunity of being included in the second sample.

In other words, in order to give each student's opinion the same opportunity of occurring in each sample, there would have to be _____ slips of paper in the basket each time a sample was obtained.

The number of students in favor of the overseas campus in each sample could be anywhere from 0 to 5. Thus, the **proportion** of students in each sample who were in favor of the overseas campus would have to equal 0.2.

Imagine that 100 samples consisting of 5 opinions each were obtained in this manner. The results could be

summarized in the following table.

Proportion of Opinions in Favor	Frequency of Such Samples
0	0
.2	5
.4	20
.6	35
.8	30
1.0	10

73. The scores 5 and 6 are the only scores deviating from the mean by $\frac{1}{2}$ or less on both examinations. The proportion of students who received scores of 5 or 6 is equal to

$$\frac{5}{20} + \frac{5}{20} = \frac{10}{20}$$

on Examination $\frac{A}{B}$.

A

74. The proportion of students who received **either** a score of 5 or 6 on Examination B is equal to _____ + _____, which equals _____.

$$\frac{4}{20} + \frac{4}{20} = \frac{8}{20}$$

75. It was slightly more **unusual** for a subject's score to have an absolute deviation from the mean of more than one-half on Examination $\frac{A}{B}$ than on the other

A

examination, since a higher proportion of students received grades of 5 or 6 on Examination $\frac{A}{B}$.

A

76. The absolute deviation of an observed value from the mean does not necessarily indicate how **unusual** that value was in relation to the rest of the distribution. If most of the values in the distribution were grouped very closely around the mean, the proportion of values whose absolute deviation from the mean was greater than 1 might be very small. On the other hand, if the distribution were quite variable, a much higher proportion of the values might have absolute deviations from the mean greater than 1.

66.

While you might be unfortunate enough to obtain a sample in which only 1 out of the 10 (.1) of the subjects were in favor of the overseas campus, you would probably find that such highly unrepresentative samples occurred infrequently. In other words, an unrepresentative sample would be more/less unusual than a representative sample.

more

67.

The time and effort you could save by taking a sample rather than a complete collection of data could be balanced against the risk of obtaining a highly unrepresentative sample. If the risk of making a large error of estimate were small compared to the amount of effort that you would save by only having to collect a sample, it would/would not be more desirable to use the sampling procedure.

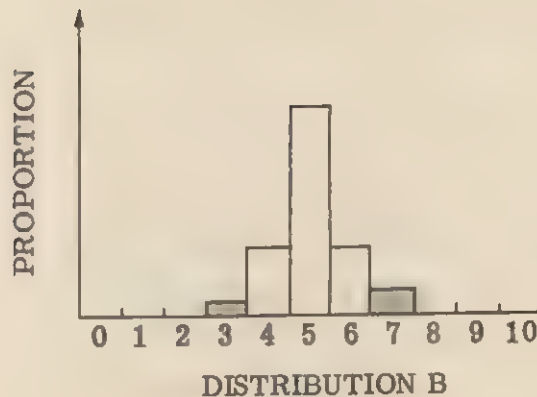
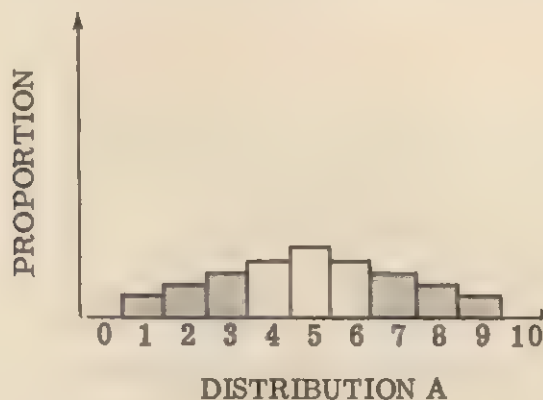
would

68.

Let's suppose the statistician suggested the following procedure as a means of clarifying his proposal. Since the data which defined the population of student opinion had already been collected, it would be possible to obtain as many samples of this population as you desired. Instead of assigning each student a number from 1 to 10,000, you could actually write each student's opinion on a piece of paper, place all the papers in a basket, shake the basket well, and draw out a sample of sample from the random/biased population of student opinions.

random

For example, consider the two distributions shown below.



The mean of both distributions is 5. However, the variability of Distribution A / B appears to be greater than the variability of the other distribution.

A

77. While the mean of both distributions is 5, absolute deviations greater than 1 occur much more frequently in Distribution A / B than they do in the other distribution.

A

64.

The statistician might propose a **random sampling** procedure of the following sort. Every student enrolled at the university would be assigned a different number from 1 to 10,000. Each of these numbers would be written on a separate slip of paper and all of these slips could be placed in a large basket. Then, the basket would be thoroughly shaken so that all the slips of paper inside were scrambled. If you wanted to obtain a sample consisting of 10 students' opinions, for example, you would simply draw out 10 slips of paper identifying 10 students. You could obtain these 10 students' opinions by contacting them directly. Their answers would represent a sample of 10 student opinions. This would be a random/biased procedure for obtaining a sample of 10 opinions, since every student's opinion had the opportunity to be included in the sample.

65.

You might then point out to the statistician that even a procedure of this sort would not preclude the possibility of obtaining an **unrepresentative** sample. For instance, you might be unlucky and obtain a sample of 10 opinions of which 9 were against the overseas campus and 1 was in favor of the overseas campus.

If .7 of the whole student body were actually in favor of the overseas campus, the population proportion would be .7, whereas the corresponding sample statistic would be _____. This would mean your **error of estimate** would be .1 minus .7, which equals _____.

- .6

.1

same

random

78. While the value 7 represents a deviation from the mean of _____ in either distribution, values with deviations that large or larger were more unusual in Distribution _____ than they were in the other distribution. 2
B
A / B
79. Statisticians describe an observed value in a way that takes into account the variability of the distribution. You saw earlier how an observed value could be represented by its deviation from the mean rather than by its actual value. In addition, it is sometimes useful to indicate the relationship between that deviation and the standard deviation of the distribution. For example, if the mean of a particular distribution were 10, the value 15 would have a deviation from the mean of _____. If the standard deviation of the distribution were 5, you could say the value 15 deviates from the mean by exactly **one standard deviation**. Similarly, since the value 20 has a deviation from the mean of 10, 20 would be exactly two/three 5

two standard deviations away from the mean if $\sigma = 5$.
80. The variance of a distribution is simply the typical (mean) squared deviation from the mean of that distribution. The **standard deviation** squared would equal the _____ variance.

sample of the students' opinions. In other words, he was suggesting that you might have viewed the 10,000 opinions as a _____ and obtained a _____ consisting of part of this complete collection of 10,000 opinions. Suppose he suggested that you could have used the proportion of favorable opinions in your **sample** as an estimate of the actual proportion of favorable opinions in the population. Or, to put it more exactly, you could have used a population/sample statistic to estimate a

sample

population

62.

You might be concerned with exactly how the sample should be obtained. If it were obtained improperly, it could be very misleading. For example, you might point out to him that the sample was obtained by questioning all the people in the French department, where many people could be expected to favor spending time in a foreign country, in which case you would more than likely overestimate/underestimate the population proportion of students in favor of an overseas branch.

overestimate

63.

The statistician would probably answer that you were describing a procedure for collecting samples which would give some students more opportunity of being included in the sample than it would other students. The statistician would suggest using a sampling procedure that gave every student's opinion an equal opportunity of being included in the sample. In other words, he would say that rather than using a biased/random sampling procedure, you should use a biased/random sampling

biased

random

procedure.

81. Consider the following relative frequency distribution.

VALUE	PROPORTION
2	1/20
3	4/20
4	10/20
5	4/20
6	1/20

- It can be shown that the variance of this distribution is 16. Therefore, the _____ is equal to $\sqrt{16}$ or 4. standard deviation
82. The mean is also 4. Thus, the values 3 and 5 _____ are not more than one standard deviation away from the mean because the absolute deviation from the mean of 4 for both the value 3 and the value 5 is _____. 1
83. You could say the value 5 is $\frac{1}{4}$ of a standard deviation away from the mean, since the value 5 represents a deviation from the mean of 1 and 1 is $\frac{1}{4}$ the size of the standard deviation 4. Similarly, the value 6 would be _____ of a standard deviation from the mean, since $\frac{1}{4} / \frac{1}{2}$ $\frac{1}{4}$
the value 6 deviates from the mean by 1 and 1 is _____ $\frac{1}{4}$
the size of the standard deviation 4.
84. We have described the distance or difference between a particular value and its mean as the _____ deviation of that value from that mean.

10,000

Yes

10,000

If a value is smaller than the mean, you say that it has a _____ deviation. If a value is larger than the mean, that value has a _____ deviation.

negative

positive

If the distribution had a mean of 6, the value 7 would represent a deviation of _____ and the value 5 would represent a deviation of _____.

1

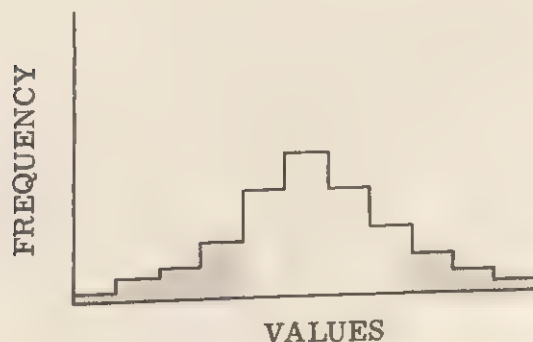
-1

85. Suppose the distribution of your data were approximately "bell shaped." You would know that most of the observed values were clustered around the mean with progressively fewer and fewer observed values farther away from the mean. In other words, the frequency of values having a deviation of 2 is probably greater than the frequency of values with a deviation of $\frac{3}{1}$.

3

86. The values farthest away from the mean in a "bell-shaped" distribution are often referred to as the **tails** of the distribution (since the distribution appears to taper into a tail at the extreme values). For example, the _____ areas in the following distribution would be referred to as the tails of the distribution.

shaded



56. Suppose you were the principal of a school and you had selected ten of your best mathematics students to compete in a contest with another school. The grades in mathematics of the ten students would/would not be a **random** sample from a population consisting of the mathematics grades of all the students in the high school.

57. The grades scored by these ten students would **not** be a random sample from the population of mathematics grades, since each of the grades in the population have the same opportunity of being included did/did not in the sample. Because of the way the sample was formed, students with good grades had the same/a better chance of being included than did those with poor grades.

58. Later you will see how it is often possible to **predict** the distribution of sample statistics (i.e., the sampling distribution) if samples are drawn in a **random** or unbiased manner. First, however, let's consider how knowledge of a sampling distribution would help you to make estimates of population statistics from sample statistics. In other words, let's consider how knowledge of the sampling distribution would help you to **estimate** statistics that describe the _____ on the basis of statistics describing the _____.

59. Suppose you were the president of a large Midwestern university who was trying to decide whether or not to open an overseas branch of the university. The object of having an overseas branch would be to provide students with an opportunity for living and studying in a foreign country sometime during their period of attendance at the university.

87. The values from the largest/smallest absolute largest deviations from the mean appear in the **tails** of a bell-shaped distribution.
88. In a so-called "bell-shaped" distribution, the farther a value is from the mean (the more it deviates from the mean), the larger/smaller will be its frequency. smaller
89. You should recall that the **standard deviation** (σ) is a deviation which, squared, would equal the . variance
If the **variance** of a distribution were 9, a deviation of 4 would be than one **standard deviation**. greater
greater/less
90. If the mean of your distribution were 10 and the standard deviation of the distribution were 4, a score of 14 would have a deviation from the mean equal to standard deviation(s). 1
 $1 / 2$
91. Another way of indicating that a particular value deviates from the mean by one standard deviation is to say that that value equals a **standard score** of one. Thus, if the value 14 represents a **standard score** of one in a distribution with a mean of 10, the standard deviation of the distribution must be . 4
92. If the distribution had a standard deviation of 2 and a mean of 10, then the value would deviate from the mean by one **standard deviation**. Therefore, 12 would represent a of one. 12
standard score

53. Of the two statements shown below, Statement A/B would more properly describe the procedure by which samples were obtained in the preceding illustration.
- Statement A: Every yearly income in the population had the same opportunity of occurring in each sample as did any other.
- Statement B: Certain yearly incomes in the population were more likely to occur in samples taken in one part of town than in the samples taken in the other part of town.
54. A procedure for selecting a sample from a population that gives some members of the population a greater opportunity of being selected than other members is called a **biased** sampling procedure. A sampling procedure in which all members of the population have the **same** opportunity of being selected is called an **unbiased or random** sampling procedure. The sampling procedure used in the previous illustration is an example of a(n) biased/random sampling procedure.
55. Consider a population consisting of 100 observations. Suppose you listed each observation on a card and then shuffled the cards very thoroughly. If you then picked ten of these cards, each of the 100 observations would have had the same opportunity of being included in the sample. Therefore, this method of selecting a sample would be an example of a random sampling procedure.

93. If a particular value were said to equal a **standard score** of 2, that value would have a deviation from the mean equal to twice the standard deviation of the distribution. If a particular value equaled a standard score of -2, that value would represent a **negative deviation** from the mean equal in size to twice the _____.

standard deviation

94. Below are a list of values forming a distribution whose mean is 10 and whose variance is 25. Since the **variance** is 25, the size of a **standard deviation** is _____.

$$5 = \sqrt{25}$$

VALUE	DEVIATION FROM 10	STANDARD SCORE
15	5	.1
5	-5	-1
10	0	0

95. Notice that the first value, 15, deviates from the mean of 10 by _____. Since the standard deviation of the distribution is 5, a deviation of 5 would be the same size as one standard deviation. This is all we mean when we indicate (as we did in the previous table) that the value 15 is equal to a **standard score** of _____.

5

1

96. The value 5 in the previous distribution represents a negative deviation from the mean of -5 and, therefore, is equivalent to a standard score of $\frac{1}{-1}$.

-1

49. The most common or frequently occurring sample mean in the distribution shown on Graph A (above) is a mean of \$_____ per year, while the modal value of the sample means on Graph B is \$_____ per year.
- \$12,000
- \$5,000
50. It would be reasonable to suppose that the distribution of twenty samples shown in Graph $\frac{A}{B}$ is based on _____
- A
- the samples collected in the lobby of the expensive hotel, while the distribution of sample means shown in Graph $\frac{A}{B}$ is more likely that of the twenty samples collected in the poor section of town.
- B
51. The distribution of sample means shown on Graph A were probably based on the samples taken in the expensive hotel (rather than in the poorest section in town), because the sample means tend to be _____ on Graph A than they are on Graph B.
- larger
- In other words, you would expect to meet more people with large incomes in the expensive hotel than you would in the poor section of town.
52. Both the twenty samples taken in the expensive hotel and the twenty samples taken in the poor section of town _____ be considered as samples from a _____
- can
- population consisting of the yearly incomes of all the people in the city. Although all the samples were from the same population, you were more likely to sample a person with a _____ yearly income in the _____
- high
- expensive hotel than you would in the poor section of town.

146. Most of the values in a normal distribution are clustered around the mean, with fewer and fewer values farther away from the mean (i.e., the distribution is "bell-shaped"). Thus, values representing a Z-score larger than 2 would be more/less frequent than values less representing Z-scores less than 2.
147. You have already seen that .95 of all the values in a normal distribution are within two standard deviations of the mean. Therefore, .95 of all the values in a normal distribution would represent Z-scores between -2 and 2.
148. Thus, a Z-score is a useful way of representing values in a normal distribution since it indicates how unusual such values are, regardless of the mean or variance of the distribution. No matter what the mean or variance of the normal distribution, you would know that a Z-score as large as 1 was more/less frequent than a more Z-score as large as 3.
149. Suppose you were told that your score on the last Psychology examination was approximately equal to a Z-score of two. This would imply that the distribution of test scores was approximately normal and that your score was about two standard deviations above the mean of the distribution.
150. Furthermore, since only .05 of the values in a normal distribution are farther than two standard deviations from the mean, and since these extreme values are divided equally between negative and positive deviations, you would know that only about .05/.025 of the students .025 had made a better/poorer grade on the test. better

a collection of data as if it had a normal distribution, even though any actual collection of data can/cannot have a perfect normal distribution.

cannot

Suppose you told someone that your data had an approximately normal distribution. He would know that approximately .95 of all the values in the distribution were within two standard deviations of the mean. If a particular observed value represented a **standard score** of 2, he would know that only about $\frac{.05}{.01}$ of the observed values were farther away from the mean of your data than was that particular value.

.05

143.

A **standard score** from a normal distribution is often called a Z-score. If a value from a normal distribution represented a Z-score of -2, you would know the value was two standard deviations above/below the mean.

below

144.

A value would represent a Z-score of 2 if it were from a **normal distribution** and had a positive/negative deviation from the mean equal to standard deviations.

positive

2

145.

Since a standard score is simply the number of standard deviations between a value and the mean, a Z-score is simply the of standard deviations between a value and the mean in a normal distribution.

number
normal
distribution

Turn the book around and continue with frame 146 on page 240.

151. To summarize then, a standard score (or Z-scores if the distribution is normal) are a convenient way of representing a value, since it indicates how many _____ standard deviations _____ that value is away from the mean.
152. This is particularly useful to you in the case of a **normal distribution**, since you know exactly what _____ proportion of the values in the distribution are within any particular number of standard deviations from the mean.

By imagining the proper dotted lines, you could

determine that approximately $\frac{.36}{.46}$ of all the

.36

If .36 of the values were within $1/2$ a standard deviation of the mean, the remaining values ($1 - .36 = .64$)

would be $1/2$ or more standard deviations from the mean. Since the normal distribution is **symmetrical**, these .64

of the values would be equally divided between each tail

of the distribution. Thus, there would be _____ of the

.32

values in one tail and _____ of the values in the other

.32

tail.

The normal distribution is referred to as a **theoretical**

distribution because it is defined in terms of a

mathematical equation or formula rather than in terms

of an actual collection of data. An actual collection of

data may have a distribution similar to a normal

distribution. However it can never have a distribution

that is exactly normal, in the same sense that an actual

circle can never be **perfectly round** since it would

always be possible to demonstrate some slight difference

between the actual object and the mathematically defined

perfect circle. It is often useful, however, to treat

circles as if they were perfectly round, even when, in

fact, they are not.

is not

A doughnut is/is not **exactly** round, but a person who

packaged doughnuts would probably find it convenient to

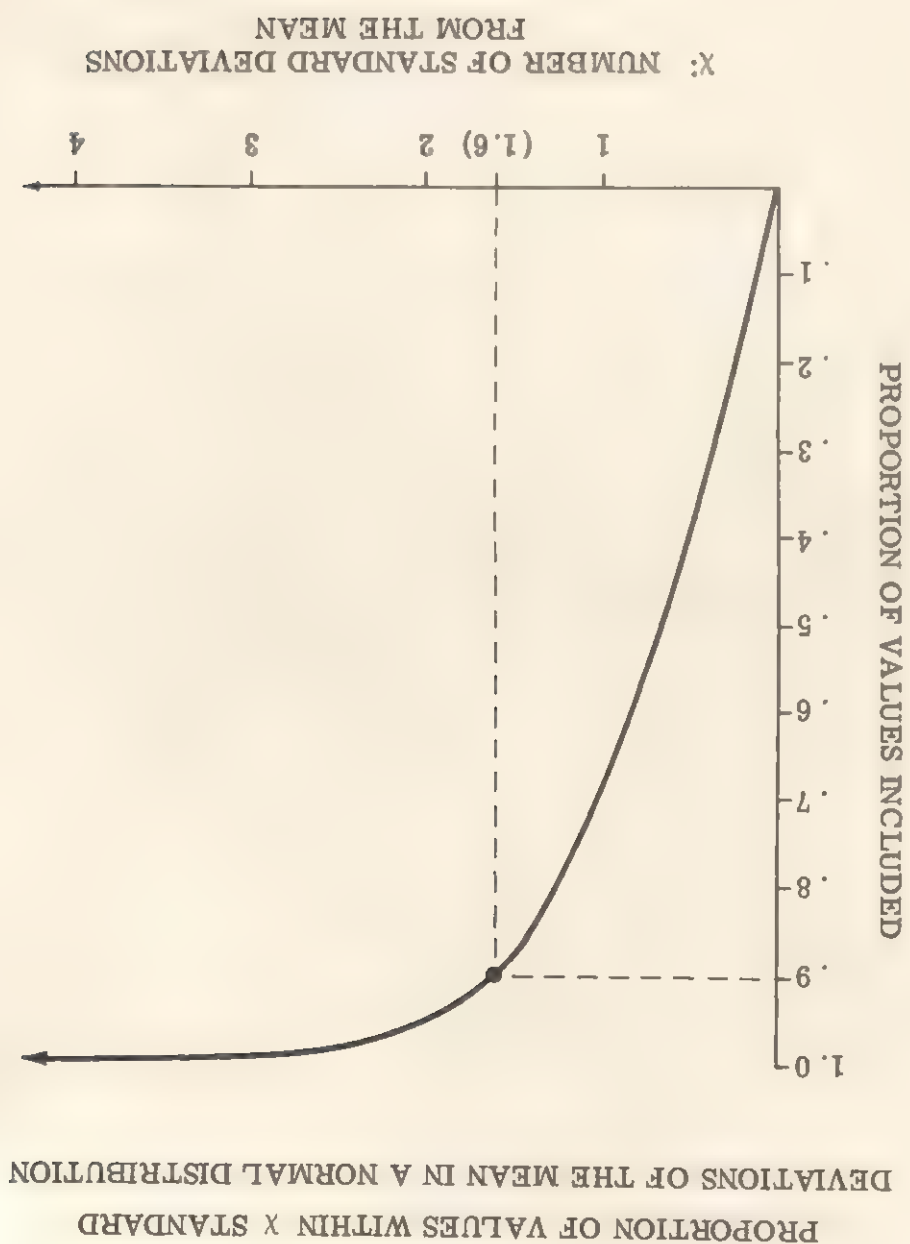
treat them as if they were round, since the difference

between their shape and a perfect circle is not important

to him. In the same sense, it is often useful to treat

Section VI: Samples and Populations

1. There are some things you can say about a collection of data even before you actually collect it. Suppose you were interested, for example, in the heights of students at a particular high school. You could determine the distribution of these heights by measuring and recording the height of each student in the high school. If there were 1, 500 students, your collection of data would consist of _____ observations of a variable called "height." 1, 500
2. Suppose you only knew the heights of ten of the 1, 500 students. Although these ten observations form a collection of data, they could also be considered as part of the larger, complete collection of data. Statisticians use the name **sample** to describe a collection of data which is viewed as part of a larger, complete collection of data. In other words, the collection of $\frac{10}{1, 500}$ heights 10 would be considered a **sample** in this illustration.
3. Statisticians refer to the **complete** collection of data (of which the sample is a part) as a **population**. Thus, the heights of the ten students would be considered a sample, whereas the heights of the 1, 500 students would be considered a _____. population



The dotted lines on the previous graph indicate that $\frac{.9}{.8}$ of the values in a normal distribution are within $\frac{1.6}{1.2}$ standard deviations of the mean.

4. Let's look at another example of the difference between a sample and a population. Suppose you wanted to know which of two candidates for some public office was most preferred by each of the 10,000 people in your city. If you asked the first 100 people you met on the street to state their preference, you would have a sample consisting of _____ observations from the complete collection of data (population) consisting of _____ observations.
- 100
10,000
5. Suppose you were interested in the yearly income of the 200 school teachers in your city. A collection of data consisting of yearly incomes of only 5 of these teachers would be a _____ if you viewed it as part of the _____ consisting of the yearly incomes of all 200 teachers.
- sample
population
6. It is important to realize that a particular collection of data could be treated as either a sample or population depending upon how you view it. In the previous illustration, for example, the complete collection of data consisting of the yearly income of each of the 200 teachers in your city was viewed as a **population**. Suppose, however, you were interested in the yearly incomes of all the teachers in your state. A **complete** collection of data would now consist of a list of yearly incomes of all the teachers in the state.

This list would now be the population, whereas the list of yearly incomes for the 200 teachers in your city now could be viewed as a _____ from this population.

sample

To find the proportion of values within some number of standard deviations from the mean, simply locate that number along the bottom of the graph. Then find a point on the curve directly above the number you found on the baseline. Finally, find the point on the left side of the graph that is the same height as the point on the curve. This procedure is illustrated by the **dotted lines** on the graph. The vertical dotted line (which goes from the bottom of the graph to the curve) indicates that the dot on the curve is directly **above** the number $\frac{1}{2}$ on the bottom of the graph.

The horizontal dotted line (which goes from the left side of the graph to the curve) indicates that the dot on the curve is at the **same height** as a proportion of $\frac{.68}{.78}$ shown on the side of the graph.

138.

The dot on the curve is directly above a particular **number of standard deviations** and at the same height as a **proportion**. This indicates that the proportion of values is within that particular number of standard deviations from the mean of a normal distribution. Thus, the point identified by the dotted lines on the previous graph indicates that the proportion of values within _____ standard deviation from the mean of a normal distribution is _____.

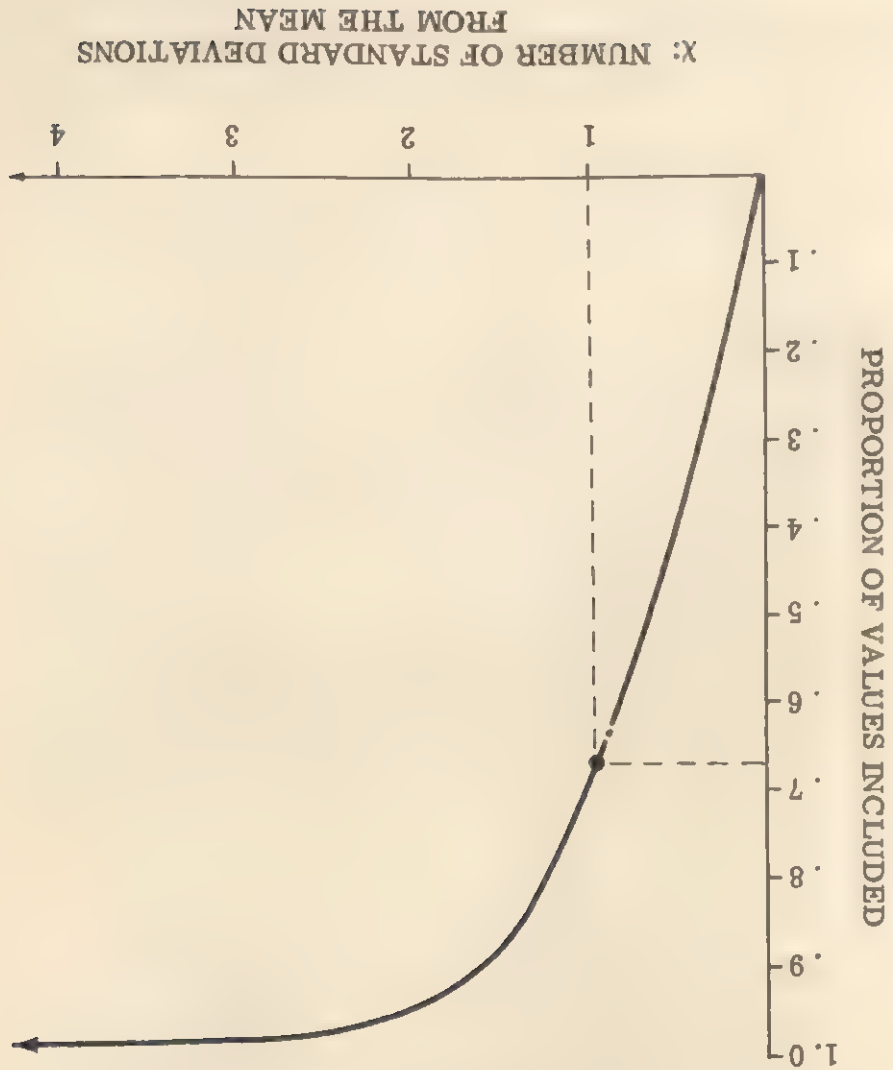
.68

one

7. Notice also that the yearly income for any 5 particular teachers in your city _____ be viewed as _____ could
could/ could not
a sample from **either** a population consisting of the incomes of all the teachers in your city, or the larger population consisting of the yearly incomes of all the teachers in your state.
8. A particular collection of data may be viewed as a _____ if you are comparing that collection _____ sample
sample/ population
to a **larger** collection of data of which it is a part, or as a _____ if you are comparing it to a _____ population
sample/ population
smaller collection of data which would be included in it.
9. To define a population, you could describe what would be included in the complete collection of data. For example, you could describe a collection of data consisting of the ages of all the Democratic presidents. You could describe another collection of data consisting of the ages of all the Republican presidents.
- Either one of these collections of data could be considered a population, but they _____ be the _____ would not
would/ would not
same population(s).
10. A particular collection of data may be a sample from more than one population. For example, a list of the ages of all the truck drivers in Detroit could be a _____ sample
s _____ from the population consisting of the ages of all the truck drivers in the United States. The same collection of data could be considered a _____ sample
from a population consisting of all the male workers in Detroit.

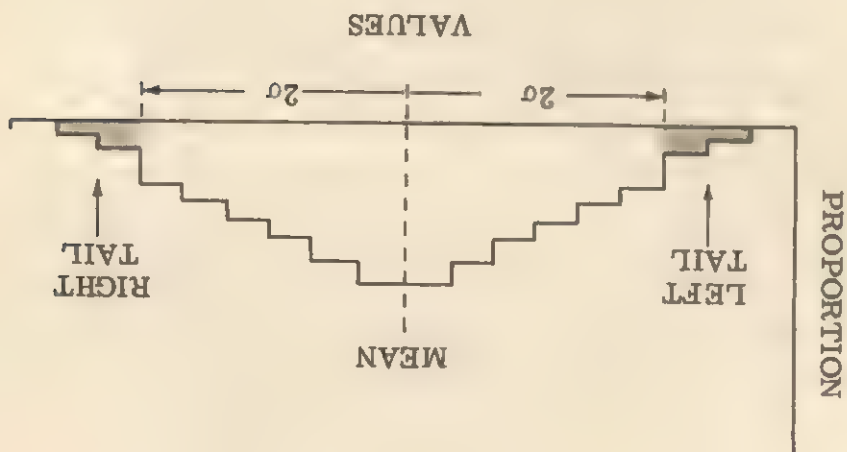
The theoretical formula or equation for a normal distribution specifies the proportion of values within any number of standard deviations from the mean. This information is summarized by the following graph.

PROPORTION OF VALUES WITHIN x STANDARD DEVIATIONS OF THE MEAN IN A NORMAL DISTRIBUTION



11. The ages of all the female school teachers in Detroit
_____ be a sample from a population _____ would not
would/ would not
consisting of the ages of all the male drivers in the
United States. However, the ages of all the female
school teachers in Detroit _____ be a sample _____ would
would/ would not
from a population consisting of the ages of all the people
employed in Detroit.
12. Up to this point, we have only considered populations as
collections of data that could actually be completely
collected. For example, we discussed a population
consisting of the heights of all the students in a
particular high school. In principle, at least, it
_____ be possible to actually collect a _____ would
would/ would not
complete list of the heights of all the students in the high
school.
13. It is sometimes useful to think of a particular collection
of data **as if** it were a sample from a larger collection
of data consisting of an **unlimited** number of observations.
An example of a population consisting of an unlimited
number of observations would be a list of the outcomes
of an unlimited number of throws of a die (one member
of a pair of dice). You could roll a die and record the
number of dots showing on the face of the die as the
"outcome" of this toss. You could then roll it again,
and again, and again, each time recording the number
of dots showing on the face of the die. In this way, you
could go on producing a list of observed values without
ever specifying where you should stop. A list of out-
comes for any specific group of tosses could be viewed
as a _____ from a longer, unlimited list of _____ sample
outcomes, since this smaller collection of data would
be part of the larger, **unlimited** collection.

This is illustrated in the following graph:



The distribution in this graph is **similar**, but not identical, to a normal distribution. We have shaded the columns representing values of 2 standard deviations or more from the mean. Since the distribution is **symmetrical** and since exactly .05 of all the values are 2 σ 's or more from the mean, the proportion represented by the shaded area in the left tail is _____ and the proportion in the other tail is _____.

.025
.025

14. If a population consists of a specific number of observations, it is called a **finite** population. The hair color of every person in the United States is a finite population since, in principle at least, you could actually list the hair color of every person in the United States.

Similarly, since there are a limited number of people in the world at any one time, a list of the heights of everyone in the world would consist of a number of observations and would therefore be a **finite** population.

limited

15. A population viewed as consisting of an **unlimited** number of observations is referred to as an **infinite** population. A population consisting of the yearly income of each person in a particular city be an **infinite** population, since the number of people in the city is limited (in principle, you could actually count them all).

would not

16. Suppose you shuffled a deck of playing cards, dealt out five cards, and counted the number of red cards among the five. You could consider this number to be a single observation of a variable whose 6 possible are: 0, 1, 2, 3, 4, and 5. You could shuffle the cards again, deal out five more cards, and count the number of red cards in this new group of five. You could continue this process **indefinitely**. (The procedure for generating this list of observations does not specify any **end** to the list.) In other words, the possible number of observations would be . Any specific

values

unlimited

133. The previous table indicates the additional values that are included as you move away from the mean in steps of _____ a standard deviation.

1/2

134. By using the formula for the **normal distribution**, it is possible to construct a table of this sort with "steps" as small as you like. This kind of table is useful because you will find that many of the distributions you encounter are very similar to the theoretical shape of a normal distribution. For example, if you knew a distribution was similar to a normal distribution, you would know that approximately $\frac{.68}{.95}$ of all the values were within one standard deviation.

.68

135. An important feature of a **normal distribution** is the fact that it is **symmetrical**. Therefore, the .68 of the values within one standard deviation of the mean are equally divided on either side of the mean. In other words, .34 of the values in the normal distribution have a positive deviation of less than one σ , whereas another _____ of the values have a negative deviation less than one σ .

.34

136. There are .05 of the values in the distribution which are two or more standard deviations away from the mean. These values are equally distributed in either "tail" of the distribution — .025 in one tail and .025 in the other.

16. (Continued)

group of observations, therefore, could be considered to be a **sample** from a(n) finite/infinite population consisting of the unlimited number of observations.

infinite

17. We have seen that every collection of data describes a distribution. The distribution can either be described in absolute terms, by listing the frequency with which each value occurred in the data, or it can be described in relative terms, by listing the p of times each value occurred in the data.

proportion

18. A distribution can be described by statistics such as the mean, the median, and the mode, all of which characterize its c t ; or a distribution can be described by statistics such as the range and the variance, which describe its v .

central tendency

variability

19. While you may be interested in the distribution of a particular collection of data, for one reason or another you will often have only **part** of that complete collection. In other words, although you are really interested in a , you may have only a population/ sample .

population

sample

129. The proportion of values within two σ 's of the mean is _____, or _____ percent, since .95 equals 95/100.

130. Only _____ of the observed values are 3 standard deviations or more from the mean. (Again, the proportion .01 equals _____ %).

131. Since .99 of the values are less than 3 σ 's from the mean, only 1 - _____, or .01, of the values are 3 or more standard deviations from the mean.

132. The normal distribution is an example of a **theoretical distribution** since its shape is defined by a mathematical equation and not simply by an actual distribution of data. The equation defining the normal distribution specifies the proportions of values within **any** number of standard deviations from the mean. Thus, the previous table could be expanded to give a more detailed description of a normal distribution.

Number of σ 's from the Mean	Proportion of Values
Less than 1/2	.38
Less than 1	.68
Less than 1 1/2	.87
Less than 2	.95
Less than 2 1/2	.98
Less than 3	.99

Notice that each proportion could be converted to a percentage by simply dropping the decimal point, since $.38 = \frac{38}{100} = 38\%$ and $.68 = \frac{68}{100} = \frac{\quad}{\quad}\%$, and so on.

20. Suppose, for example, you were interested in the number of people in the United States with a college education. It would probably be too difficult, too time-consuming, and too expensive to actually list whether or not each person in the United States held a college degree. Suppose, however, you stopped twenty people on a street corner and asked them if they held a college degree. The data you collected would be only **part** of the data in which you were interested. Therefore, the twenty observations would be a _____ from the _____ consisting of the education of all the people in the United States.

sample
population

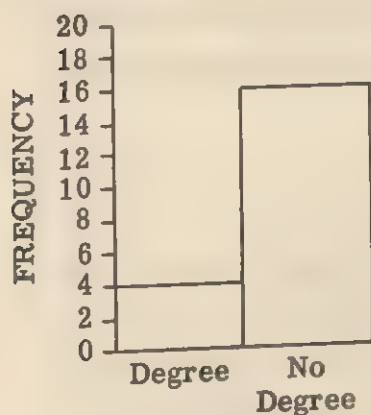
21. Since there is a **specific number** of people in the United States, your list of twenty observations would be a sample from a _____ population.

finite

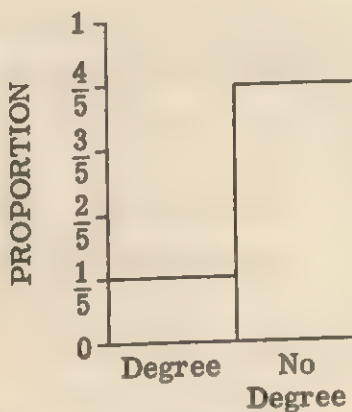
22. Suppose you found that four of the twenty people you had stopped on the street did hold college degrees. You could describe the distribution of your sample by the **absolute** frequency distribution shown in Graph A/B, or by the **relative** frequency distribution shown in Graph A/B.

A

B



GRAPH A



GRAPH B

This table indicates how many more observations are included as you proceed in $1/2$ standard deviation steps away from the mean. Thus, the first row indicates $4/10$ of the values were within _____ a standard deviation of the mean.

The second row indicates _____ tenths of the observed values were within _____ standard deviation of the mean.

No additional values were included when the limit was extended to include values less than $1\frac{1}{2}$ standard deviations from the mean. However, all of the values are within _____ standard deviations of the mean.

128. Statisticians often describe the **shape** of a distribution by indicating the proportion of values within different numbers of standard deviations of the mean. For example, a very important type of distribution, **normal distribution**, can be described in this manner.

Number of Standard Deviations from the Mean	Proportion of Observations
Less than one	.68
Less than two	.95
Less than three	.99

Notice that .68 of all the values in this type of distribution are within _____ standard deviation of the mean. (The proportions in the table above have been rounded off to the nearest hundredth).

23. If you felt that the sample of 20 people was **typical** or **representative** of the whole population, you might guess that the proportion of people in the United States with college degrees was about _____.

$$\frac{1}{5} / \frac{4}{5}$$

$$\frac{1}{5}$$

24. In other words, while you don't actually have a complete collection of data, you might feel that the **sample** is **similar** to the _____.

population

25. A guess at some statistic describing the population on the basis of a sample from that population is called a **statistical inference**. You made a _____ about the proportion of college graduates in the United States on the basis of a sample of 20 people.

statistical
inference

26. A **statistic** describing a **population** is often called a **population parameter**. For example, if you had a complete collection of data listing whether or not each person in the United States had a college degree, you could calculate the true proportion of people with college degrees in this population. Since the proportion would be a statistic describing a population, it would be called a population p _____.

parameter

27. Any statistic describing a sample is called a **sample statistic**. Thus, the **proportion** of the people with degrees in your **sample** would be a _____.

sample statistic / population parameter

sample statistic

One way of **describing** the previous distribution would be to list the **proportion** of values within different distances from the mean. Consider, for example, the following table:

Number of Standard Deviations from the Mean	Proportion of Observed Values	
	Less than 1	Less than 2
	8/10	10/10

The first row indicates that 8/10 of the observed values were closer to the mean than one standard deviation.

The second row indicates that _____ of the observed values were within two standard deviations from the mean.

We _____ have described the collection of ten

observations shown in Frame 116 in this manner.

The data consisted of ten observed values of which only two values were more than one σ away from the mean.

Those values were _____ and _____.

0, 20

You could describe the same data in more detail if you expanded the previous table as follows:

Number of Standard Deviations from the Mean	Proportion of Observed Values			
	Less than 1/2	Less than 1	Less than 1 1/2	Less than 2
	4/10	8/10	8/10	10/10

120. X_4 is below the mean. Therefore, even though it is the same distance from the mean as is X_2 , it has a standard score of _____, instead of 1.82.

-1.82

121. Both the observed value of 20 and the observed value of _____ have the same **absolute deviation** from the mean, even though one deviation is positive and the other is negative.

0

122. The only two observed values whose **absolute deviation** from the mean was **greater than** one standard deviation are _____ and _____.

20, 0

123. Since there were only _____ observed values farther away from the mean than one standard deviation, you would say that $\frac{2}{8} / \frac{10}{10}$ of all ten observations had values **less than** one standard deviation from the mean.

two

 $\frac{8}{10}$

124. Any standard score **greater than +1 or less than -1** would represent a value more than _____ standard deviation away from the mean.

one

125. Out of the ten observed values in the previous table, there were _____ **within** (less than) $\frac{1}{2}$ a standard deviation from the mean.

4

There were _____ values within one σ of the mean.

8

31. The difference between the largest and the smallest shoe size in the population of all potential customers is a statistic that describes the population/sample and population would, therefore, be called a population statistic.
32. The variance of your **sample** would be a sample statistic/population parameter sample statistic
33. If you had a complete collection of data concerning the shoe sizes of all potential customers, you could actually calculate the true variance of the population. This variance would be a population statistic or parameter.
34. Suppose you guessed that the population (parametric) mean was the same as the mean of your sample. You would be using a population/sample statistic as an sample estimate of a population/sample statistic. population
35. You would be making an **inference** about the population on the basis of a sample. The difference between your **estimate** of the population mean and the **true** population mean would be your **error of estimate**.

In other words, if the true mean were 10 and your estimate were 9, the **error of estimate** would be $10 - 9$, or 1 .

9, 1

116.

The following table contains the collection of data we considered previously. The fourth column contains the standard scores corresponding to these values.

In other words, the last entry in each row indicates the number of _____ between the observed value in that row and the mean. In this table,

$$\bar{x} = 10 \text{ and } \sigma = 5.5.$$

OBSERVATION	VALUE	$X - \bar{x}$	STANDARD SCORE
1	15	5	.91
2	20	10	1.82
3	10	0	0
4	0	-10	-1.82
5	15	5	.91
6	10	0	0
7	5	-5	-.91
8	10	0	0
9	5	-5	-.91
10	10	0	0

117.

Notice that observation one (X_1) corresponds to a standard score of .91. This means that 15 was slightly less than one standard deviation from the mean (where $\bar{x} = 10$ and $\sigma = 5.5$).

118.

Observation two (X_2) corresponds to a standard score of _____, which means that the deviation of X_2 from \bar{x} equals _____ standard deviations.

1.82
1.82

119.

X_3 corresponds to a standard score of _____, since the deviation of observation _____ from the mean was _____ standard deviations.

0
three
0

36. Consider the two collections of data shown below:

Observation	Value
1	10
2	40
3	20
4	25
5	20
6	20
7	40
8	20
9	15
10	25

TABLE A

Observation	Value
1	25
2	10
3	25

TABLE B

- Suppose the data in Table A represented a **population** and the data in Table B represented a **sample** from that population. The mode of the population is _____, 20
 whereas the mode of the sample is _____. 25
37. If you used the sample mode as an estimate of the population mode, your **error of estimate** would equal 20 - _____, or _____. 25, -5
38. If the sample were **typical**, or **representative**, of the population, the difference between a sample statistic and the population statistic would be small. In other words, your e_____ of e_____ would error, estimate be small if you used this sample statistic as your estimate of the population statistic.
39. However, the sample statistic might be quite different from the population statistic and the error of estimate would then be large/small. large

112.

Another way of saying that the value 21 is two standard deviations **above** the mean would be to say that the value 21 represents a _____ of $\frac{2}{-2}$.

113.

If the $\bar{x} = 10$ and $\sigma = 5.5$, a **standard score** of -1 would correspond to an observed value of _____.

4.5

114.

A formula for the standard score of any distribution can be written as follows:

$$\text{Standard score} = \frac{X - \bar{x}}{\sigma}$$

According to this formula, you simply divide the deviation of the value by the standard deviation. Your answer is the number of standard _____ between the observed value and the mean.

deviations

Therefore, the number you obtain by dividing $(X - \bar{x})$ by _____ score.

standard

115.

In the example you considered earlier, you saw that the value 21 would represent a **standard score** of 2 in a distribution where $\bar{x} = 10$ and $\sigma = 5.5$.

This is consistent with the formula, since

$$\frac{X - \bar{x}}{\sigma} = \frac{21 - 10}{5.5} = \frac{11}{5.5} = \underline{\hspace{2cm}}$$

2

40. Some samples you obtained could be quite representative of the population because the sample statistic was very similar to the population statistic. However, you could also obtain samples which gave you a poor or distorted picture of the population. It would be useful to know how often different values of the sample statistic would occur if you collected one sample after another. Suppose there were 1000 students at the high school and you knew the height of five particular students. The collection of 1000 heights would be your sample/population, whereas the collection of five heights would be a sample/population.

population
sample

41. Suppose you had twenty such samples from this population, where each sample was a collection of the heights of five students. If you calculated the mean of each of these samples, the list of these twenty means would be a collection of twenty sample/population statistics.

sample

42. The frequencies with which each possible value of the sample mean occurred in this collection of twenty sample statistics define a distribution of sample means. The distribution of these twenty sample means is called a **sampling distribution**, since it is a distribution of sample statistics.

Instead of calculating the mean of each of the twenty samples, you could calculate the variance of each sample. The list of twenty sample variances would also be a collection of twenty sample statistics. Therefore, the **distribution** of these twenty sample variances would be a sampling distribution.

sample
sampling

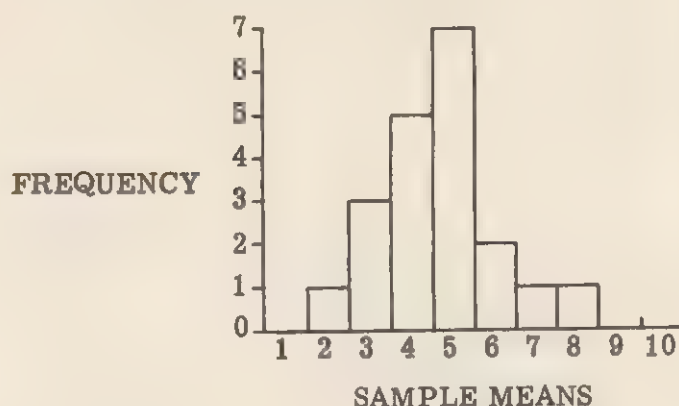
107. The deviation of each observed value from the _____ is shown in the third column of the table.
- Naturally, according to the definition of the mean, the total of all the entries in the third column of the table is _____.
108. The _____ of each of the deviations from the mean is shown in the fourth column of the table.
109. To find the _____ of the previous data, you would add all the squared deviations from the mean and divide this total by the number of observations.
110. The sum of all the squared deviations from the mean in the previous data is simply the total of column 4. Thus, $\sum (X - \bar{x})^2$ is equal to $\frac{300}{400}$.
- 300
- According to the following formula, the variance of this data is equal to _____ divided by _____.
- $$\sigma^2 = \frac{\sum (X - \bar{x})^2}{N}$$
- Thus, the variance (σ^2) of the previous data is simply 300 divided by 10, which indicates that the average squared deviation from the mean is _____.
- 30
111. The _____ of the previous data is equal to $\sqrt{30}$, or approximately 5.5.
- A value of 21 would be exactly _____ standard deviations away from the mean, if $\sigma = 5.5$ and $\bar{x} = 10$.
- two
- standard deviation

43. Similarly, you could calculate the median, the mode, the range, or the standard error (or any other sample statistic) of these twenty samples. In each case, the distribution of the resulting twenty sample statistics would be called a s _____ distribution.

sampling

44. If you collected several samples from some population and calculated the mean of each sample, the following graph might be the frequency distribution of these sample means and would therefore be a _____ distribution of sample means.

sampling



According to this graph, there were $\frac{10}{20}$ samples, since you calculated a mean for each sample.

20

45. The largest (not the most frequent) sample mean was _____ and the smallest sample mean was _____.

8, 2

46. The most frequently occurring sample mean was _____, since _____ of the twenty samples had that mean. Therefore, the modal value of the sample mean was _____.

5

7

5

102. Converting values to **standard scores** is useful when you wish to indicate how many _____ each value is from the mean.

standard deviations

103. In distributions that are "bell shaped," standard scores of 2 or -2 are _____ frequent than are standard scores of 1 or -1.

less

104. Deviations of two standard deviations (standard scores of 2 or -2) are less frequent than deviations of one standard deviation, since most of the values are clustered around the mean in a "_____ bell-shaped distribution."

105. Suppose you collected the data shown in the following table:

OBSERVATION	VALUE	$X - \bar{X}$	$(X - \bar{X})^2$
1	15	5	25
2	20	10	100
3	10	0	0
4	0	-10	100
5	15	5	25
6	10	0	0
7	5	-5	25
8	10	0	0
9	5	-5	25
10	10	0	0

Since the total of all observed values is _____, you could divide this by _____ to find the mean. Therefore, \bar{X} is equal to _____.

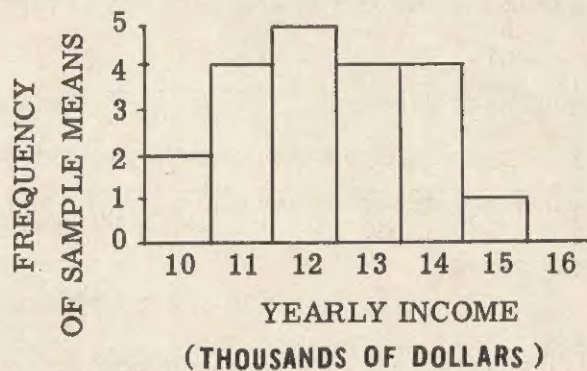
100
10
10

106. The deviation of observation 1 from the mean is simply $X_1 - \bar{X}$ or, in this case, _____, which equals _____.

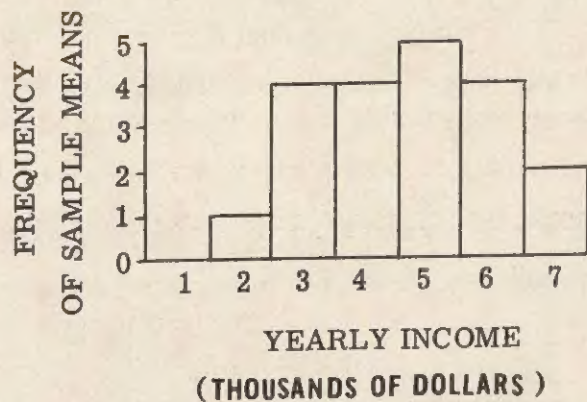
15, 10
5

47. Suppose you were interested in a population consisting of the yearly incomes of all of the people in a particular city. If you determined the yearly income of ten people you stopped on the street in that city, the resulting list of ten incomes would be considered a _____ from that population. sample

48. Imagine you collected twenty samples, where each sample consisted of a list of the yearly incomes (to the nearest thousand dollars) of ten people. Furthermore, imagine you collected these twenty samples by stopping people in the lobby of the most expensive hotel in town. Suppose you collected **another** 20 samples on a street corner in the poorest section of town. The graphs shown below could indicate the distribution of sample means in each group of twenty samples.



GRAPH A



GRAPH B

97.

A **standard score** is simply the number of standard deviations between a particular value and the mean.

If the value is greater than the mean, the standard score is $\frac{\text{positive/negative}}{\text{positive/negative}}$. If a value is less than the mean,

the standard score is $\frac{\text{positive/negative}}{\text{positive/negative}}$.

A value identical to the mean has a deviation of 0. Such a value, therefore, is equivalent to a **standard score**

of _____.

98.

If a value represented a standard score of -2, you would know it was 2 standard deviations $\frac{\text{above/below}}{\text{the mean}}$.

99.

If $\bar{x} = 10$ and $\sigma = 2$, the value _____ would represent a standard score of 2.

14

100.

The value $\frac{6/14}{6/14}$ would represent a standard score of -2

6

in a distribution with $\bar{x} = 10$ and $\sigma = 2$, since _____ minus 10 equals -4 and -4 is equal in absolute size to

6

two standard deviations.

101.

Standard scores of -1 or 1 would **both** represent values which deviated from the mean by one standard deviation. A standard score of $\frac{-1/1}{-1/1}$ would represent a value one

-1

standard deviation below the mean, whereas the other standard score would represent a value one standard

deviation above the mean.



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